Tensor Decomposition Based DOA Estimation for Transmit Beamspace MIMO Radar

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***Abstract*—The detection and localization of multiple targets is a fundamental research area for multiple input multiple output (MIMO) radar. In many civilian applications of MIMO technology, for example, automotive radar, high resolution direction of arrival (DOA) estimation is required. In this paper, a novel DOA estimation algorithm based on tensor decomposition is proposed for collocated transmit beamspace MIMO radar. First, we introduce the flipped-conjugate version of the transmit beamspace matrix, which focuses the transmit energy into fixed region. This can increase the signal to noise ratio (SNR) of targets. Then we reshape the received data into a tensor form, the structure of which provides the estimations of the transmit and receive steering matrices. The alternating least squares (ALS) algorithm is applied to find the tensor components. The DOA estimation is conducted in transmitters via the rotational invariance property achieved by beamspace matrix. It is proved that at most $M-2$ grating lobes exist during the process of DOA estimation, where M is the number of the transmitters. These grating lobes can be eliminated by finite trials of spectrum search. The performance of our proposed DOA estimation method surpasses several conventional algorithms in terms of accuracy and resolution.**

***Index Terms*—Collocated MIMO radar, DOA estimation, Grating lobes, Localization, Tensor decomposition**

I. Introduction

Multiple input multiple output (MIMO) radar has been the focus of intensive research [1]–[3], and has found increasingly many applications in civilian radar technology [4], [5]. Among these applications, direction of arrival (DOA) estimation is one of the most fundamental research topics [6]–[9]. Much of the literature has generalized classic DOA estimation algorithms, such as multiple signal classification (MUSIC) [10]–[12] and estimation of signal parameters via rotational invariance technique (ESPRIT) [7], [13]–[15] from conventional phased array radar to MIMO radar. However, the problem of achieving higher resolution and improved estimation accuracy still remains. In some scenarios like automotive radar, it is even necessary to relax the requirement of using regular array geometries [16], but still be able to achieve multiple targets detection and localization with high resolution and accuracy.

In collocated MIMO radar, it has been shown that by using less number of waveforms than the number of transmitters the transmitting energy can be focused in fixed region [11], [16]. Specifically, at some number of waveforms, the gain from using more waveforms begins to degrade estimation performance. This trade-off between waveform diversity and spatial diversity implies that the performance of DOA estimation in MIMO radar can be further improved. Meanwhile, the multi-linear structure of received echoes in MIMO radar has been studied [12], [17]–[19]. Methods like parallel factor (PARAFAC) analysis [12] have been applied to find each component of received signal tensor and thus perform DOA estimation for multiple targets simultaneously. It has been shown that the performance of tensor decomposition methods is better than covariance matrix-based algorithms [12], [19].

In this paper, a DOA estimation method for collocated MIMO radar with special transmit beamspace matrix is proposed. First, we introduce the flipped-conjugate version of the transmit beamspace matrix, which focuses the transmit energy into fixed region in order to achieve better spatial diversity [11], [16]. Then we reshape the received signal into a 3-order tensor form, whose structure distinctly provides estimations of steering matrices. The transmit steering matrix takes the advantage of rotational invariance property (RIP) achieved due to the beamspace matrix, and DOA estimation can be then conducted using the phase rotations. Owing to the conjugate symmetry property enforced by the transmit beamspace matrix, at most $M-2$ grating lobes exist for each target. These grating lobes can be mitigated by MUSIC algorithm with finite trails. It is worth noting that the receive array geometry can be arbitrary since DOA estimation is performed thanks to the processing in the transmit side. Simulations verify the DOA estimation performance of our proposed method in terms of accuracy and resolution.

II. Signal Model

We start by presenting the following result that will be useful for understanding the findings of this paper. For matrices $\mathbf{A} \in \mathbb{C}^{M\times N}$, $\mathbf{C} \in \mathbb{C}^{N\times P}$, and diagonal matrix $\mathbf{B} = \mathrm{diag}(\mathbf{b}) \in \mathbb{C}^{N\times N}$, the following Lemma holds.

