SPARSE ARRAY RECEIVER BEAMFORMER DESIGN FOR MULTI-FUNCTIONAL ANTENNA

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ABSTRACT

Planning sensor locations can potentially economize the receiver cost by minimizing the hardware and computational needs, while satisfying a predetermined design criterion. In this paper, we consider a sparse array receive beamformer design approach for Multi-Functional Antennas (MFA), operating in different frequency bands. In this approach, antenna positions are selected from uniformly spaced locations that are served by a limited number of transceiver chains. The design objective is to maximize the beamformer output Signal-to-Interference-plus-noise-ratio (MaxSINR) for desired sources with disjoint frequency bands operating in a wideband jamming environment. The problem is solved efficiently through alternating direction method of multipliers (ADMM) and simplified to parallel quadratically constraint quadratic programs (QCQP) with a single associated constraint. The re-weighted group sparsity is adopted to ensure a common sparse configuration across all frequency bands. The efficacy of the proposed algorithm is demonstrated with the help of a design example.

***Index Terms*—** Sparse arrays, QCQP, ADMM

I. INTRODUCTION

Multi-functional antennas find applications in various modern wireless radios, like Cognitive Radio (CR) and Software Defined Radio (SDR) as well as radar and communications multiple-input multiple-output (MIMO) technology [1], [2]. Recent design techniques have enabled deriving multiple antenna functionalities from a single antenna. The emergence and developments of such antenna technology begs the question of how to best utilize multifunctional antennas in optimum sparse array design with the emphasis that a multifunction antenna can be housed in a single antenna location.

Non-uniform linear arrays can efficiently use hardware resources for improved system performance. This is made possible by the additional degrees of freedom (DOF) that are provided by incorporating sensor locations as part of the design objective. Depending on the application and the performance criteria, array design can be either environment-independent or strongly guided by the operating environment. Sparse array design that pursues filled co-arrays to enable direction of arrival (DOA) estimation for more sources than physical sensors is an environment-independent design [3], [4]. On the other hand, environment-sensitive design criteria, such as MaxSINR, improves the beamformer performance for the underlying constellation of sources or interferences in the array field of view (FOV) [5], [6], [7]. The environment-sensitive design objectives have recently become more realizable due to advances in efficient sensor switching technologies that readily activate a subset of sensors on a predefined grid points [8], [9]. Hence, the system cost can significantly be reduced by limiting the number of expensive transceivers chains.

In this paper, we consider sparse array design for MaxSINR receive beamforming. Specifically, we pursue multi-functional modality equipped with multiband antennas that concurrently handle a variety of radio services [1], [2]. This is in contrast to the MaxSINR subarray design for multitask receivers, discussed in [10], where the subarrays share aperture and each has its own antenna type, with different subarrays serving different functions. Sparse array design for multiband antennas translates to a common sparse array configuration across all bands while lending separate beamforming weights. Designing multiple sets of beamformers using a common sparse array configuration also arises while assigning a separate beams to service different sources in the FOV or processing wideband sources jointly in the space-time domain [11], [12].

The sparse antenna selection is considered for selecting K sensor locations out of N uniformly spaced perspective sensor locations, where each antenna can process M different subbands. Achieving MaxSINR design for K antenna selection amounts to maximizing the principal eigenvalue of the product of the inverse of data correlation matrix and the desired source correlation matrix over all possible sparse combinations and is clearly an NP hard optimization problem. Our approach for MaxSINR design directly operates on the received data and requires the knowledge of DOA of the desired source. It does not assume explicit information regarding the locations or powers of the interfering signals. We initially formulate the problem as QCQP with re-weighted mixed $l_{1,2}$-norm to promote group sparsity in seeking common sparse configuration. The problem is decoupled into two sets of variables, with separable objectives. One set of variables deals with M QCQPs, each with a single constraint that can be efficiently solved [13]. The unconstrained minimization of the re-weighted mixed $l_{1,2}$-norm is carried out over the other set of variables. The convergence of the proposed approach is guaranteed by the ADMM algorithm [14].