Lemma 1: $vec\{\mathbf{ABC}\} = \left(\mathbf{C}^H \odot \mathbf{A}\right)\mathbf{b}$

Proof 1:

$$\begin{aligned} vec\{\mathbf{ABC}\} &= \sum_{n=1}^{N} \mathbf{B}(n,n)\left(\mathbf{c}_n^H \otimes \mathbf{a}_n\right) \\ &= \left(\mathbf{C}^H \odot \mathbf{A}\right)\mathbf{b} \end{aligned} \tag{1}$$

where $\mathbf{B}(n,n)$ is the (n,n)-th element of $\mathbf{B}$, $\mathbf{a}_n$ is the nth column of matrix $\mathbf{A}$, $\otimes$ is the Kronecker product, and $\odot$

is the Khatri-Rao product (column-wise Kronecker product). Note that the Khatri-Rao product and Kronecker product are identical for vectors.

Consider a collocated MIMO radar system with M transmitters organized in an uniform linear array (ULA) and N receivers with arbitrary array geometry within a fixed aperture. The distance between transmitters is denoted by d_t. The $M \times 1$ transmit steering vector is given by $\boldsymbol{\alpha}(\theta) \triangleq [1, e^{-j\frac{2\pi}{\lambda}d_t \sin\theta}, \cdots, e^{-j\frac{2\pi}{\lambda}(M-1)d_t \sin\theta}]^T$, where $(\cdot)^T$ denotes the transpose, θ represents the target direction. Similarly, the $N \times 1$ receive steering vector of N receivers with arbitrary geometry is given by $\boldsymbol{\beta}(\theta) \triangleq [1, e^{-j\frac{2\pi}{\lambda}x_n \sin\theta}, \cdots, e^{-j\frac{2\pi}{\lambda}x_N \sin\theta}]^T$, where $\{x_n | 0 \leq x_n \leq D_r, n = 1, \cdots, N\}$ are the coordinates of the receivers and D_r is the aperture of the receive array.

Let $\mathbf{S}(t) \triangleq [S_1(t), S_2(t), \cdots, S_M(t)]^T$ be the $M \times 1$ vector of pre-designed waveforms. It needs to satisfy the orthogonality property, i.e., $\int_T \mathbf{S}(t)\mathbf{S}^H(t)dt = \mathbf{I}_M$, where T denotes the radar pulse duration, $(\cdot)^H$ represents the Hermitian transpose, and $\mathbf{I}_M$ is the $M \times M$ identity matrix. The matrix of transmit waveforms is denoted by $\mathbf{X}(t) = \boldsymbol{\alpha}^H(\theta)\mathbf{S}(t)$. Assuming the presence of L targets at angles θ_l, $l = 1, 2, \cdots, L$, the received signal of all reflections from the targets can be represented as

$$\mathbf{y}(t) = \sum_{l=1}^{L} \sigma_l^2 \boldsymbol{\beta}(\theta_l)\boldsymbol{\alpha}^H(\theta_l)\mathbf{S}(t) + \mathbf{n}(t) \tag{2}$$

where σ_l^2 is the radar cross section (RCS) and $\mathbf{n}(t)$ is the zero-mean white Gaussian noise. After right multiplication of (2) by $\mathbf{S}^H(t)$, the matched-filter output is

$$\mathbf{Y} = \mathbf{B}\boldsymbol{\Sigma}\mathbf{A}^H + \mathbf{N} \tag{3}$$

where $\boldsymbol{\Sigma} = diag(\mathbf{c})$ is a diagonal matrix formed from $\mathbf{c} \triangleq [\sigma_1^2, \cdots, \sigma_L^2]^T$, $\mathbf{B} \triangleq [\boldsymbol{\beta}(\theta_1), \cdots, \boldsymbol{\beta}(\theta_L)]_{N\times L}$, $\mathbf{A} \triangleq [\boldsymbol{\alpha}(\theta_1), \cdots, \boldsymbol{\alpha}(\theta_L)]_{M\times L}$, and $\mathbf{N}$ is the noise residue. The operator $diag(\cdot)$ here returns a square diagonal matrix with diagonal elements equal to its vector argument.

III. Proposed Tensor Decomposition based DOA Estimation Method

In many cases, the omnidirectional transmit beampattern of MIMO radar leads to the deterioration of signal to noise ratio (SNR) of targets. This can be improved by applying beamspace matrix at the transmitters to focus energy on several directions or special sectors of particular interest. It is a trade-off between waveform diversity and spatial diversity [11], [16].

Let $\mathbf{W}$ be the transmit beamspace matrix of dimension $M \times 2K$, K is the number of waveforms used at the transmitters (we refer to [16] for design details). Define $\mathbf{W}$ as follows

$$\begin{aligned} \mathbf{W} &\triangleq [\mathbf{W}_1, \mathbf{W}_2]_{M\times 2K} \\ \mathbf{W}_1 &\triangleq [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_K]_{M\times K} \\ \mathbf{W}_2 &\triangleq [\bar{\mathbf{w}}_1, \bar{\mathbf{w}}_2, \cdots, \bar{\mathbf{w}}_K]_{M\times K} \end{aligned} \tag{4}$$

where $\bar{\mathbf{w}}_k(m) \triangleq \mathbf{w}_k^*(M - m + 1)$, $m = 1, 2, \cdots, M$, $k = 1, 2, \cdots, K$, i.e., $\bar{\mathbf{w}}$ is the flipped-conjugate version of $\mathbf{w}$. The inner products of $\mathbf{w}^H\boldsymbol{\alpha}(\theta)$ and $\bar{\mathbf{w}}^H\boldsymbol{\alpha}(\theta)$ are related through the RIP and share identical transmit beampattern, denoted by

$$\begin{aligned} \bar{\mathbf{w}}^H\boldsymbol{\alpha}(\theta) &= \sum_{m=1}^{M} w(M - m + 1)e^{-j\frac{2\pi}{\lambda}(m-1)d_t \sin\theta} \\ &= \kappa\big(\mathbf{w}^H\boldsymbol{\alpha}(\theta)\big)^* \end{aligned} \tag{5}$$

where $\kappa \triangleq e^{-j\frac{2\pi}{\lambda}(M-1)d_t \sin\theta}$. This relationship can be generalized to L targets, given by

$$\mathbf{W}_2^H\mathbf{A} = \big(\mathbf{W}_1^H\mathbf{A}\big)^*\mathbf{T} \tag{6}$$

where $\mathbf{T} = diag(\boldsymbol{\Phi})$ is a $L \times L$ diagonal matrix. Each element of $\boldsymbol{\Phi}$ contains the angle information of one target, where

$$\boldsymbol{\Phi} \triangleq [e^{-j\frac{2\pi}{\lambda}(M-1)d_t \sin\theta_1}, \cdots, e^{-j\frac{2\pi}{\lambda}(M-1)d_t \sin\theta_L}]^T \tag{7}$$

Substituting this result into (3), we obtain

$$\mathbf{Y} = \mathbf{B}\boldsymbol{\Sigma}(\mathbf{W}^H\mathbf{A})^H + \mathbf{N}. \tag{8}$$

Using Lemma 1 and vectorizing (3) into a $2KN \times 1$ vector, we can write

$$\mathbf{y}_w = \big[\big(\mathbf{W}^H\mathbf{A}\big) \odot \mathbf{B}\big]\,\mathbf{c}^T + \tilde{\mathbf{n}}. \tag{9}$$