The rest of the paper is organized as follows: In Section II, we state the problem formulation for maximizing the beamformer output SINR. Section III deals with the sparse array design by iterative ADMM methodology for designing K-sensor sparse array for multiband antenna system. In section IV, we demonstrate the usefulness of MaxSINR by a design example followed by concluding remarks.

II. PROBLEM FORMULATION

Assume the multifunction antennas have M narrow bands centered at $\omega_i (i \in 1, 2, ..., M)$. Consider a desired source operating in the ith band, i.e., at frequency ω_i, in the presence of P_i interfering sources. The signals impinge on an N uniformly spaced sensors locations. The baseband signal received at the array is given by;

$$\mathbf{x}_{\omega_i} = \alpha_i \mathbf{s}(\theta_{s_i}, \omega_i) + \sum_{p_i=1}^{P_i} \beta_{p_i} \mathbf{i}(\theta_{p_i}, \omega_i) + \mathbf{v}_i, \qquad (1)$$

where, $(\alpha_i,\ \beta_{p_i}) \in \mathbb{C}$ are the complex amplitudes of the incoming baseband signals, $\mathbf{s}(\theta_{s_i}, \omega_i)$ and $\mathbf{i}(\theta_{p_i}, \omega_i) \in \mathbb{C}^N$ are the frequency dependent steering vectors with respect to DOAs of the desired source θ_{s_i} and the interfering sources θ_{p_i} active in the ω_i frequency band;

$$\mathbf{s}(\theta_{s_i}, \omega_i) = [1 \;\; e^{j\frac{2\pi}{\lambda_{\omega_i}} d_{\omega_m} cos\theta_{s_i}} \ldots e^{j\frac{2\pi}{\lambda_{\omega_i}} d_{\omega_m}(N-1) cos\theta_{s_i}}]^T \qquad (2)$$

To avoid spatial aliasing, the inter-element spacing d_{ω_m} is designed at $\frac{\lambda_{\omega_m}}{2}$, corresponding to the highest operating frequency ω_m, simplifying (2) as follows,

$$\mathbf{s}(\theta_{s_i}, \omega_i) = [1 \;\; e^{j\pi \frac{\omega_i}{\omega_m} cos\theta_{s_i}} \ldots \quad e^{j\pi \frac{\omega_i}{\omega_m}(N-1) cos\theta_{s_i}}]^T \quad (3)$$

The variance of additive Gaussian noise $\mathbf{v}_i \in \mathbb{C}^N$ is $\sigma_{v_i}^2$ at the receiver output. The received signal vector $\mathbf{x}_{\omega_i}$ is linearly combined at the receiver to maximize the output SINR. The output signal y_{ω_i} of the optimum beamformer for MaxSINR is given by [15],

$$y_{\omega_i} = \mathbf{w}_{o_i}^H \mathbf{x}_{\omega_i}, \quad \forall i = 1, 2, ..., M. \qquad (4)$$

where, $\mathbf{w}_{o_i}$ are given by the following optimization problem;

$$\begin{aligned} \underset{\mathbf{w}_1, \mathbf{w}_2, \ldots \mathbf{w}_M \in \mathbb{C}^N}{\text{minimize}} \quad & \sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{R}_{n_i} \mathbf{w}_i, \\ \text{s.t.} \quad & \mathbf{w}_i^H \mathbf{R}_{s_i} \mathbf{w}_i = 1, \quad \forall i \in \ 1, \ldots M. \end{aligned} \qquad (5)$$

The desired source correlation matrix $\mathbf{R}_{s_i} = \sigma_i^2 \mathbf{s}(\theta_{s_i}, \omega_i)$ $\mathbf{s}^H(\theta_{s_i}, \omega_i)$, where $\sigma_i^2 = E\{\alpha_i \alpha_i^H\}$ is the average source power in the ith frequency. Likewise, we define the interference and noise correlation matrix $\mathbf{R}_{n_i} = \sum_{p_i=1}^{P_i} (\sigma_{p_i}^2 \mathbf{i}(\theta_{p_i}, \omega_i) \mathbf{i}^H(\theta_{p_i}, \omega_i)) + \sigma_{v_i}^2 \mathbf{I}$. Although, the M sources are served with M different beams, all beams emanate from the receiver array with one configuration. This configuration is implicit in the correlation matrices in (5). Equation (5) can equivalently be written by replacing $\mathbf{R}_{n_i}$ by the data covariance matrix, $\mathbf{R}_{\mathbf{x}_i} = \mathbf{R}_{s_i} + \mathbf{R}_{n_i}$ as follows [15],