Assuming that Q pulses are utilized in a single coherent processing interval (CPI), the overall echoes can be denoted by $\mathbf{Y}_w \triangleq [\mathbf{y}_{w1}, \mathbf{y}_{w2}, \cdots, \mathbf{y}_{wQ}]$, or equivalently, by

$$\mathbf{Y}_w = \big[\big(\mathbf{W}^H\mathbf{A}\big) \odot \mathbf{B}\big]\,\mathbf{C}^T + \tilde{\mathbf{N}} \tag{10}$$

where $\mathbf{C} \triangleq [\mathbf{c}_1, \cdots, \mathbf{c}_Q]^T$, $\tilde{\mathbf{N}} \triangleq [\tilde{\mathbf{n}}_1, \cdots, \tilde{\mathbf{n}}_q]$. This is the matrix form of a 3-order tensor $\mathcal{Y}(k, n, q)$ unfolded across the third dimension. Alternating least square (ALS) algorithm [18] can be directly used to estimate each component of this tensor simultaneously. This process is given as follows

$$\begin{aligned} &\min_{\hat{\mathbf{X}}} \Big\| [\mathcal{Y}]_{(1)} - \Big[(\mathbf{C} \odot \mathbf{B})\hat{\mathbf{X}}^T\Big] \Big\|_F^2 \\ &\min_{\hat{\mathbf{B}}} \Big\| [\mathcal{Y}]_{(2)} - \Big[(\mathbf{C} \odot \mathbf{X})\hat{\mathbf{B}}^T\Big] \Big\|_F^2 \\ &\min_{\hat{\mathbf{C}}} \Big\| [\mathcal{Y}]_{(3)} - \Big[(\mathbf{B} \odot \mathbf{X})\hat{\mathbf{C}}^T\Big] \Big\|_F^2 \end{aligned} \tag{11}$$

where $\mathbf{X} \triangleq \mathbf{W}^H\mathbf{A}$, $[\mathcal{Y}]_{(i)}$, $i = 1, 2, 3$ denotes the mode-i unfolding of $\mathcal{Y}(k, n, q)$, and $\hat{\mathbf{X}}$ denotes the estimation of $\mathbf{X}$.

Substitute (4) into (10), we have

$$\tilde{\mathbf{Y}} = \left[\begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} \odot \mathbf{B}\right]\mathbf{C}^T + \tilde{\mathbf{N}} \tag{12}$$

where $\mathbf{X} \triangleq (\mathbf{X}_1^H, \mathbf{X}_2^H)^H$, $\mathbf{X}_1 \triangleq \mathbf{W}_1^H\mathbf{A}$, and $\mathbf{X}_2 \triangleq \mathbf{W}_2^H\mathbf{A}$. Using (6), (12) can be reformulated as

$$\tilde{\mathbf{Y}} = \left\{\begin{bmatrix} \mathbf{W}_1^H\mathbf{A} \\ (\mathbf{W}_1^H\mathbf{A})^*\mathbf{T} \end{bmatrix} \odot \mathbf{B}\right\}\mathbf{C}^T + \tilde{\mathbf{N}}. \tag{13}$$

It can be seen from (13) that the special structure of the beamspace matrix is maintained in the first component. The ALS algorithm can estimate $\mathbf{X}$, and the estimation of $\mathbf{T}$ can be found by least-squares method, i.e., $\hat{\mathbf{T}} = (\mathbf{X}_1^*)^\dagger\mathbf{X}_2$, knowing that $\mathbf{T}$ is full rank. Here $(\cdot)^\dagger$ is the pseudoinverse of a matrix.

TABLE I
GRATING LOBES ELIMINATION

$\hat{\theta}_l = CancelGratingLobes(\hat{\mathbf{\Phi}}, \mathbf{R}_{\tilde{y}})$
• For each column $l = \{1, 2, .., L\}$
1) Set $\hat{\mathbf{\Phi}}(l) \triangleq e^{-j\frac{2\pi}{\lambda}(M-1)d_t \sin\theta_l}$
2) Let $\hat{\theta}_{lk}, k = 1, ..., M-1$ be the possible estimations, with $\sin\hat{\theta}_{lk} = \frac{jIn(\hat{\mathbf{\Phi}}(l))}{2\pi d_t(M-1)} - \frac{k'\lambda}{d_t(M-1)}, k' = k - \left\lceil \frac{M-1}{2} \right\rceil$
3) Compute the MUSIC spectrum of $\hat{\theta}_{lk}$ by (14)
4) Keep the one with highest peak as the estimation of $\hat{\theta}_l$

Then the eigenvalues of $\hat{\mathbf{T}}$ are computed as the estimation of $\mathbf{\Phi}$ via singular value decomposition (SVD).