$$\begin{aligned} \underset{\mathbf{w}_1, \mathbf{w}_2, \ldots \mathbf{w}_M \in \mathbb{C}^N}{\text{minimize}} \quad & \sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{R}_{\mathbf{x}_i} \mathbf{w}_i, \\ \text{s.t.} \quad & \mathbf{w}_i^H \mathbf{R}_{s_i} \mathbf{w}_i = 1, \quad \forall i \in \ 1, \ldots M. \end{aligned} \qquad (6)$$

The formulation in (6) is more practical to handle as, $\mathbf{R}_{\mathbf{x}_i} \approx \frac{1}{T} \mathbf{x}_{\omega_i} \mathbf{x}_{\omega_i}^H$, is readily estimated from the T received data snapshots. The solution of the optimum weights in (6) is given by $\mathbf{w}_{o_i} = \mathscr{P}\{\mathbf{R}_{\mathbf{x}_i}^{-1} \mathbf{R}_{s_i}\}$, which depends on the array configuration to generate both the correlation matrices. The operator $\mathscr{P}\{.\}$ denotes the principal eigenvector of the input matrix. The optimum output SINR_{o_i} associated with the band ω_i is then given by substituting $\mathbf{w}_{o_i}$ into (4) yielding;

$$\text{SINR}_{o_i} = \Lambda_{max}\{\mathbf{R}_{n_i}^{-1} \mathbf{R}_{s_i}\}. \qquad (7)$$

The optimum output SINR_{o_i} is the maximum eigenvalue (Λ_{max}) of the product of the two matrices, the inverse of interference plus noise correlation matrix and the desired source correlation matrix. The resulting solution, maximizes the average $\text{SINR}_o = \frac{1}{M} \sum_{i=1}^{M} \text{SINR}_{o_i}$, over all frequency bands. With multifunctional antennas, one single array configuration can serve all functions at all frequencies. This is the case whether the array is uniform or sparse. For the latter, MaxSINR beamformer performance is intrinsically tied to the array configuration.

III. SPARSE ARRAY DESIGN

Sparse array for MaxSINR beamformer design is combinatorial optimization problem and, therefore, cannot be solved in polynomial time [16]. We resort to convex relaxation in order to solve the sparse sensor selection rather efficiently. Proceeding further, we assume that an estimate of the correlation matrix corresponding to a full sensor array is available. For a fewer active sensors at any given time, the aforementioned assumption is practically realized by adopting the hybrid sparse design approach or performing matrix completion prior to carrying out the sparse optimization [17], [18].

In order to recover a common sparse configuration over all frequencies, we introduce an additional regularization term in (6) to invoke the group sparsity in the designed beamformer,

$$\begin{aligned} \underset{\mathbf{w}_1, \mathbf{w}_2, \ldots \mathbf{w}_M \in \mathbb{C}^N}{\text{minimize}} \quad & \sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{R}_{\mathbf{x}_i} \mathbf{w}_i + (\sum_{k=1}^{N} \mathbf{f}(k) ||\mathbf{w}^{(k)}||_2), \\ \text{s.t.} \quad & \mathbf{w}_i^H \mathbf{R}_{s_i} \mathbf{w}_i = 1, \quad \forall i \in \ 1, \ldots M. \end{aligned} \qquad (8)$$

The vector $\mathbf{w}^{(k)} \in \mathbb{C}^M$ represents the beamformer weights for all frequencies corresponding to the kth sensor. The l_2-norm ($||.||_2$) alongside with the summation in the regularization term implements the mixed $l_{1,2}$ norm, which is a well known group sparsity encouraging regularizer [19]. The kth element $\mathbf{f}(k)$ of the re-weighting vector $\mathbf{f}$ is iteratively updated to further promote sparse solutions [20]. The beamforming weight vectors are generally complex valued, however, the problem is transformable to real domain by substituting the correlation matrices $\mathbf{R}_{s_i}$, $\mathbf{R}_{n_i}$ by $\tilde{\mathbf{R}}_{s_i}$, $\tilde{\mathbf{R}}_{n_i}$, and concatenating the real and imaginary (img) parts of the respective beamforming weight vectors,

$$\tilde{\mathbf{R}}_{s_i} = \begin{bmatrix} \text{real}(\mathbf{R}_{s_i}) & -\text{img}(\mathbf{R}_{s_i}) \\ \text{img}(\mathbf{R}_{s_i}) & \text{real}(\mathbf{R}_{s_i}) \end{bmatrix}, \tilde{\mathbf{w}}_i = \begin{bmatrix} \text{real}(\mathbf{w}_i) \\ \text{img}(\mathbf{w}_i) \end{bmatrix} \quad (9)$$

The real domain problem has twice the dimensionality of the original complex valued variables, as is $\tilde{\mathbf{w}}^{(k)} \in \mathbb{R}^{2M}$, because the deselected sensor implies the simultaneous removal of the real and imaginary entries of the corresponding beamformer weights [21],

$$\begin{aligned} \underset{\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2, \ldots \tilde{\mathbf{w}}_M \in \mathbb{R}^{2N}}{\text{minimize}} \quad & \sum_{i=1}^{M} \tilde{\mathbf{w}}_i^T \tilde{\mathbf{R}}_{\mathbf{x}_i} \tilde{\mathbf{w}}_i + \sum_{k=1}^{N} \mathbf{f}(k) ||\tilde{\mathbf{w}}^{(k)}||_2, \\ \text{s.t.} \quad & \tilde{\mathbf{w}}_i^T \tilde{\mathbf{R}}_{s_i} \tilde{\mathbf{w}}_i = 1, \quad \forall i \in \ 1, \ldots M. \quad (10) \end{aligned}$$

The problem is transformed to ADMM form. Duplicating the variables $\tilde{\mathbf{w}}_i$ by introducing auxiliary variables $\mathbf{z}_i$, and adding an equality constraint among the variables, keeping the above problem invariant as follows (^{T} denotes transpose) [14],

$$\begin{aligned} \underset{\{\tilde{\mathbf{w}}_i, \mathbf{z}_i \in \mathbb{R}^{2N}\}_{i=1}^{M}}{\text{minimize}} \quad & \textstyle\sum_{i=1}^{M} \tilde{\mathbf{w}}_i^T \tilde{\mathbf{R}}_{\mathbf{x}_i} \tilde{\mathbf{w}}_i + \sum_{k=1}^{N} \mathbf{f}(k) ||\mathbf{z}^{(k)}||_2, \\ \text{s.t.} \quad & \tilde{\mathbf{w}}_i^T \tilde{\mathbf{R}}_{s_i} \tilde{\mathbf{w}}_i = 1, \quad \forall i \in \ 1, \ldots M. \\ & \tilde{\mathbf{w}}_i = \mathbf{z}_i, \quad \forall i \in \ 1, \ldots M. \quad (11) \end{aligned}$$

Similarly to $\tilde{\mathbf{w}}^{(k)}$, the vector $\mathbf{z}^{(k)}$ now equivalently represents the beamformer weights for all frequencies corresponding to the kth sensor. The iterative ADMM implementation of the above problem is expressed as follows,