Note that each element in $\mathbf{\Phi}$ is equivalent with one element in the steering vector of a virtual array whose spacing between elements is $(M-1)d_t$. The target DOAs are very likely to be mixed because of other $M-2$ grating lobes (assume $d_t = \lambda/2$). However, the energy is actually distributed in only one direction. This true direction can be resolved by comparing the spectrum of each possible direction

$$P(\hat{\theta}_l) = \frac{1}{\left(\boldsymbol{\alpha}(\hat{\theta}_l) \otimes \boldsymbol{\beta}(\hat{\theta}_l)\right)^H \mathbf{R}_{\tilde{y}}^{-1} \left(\boldsymbol{\alpha}(\hat{\theta}_l) \otimes \boldsymbol{\beta}(\hat{\theta}_l)\right)} \tag{14}$$

where $\mathbf{R}_{\tilde{y}} = \tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^H$. Table I describes the process of eliminating the grating lobes.

By introducing the transmit beamspace matrix $\mathbf{W}$ and applying the tensor decomposition method, we get two advantages. First, the focused energy in transmit beampattern increases the targets SNR and thus improves the performance of DOA estimation. Second, tensor decomposition method (like ALS here) exploits the multi-linear structure of the received signal. This structure directly shows that the associated rank-1 factors of the decomposition are the estimations of the transmit and receive steering matrices. This relationship provides better DOA estimation, especially for detection and localization of multiple targets.

It is worth noting that our proposed method requires uniqueness of tensor decomposition, which is given by [17], [18]

$$\sum_{i=1}^{I} k_i \geq 2L + (I-1) \tag{15}$$

where k_i is the Kruskal-rank of each factor matrix, I is the number of tensor dimension and L is the rank. In our case, $I = 3$ and each factor matrix is column full rank. Thus, (15) can be achieved easily. Given that each component of tensor corresponding to (13) cannot keep the Vandermonde structure in our model, (15) is almost surely the optimal boundary condition of uniqueness. We refer to [19] for more discussions about uniqueness of tensor decomposition.

Finally, the proposed DOA estimation algorithm can be conducted without knowing the receiver coordinates x_n, $n = 1, \cdots, N$, Thus, the receive array configuration is flexible. However, this property indicates that it is quite difficult to withdraw the angle information merely in factor matrix $\mathbf{B}$.

TABLE II
COMPUTATIONAL COMPLEXITY FOR SEVERAL ALGORITHMS

Method	Complexity
MUSIC*	$\mathcal{O}((MN)^3 + M^2N^2QZ)$
ESPRIT	$\mathcal{O}((MN)^3 + M^2N^2Q)$
[12]	$\mathcal{O}(2(I+1)LMNQP)$
[16]	$\mathcal{O}((2KN)^3 + (2K)^2N^2Q)$
Proposed	$\mathcal{O}(4(I+1)LKNQP + K^3 + K^2L)$

* Z is the number of grids for spectrum search

A. Computational Complexity Analysis

The proposed tensor decomposition based DOA estimation algorithm mainly contains two steps. The first is the ALS algorithm to estimate factor matrices. During each iteration, the number of flops is $\mathcal{O}(4(I+1)LKNQ)$ [20]. The estimation of $\mathbf{T}$ in second step requires $\mathcal{Q}(K^3 + K^2L)$ flops. In total, the number flops needed by our algorithm is approximately $\mathcal{O}(4(I+1)LKNQP + K^3 + K^2L)$, where P is the number of iterations. Table II gives the computational complexity overview for our algorithm as well as for other popular algorithms used in the next section.