$$\begin{aligned} \tilde{\mathbf{w}}_i \quad & \longleftarrow \underset{\tilde{\mathbf{w}}_i \in \mathbb{R}^{2N}}{\text{minimize}} \quad \tilde{\mathbf{w}}_i^T \tilde{\mathbf{R}}_{\mathbf{x}_i} \tilde{\mathbf{w}}_i + \rho ||\tilde{\mathbf{w}}_i - \mathbf{z}_i + \mathbf{u}_i||_2^2, \\ & \qquad \text{s.t.} \qquad \tilde{\mathbf{w}}_i^T \tilde{\mathbf{R}}_{s_i} \tilde{\mathbf{w}}_i = 1 \\ \mathbf{z}_i \quad & \longleftarrow \underset{\mathbf{z}_i \in \mathbb{R}^{2N}}{\text{minimize}} \quad (\sum_{k=1}^{N} \mathbf{f}(k) ||\mathbf{z}^{(k)}||_2) + \rho ||\tilde{\mathbf{w}}_i - \mathbf{z}_i + \mathbf{u}_i||_2^2 \\ \mathbf{u}_i \quad & \longleftarrow \mathbf{u}_i + \tilde{\mathbf{w}}_i - \quad \mathbf{z}_i, \quad (12) \end{aligned}$$

The M updates of $\tilde{\mathbf{w}}_i$ are clearly decoupled because the beamformer weights in the original problem are only coupled with the group sparsity regularization term. This is evident for the $\mathbf{z}_i$ updates which are coupled through the $\mathbf{z}^{(k)}$ term. By expanding the regularization term ($\rho > 0$ and $\mathbf{u}_i$ is the dual variable against $\tilde{\mathbf{w}}_i = \mathbf{z}_i$), the $\tilde{\mathbf{w}}_i$ updates can be written as,

$$\begin{aligned} \underset{\tilde{\mathbf{w}}_i \in \mathbb{R}^{2N}}{\text{minimize}} \quad & \tilde{\mathbf{w}}_i^T (\tilde{\mathbf{R}}_{\mathbf{x}_i} + \rho \mathbf{I}) \tilde{\mathbf{w}}_i - 2\rho \tilde{\mathbf{w}}_i^T (\mathbf{z}_i - \mathbf{u}_i), \\ \text{s.t.} \quad & \tilde{\mathbf{w}}_i^T \tilde{\mathbf{R}}_{s_i} \tilde{\mathbf{w}}_i = 1 \quad (13) \end{aligned}$$

The problem in (13) is a standard QCQP with a single constraint and can be solved efficiently. Although, the QCQPs can also be handled with a semidefinite relaxation but the ADMM iterations doesn't make it a viable option. As the correlation matrix in the constraint is also part of the objective function, the problem in (13) can equivalently be written by swapping the correlation matrices as follows,

$$\begin{aligned} \underset{\tilde{\mathbf{w}}_i \in \mathbb{R}^{2N}}{\text{maximize}} \quad & \tilde{\mathbf{w}}_i^T (\tilde{\mathbf{R}}_{s_i} - \rho \mathbf{I}) \tilde{\mathbf{w}}_i + 2\rho \tilde{\mathbf{w}}_i^T (\mathbf{z}_i - \mathbf{u}_i), \\ \text{s.t.} \quad & \tilde{\mathbf{w}}_i^T \tilde{\mathbf{R}}_{\mathbf{x}_i} \tilde{\mathbf{w}}_i = 1 \quad (14) \end{aligned}$$

The equality constraint in (14) can be relaxed by '$\leq$' and solved rather economically through the successive convex relaxation (SCA) by iterative linearization of the objective function. However, more efficient approach is to trace the analytical solution of the above problem by adopting the following steps. Since, $\tilde{\mathbf{R}}_{\mathbf{x}_i}$ is a symmetric positive definite matrix, there exist $\tilde{\mathbf{R}}_{\mathbf{x}_i}^{\frac{1}{2}}$ and $\tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}}$. We do the change of variables, $\mathbf{w}_i = \tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}} \tilde{\mathbf{y}}_i$, limiting the constraint to a unit circle as follows,

$$\begin{aligned} \underset{\tilde{\mathbf{y}}_i \in \mathbb{R}^{2N}}{\text{maximize}} \quad & \tilde{\mathbf{y}}_i^T \tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}} (\tilde{\mathbf{R}}_{s_i} - \rho \mathbf{I}) \tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}} \tilde{\mathbf{y}}_i + 2\rho \tilde{\mathbf{y}}_i^T \tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}} (\mathbf{z}_i - \mathbf{u}_i) \\ \text{s.t.} \quad & \tilde{\mathbf{y}}_i^T \tilde{\mathbf{y}}_i = 1 \quad (15) \end{aligned}$$

Denote $\mathbf{R}_i = \tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}} (\tilde{\mathbf{R}}_{s_i} - \rho \mathbf{I}) \tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}}$ and $\mathbf{b}_i = \rho \tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}} (\mathbf{z}_i - \mathbf{u}_i)$, simplifying the above problem,

$$\begin{aligned} \underset{\tilde{\mathbf{y}}_i \in \mathbb{R}^{2N}}{\text{maximize}} \quad & \tilde{\mathbf{y}}_i^T \mathbf{R}_i \tilde{\mathbf{y}}_i + 2\mathbf{b}_i^T \tilde{\mathbf{y}}_i, \\ \text{s.t.} \quad & \tilde{\mathbf{y}}_i^T \tilde{\mathbf{y}}_i = 1 \quad (16) \end{aligned}$$

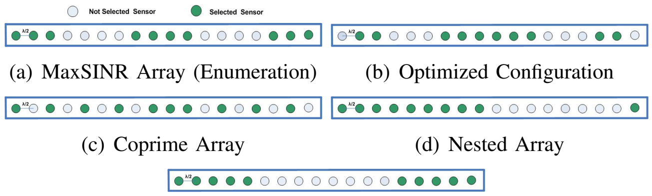
Since, $\mathbf{R}_i$ is a symmetric matrix, it is orthogonally diagonalizable, $\mathbf{R}_i = \mathbf{V}_i^T \mathbf{\Sigma}_i \mathbf{V}_i$, with the columns of $\mathbf{V}_i$ being the eigenvectors and ζ_{k_i} are the corresponding real eigenvalues. The solution in (16) can be found by solving the following system of equations [22],

$$\begin{aligned} (\gamma_i \mathbf{I} - \mathbf{R}_i) \mathbf{y}_i =& \mathbf{c}_i \\ \mathbf{y}_i^T \mathbf{y}_i =& 1 \quad (17) \end{aligned}$$

For $\mathbf{c}_i = \mathbf{V}_i^T \mathbf{b}_i$ and γ_i is the largest solution satisfying the secular equation given below,

$$\sum_{k=1}^{2N} \frac{|\mathbf{c}_i(k)|^2}{|\gamma_i - \zeta_{k_i}|^2} = 1 \quad (18)$$

The equation can be efficiently solved through various algorithms [23]. It is also noted that the eigen decomposition is only required for the initial iteration and can be used for the next iterations as $\mathbf{R}_i$ is fixed for the problem. Therefore, only



(a) MaxSINR Array (Enumeration) (b) Optimized Configuration

(c) Coprime Array (d) Nested Array

(e) Worst case Configuration

Fig. 1. Various sparse array configurations

$\mathbf{c}_i$ is updated iteratively, solving (18) and (17) subsequently. Moreover, all the M QCQPs are independent of each other and can be implemented efficiently in parallel, further optimizing the run time. After finding $\tilde{\mathbf{y}}_i$ the beamformer weights are given by, $\mathbf{w}_i = \tilde{\mathbf{R}}_{\mathbf{x}_i}^{-\frac{1}{2}} \tilde{\mathbf{y}}_i$.

The solution of the unconstrained minimization in (12) for the $\mathbf{z}_i$ updates is given by the shrinkage formula for the reweighted group sparsity regularization [24],

$$\mathbf{z}^{(k)} = \max\left\{0, ||\tilde{\mathbf{w}}_i + \mathbf{u}_i||_2 - \frac{\mathbf{f}(k)}{\rho}\right\} \frac{\tilde{\mathbf{w}}_i + \mathbf{u}_i}{||\tilde{\mathbf{w}}_i + \mathbf{u}_i||_2} \quad (19)$$

The kth element of re-weighting vector $\mathbf{f}(k)$ is iteratively updated for jth iteration as follows,

$$\mathbf{f}^{j+1}(k) = \frac{1}{t(||\mathbf{z}^{(k)^j}||_2 + \epsilon)}. \quad (20)$$

The parameter ϵ prevents the unwanted case of division by zero and also avoids the solution to converge to local minima. The re-weighting promotes group sparse solutions and can also control the desired sparsity in the final solution by finding the appropriate values of t, typically found through binary search.