IV. SIMULATION RESULTS

In this section, we evaluate the DOA estimation performance of the proposed method in terms of the root mean square error (RSME) and probability of resolution. MUSIC algorithm, ESPRIT algorithm [13], method of [12], method of [16], and the Cramer-Rao lower bound (CRLB) [21] are also shown for comparison. Throughout the simulations, a MIMO radar with $M = 10$ transmit, and $N = 11$ receive elements is used. The spacing of the transmitters is $d_t = \lambda/2$. The receivers in transmit beamspace MIMO case are randomly spaced in an array with aperture of 5λ, while its counterparts in conventional MIMO case is a ULA with $d_r = \lambda/2$. Two targets with different angles are assumed. The RCS of the targets are chosen randomly from Gaussian distribution, and they obey Swerling II model. The Doppler of the targets are fixed as zero since they have no influence on the DOA estimation. The number of pulses is $Q = 50$, and the number of Monte Carlo trials is 500. The orthogonal waveforms used here are $S_k(t) = \sqrt{\frac{1}{T_p}} e^{j2\pi\frac{k}{T_p}t}$, $k = 1, \cdots, K$.

For applying MUSIC, ESPRIT, and the method of [12] for conventional MIMO case, the beamspace matrix is just the identity matrix. The method of [16] and our proposed algorithm, representing beamspace MIMO case, use the same beamspace matrix $\mathbf{W}$ to focus the transmitted energy in the region $\Theta : [-15^\circ, 15^\circ]$ (see also Fig. 5 in [16]).

In the first example, two targets are placed at $\theta_l = [10^\circ, -30^\circ]$. Fig. 1 shows the results of grating lobes elimination in a single Monte Carlo trial. Numbers nearby those bars denote the potential $M-1$ positions of the targets, which are computed at step 2 in Table I. After finite trials of spectrum search via (14), we can easily distinguish the true angles from grating lobes. The highest two peaks denote the true directions of two targets. Note that the peak value of the

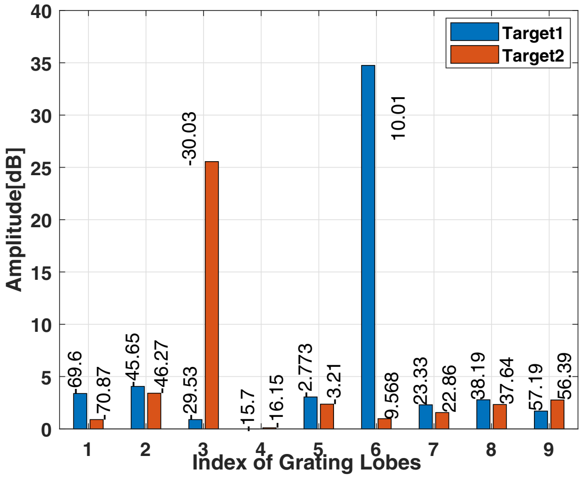

Fig. 1. Spectrum Search Results of all Gating Lobes, SNR = 5 dB, two targets locate in $\theta_l = [10°, -30°]$.

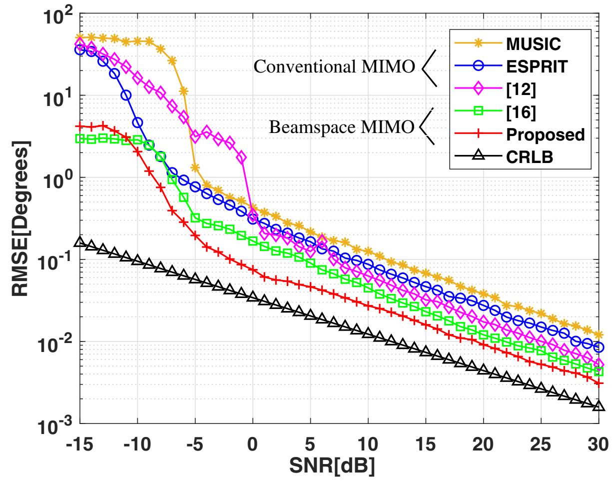

Fig. 2. RMSE of DOA estimation, 500 trials, from -15 dB to 30 dB, two targets locate in $\theta_l = [10°, -10°]$.

second target is lower, which is caused by the synthesized transmit beampattern.