IV. SIMULATIONS

We demonstrate designing sparse array with $K = 10$ sensor locations selected from $N = 18$ possible locations which are uniformly spaced with the minimum separation of of $\frac{\lambda_{\omega_m}}{2}$. We consider $M = 4$ frequencies, namely, ω_m, $0.925\omega_m$, $0.825\omega_m$ and $0.8\omega_m$. The desired source is located at 50^0. There are five interferers with DOAs of 60^0, 40^0, 95^0, 120^0 and 150^0. The SNR for the desired source is 0 dB and INR of each interference is 20 dB. For simple illustration and ease of presentation, we choose the four sources with the same DOA. That is, one source with subbands. In this case, all four optimum beamformers point to the same direction. The jammers are chosen as wideband, and as such, are present in all frequencies considered.

Selecting $K = 10$ locations from $N = 18$, yields to 43758 possible sparse configurations, rendering exhaustive search computationally expensive. The sparse configuration offering maximum average SINR of 9.67 dB is found through exhaustive search and is shown in Fig. 1a. The array configuration optimized by the proposed algorithm is shown

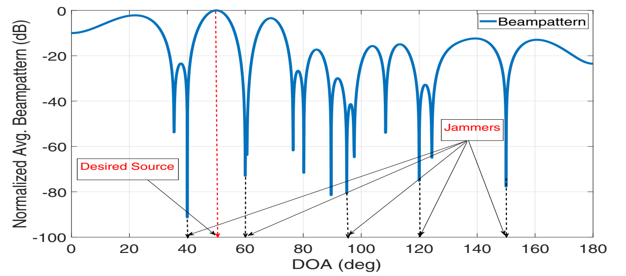


Fig. 2. Average beampattern of the optimized configuration

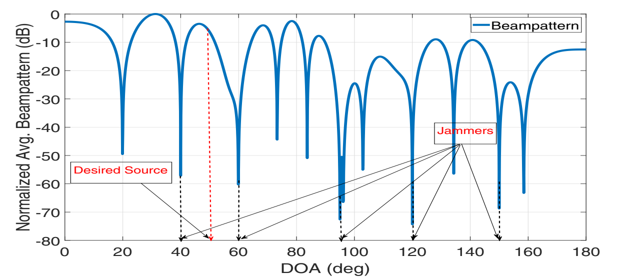


Fig. 3. Average beampattern of the worst case configuration

in the Fig. 1b, performing close to the enumerated design with an average output SINR of 9.61 dB. The structure arrays, such as coprime array and nested array [4], shown in Figs. 1c & 1d, perform inferior to the optimized configuration with respective SINRs of 8.4 dB and 4 dB. The 10 element compact uniform linear array has an average output SINR of only 5.13 dB. It is found that the compact ULA (uniform linear array) needs to employ 3 additional sensors to match the optimized design, for the underlying scenario. The performance deterioration of the compact ULA is due to the rather smaller aperture which cannot resolve the desired source for all frequencies. However, the larger array aperture does not trivially imply the SINR improvement. Case in point is the worst case beamformer design which turns out to occupy the full aperture, as shown in Fig. 1e. It has an average output SINR of only 3.23 dB. This underscores the importance of avoiding random arrays and pursuing sparse array design. The improved SINR performance of the optimized configuration is depicted in Fig. 2, showing the average beampattern across all frequencies. The optimized configuration uses its DOF to cancel the jammers efficiently. In contrast, the beampattern of worst sensor topology shown in Fig. 3, struggles to mitigate the high power jammers while maintaining maximum gain towards the desired source.

V. CONCLUSION

This paper considered MaxSINR sparse array design for multifunctional antennas. It proposed an ADMM based approach employing group sparsity to optimize a common sparse configuration across all frequencies. The proposed approach divides the problem to parallel QCQPs with a single associated constraint. It was shown that the resulting QCQPs can be solved in an efficient manner and permits parallel implementations. The proposed algorithm and enumeration showed strong agreement in performance.

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