In our second example, the second target is moved to $\theta_2 = -10°$. The RMSEs of the methods tested are shown in Fig. 2. The RMSE decreases steadily with the rise of SNR for all algorithms. Results of MUSIC and ESPRIT are quite similar, whereas ESPRIT method gets better convergence and a few improvements when SNR is large. The method of [12] applies ALS algorithm. The corresponding RMSE declines significantly when SNR is about -5 dB, then decreases gradually. After applying beamspace matrix, the DOA estimation performance is improved due to the increased SNR, which can be found also for the method of [16]. Note that the method of [16] exploits the phase rotations ensured by the beamspace matrix to conduct angle estimation at the transmitter, which can be regarded as a generalized ESPRIT for beamspace MIMO. The RMSE of our proposed method is substantially lower than other methods, since the advantages of the tensor decomposition and beamspace matrix are combined together. The performance is improved especially in low SNR region.

In the last example, the probability of resolution of two closely spaced targets is investigated, where $\theta_l = [5°, 6°]$. It can be seen in Fig. 3 that the probability of target resolution rises steadily from very low values (i.e., resolution fail) to values close to one (i.e., resolution success) when the SNR increases. Our proposed algorithm surpasses the other algorithms and demonstrates the lowest threshold. Therefore, it can be concluded that the proposed DOA estimation algorithm achieves better accuracy and higher resolution.

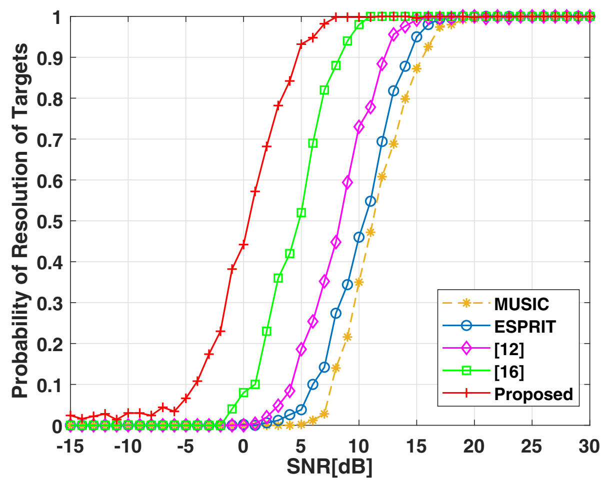

Fig. 3. Probability of Resolution of closely spaced targets, 500 trials, from -15 dB to 30 dB, two targets locate in $\theta_l = [5°, 6°]$.

V. Conclusion

A DOA estimation algorithm based on tensor decomposition for collocated MIMO radar with special transmit beamspace matrix and arbitrary receive array geometry has been proposed. The introduction of the transmit beamspace matrix leads to additional SNR gain for multiple targets, and thus, to improved DOA estimation performance. We have reshaped the received signal into a 3-order tensor form, whose structure distinctly provides estimations of the transmit and receive steering matrices via ALS algorithm. The RIP is maintained and applied to conduct DOA estimation. Owing to the conjugate symmetry property enforced by the transmit beamspace matrix, at most $M-2$ grating lobes exist for each target. These grating lobes can be mitigated by MUSIC algorithm with finite search trials. It is worth stressing that the receive array geometry can be arbitrary. The proposed method improves the DOA estimation performance in terms of resolution and accuracy with a gap of about 5 dB SNR compared to other state of the art methods.

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