Sub-Nyquist Sampling in Shift-Invariant Spaces

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***Abstract*—We introduce a novel framework for acquisition of analog signals by combining compressive sensing (CS) and the shift-invariant (SI) reconstruction procedure. We reinterpret the random demodulator as a system that acquires a linear combination of the samples in the conventional SI setting with the box function as the sampling kernel. The SI samples are recovered by solving the CS optimization problem and subsequently filtered by a correction filter in order to obtain expansion coefficients of the signal. The underlying model is inherently infinite dimensional, but the SI property allows for formulation of the problem within finite-dimensional CS. We provide experimental results of the proposed system at the end of the paper.**

***Index Terms*—B-splines, inverse problems, sampling, sparsity**

I. Introduction

A conventional assumption underlying most analog-to-digital converters (ADCs) is that a signal must be sampled at least at the Shannon-Nyquist rate [1], [2] which corresponds to twice the highest frequency contained in the signal. Even though the bandlimited assumption is usually approximately met, signals can often be much better represented in alternative bases [3], [4] other than the Fourier. This resulted in several extensions of the Shannon theorem, which treat more general classes of signals. In particular, signals that lie in shift-invariant (SI) spaces play an important role in sampling theory. Such signals are expressed as linear combinations of shifts of a generator with period T. Sampling theory developed for signals in SI spaces is a generalization of the Shannon theorem and is based on linear filtering [5]–[7].

Another sampling paradigm that received growing attention in recent years is compressive sensing (CS) [8]–[10]. In discrete CS problems, the aim is to recover a signal $\mathbf{x} \in \mathbb{R}^N$ from a set of linear measurements $\mathbf{y} \in \mathbb{R}^M$, where $M < N$. In order to be reconstructed from such an ill-posed inverse problem, the signal is assumed to be sparse in a certain transform basis. Many efficient algorithms have been developed in order to recover signals from this type of problems [11].

Due to the realization complexity and underlying infinite-dimensional model, only a few practical implementations of CS exist. Several works attempt to extend the theory of discrete CS to the analog domain [12]–[18]. Hardware realizations for analog compression of signals followed by sub-Nyquist sampling are proposed in [19]–[21]. In [22], Eldar introduces a framework for CS of analog signals in a union of SI subspaces. The sparsity is modeled by treating the case in which only k out of n SI generators are active. The CS setting is used to find the k active generators of a union of SI subspaces by filtering the signal with m instead of the conventional n filters, leading to a sampling rate m/T, where $2k \leq m < n$. This method can be extended to a special case of sampling of signals that lie in a single SI subspace and have periodic sparsity pattern in the time domain. That is, only k out of n expansion coefficient in an interval are active and the same sparsity pattern repeats. The method requires a sampling kernel that is biorthogonal to a generator, which may be quiet difficult to implement.

In this paper, we propose a novel framework for CS of analog signals that lie in a single SI subspace. Contrarily to [22], SI samples of the signals are assumed to be sparse in a certain transform domain, which allows for signals to have all expansion coefficients active. Additionally, same sampling kernel is used in order to reconstruct signals generated by different SI generators. That is, a single acquisition hardware suits all kinds of signals that satisfy the SI property. The front-end of the proposed system is a parallel version of the random demodulator (RD) [19], a CS hardware for acquiring of sparse bandlimited signals consisting of a mixer, an integrator and a low-rate ADC. An input signal is demodulated by a high-rate pseudorandom chipping sequence of ± 1s, which can be seen as a multiplication by a signal that lies in an SI subspace. The following integration and low-rate sampling of the signal are in fact a process of acquiring a linear combination of N samples in the standard SI setting. The front-end consist of M channels, where $M < N$, leading to a system with sampling rate $M/(NT)$. The model we treat is inherently infinite dimensional as it involves an infinite sequence of signal expansion coefficients in SI subspace. In the sequel, it will be shown how to transform it to a finite-dimensional model in an exact way. A CS setting recovers SI samples from a reduced set of measurements and the recovered samples are subsequently filtered by a discrete-time correction filter in order to reconstruct the signal.

In Section II, we provide a short review of sampling in SI spaces. An infinite-dimensional measurement model for CS of signals in an SI space is proposed in Section III. In Section IV, we present a strategy for reconstruction of the signals from CS measurements. Finally, we validate the proposed framework with experimental results in Section V.

II. Sampling of Signals in SI Spaces

The most of the results given in this section can be found in the literature, e.g. [5]–[7]. However, we provide a short review

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of sampling in SI spaces since our interpretation is essential in expanding it to the CS setting.

An SI subspace $\mathcal{A}$ of L_2 is spanned by shifts of a generator $a(t)$ with period T [7]. Any signal $x(t) \in \mathcal{A}$ can be expressed as a linear combination of the shifts of $a(t)$ [6]:

$$x(t) = \sum_{m\in\mathbb{Z}} d[m]a(t - mT), \tag{1}$$

where $d[m]$ are coefficients. Note that $d[m]$ are not necessarily samples of the signal. By analogy with the Shannon theorem, which is a special case of SI sampling, $a(t)$ corresponds to the *sinc* function. However, a much broader class of generators can be defined that are easier to handle numerically [3], [23].

A unique signal representation in an SI subspace $\mathcal{A}$ and a stable sampling theorem are achieved when the shifts of a generator $a(t)$ form a Riesz basis or a frame [3]. A countable set of vectors $\{a(t - mT)\}$ generate a Riesz basis if it is complete and there exist two strictly positive constants $\beta > 0$ and $\gamma < \infty$ such that [6], [7]

$$\beta\|\mathbf{d}\|_{l_2}^2 \le \left\| \sum_{m\in\mathbb{Z}} d[m]a(t - mT) \right\|^2 \le \gamma\|\mathbf{d}\|_{l_2}^2, \tag{2}$$

where $\|\mathbf{d}\|_{l_2}^2 = \sum_m |d[m]|^2$ is the squared l_2 norm of $d[m]$. As a consequence, the basis functions are linearly independent and a small modification of the expansion coefficients $d[m]$ results in a small distortion of the signal [6], [7].

Sampling in SI spaces is practical and retains the shift-invariant property of the classical theory. Fig. 1 shows a sampling and reconstruction scheme for signals in SI spaces. In general, an input signal $x(t)$ is prefiltered with a sampling filter $s(-t)$ before being uniformly sampled at a rate $1/T$ [5]–[7]. In a bandlimited scenario, $s(-t)$ corresponds to an antialiasing filter. The samples $c[n]$ in a general SI sampling scheme can be expressed as:

$$c[n] = \int_{-\infty}^{\infty} x(t)s(t - nT)dt \triangleq \langle x(t), s(t - nT)\rangle, \tag{3}$$

where $\langle\cdot,\cdot\rangle$ is the conventional L_2-inner product. Let us define the sampled cross-correlation sequence between the generator $a(t)$ and the sampling kernel $s(t)$ with

$$r_{sa}[n] = \langle a(t), s(t - nT)\rangle. \tag{4}$$

By applying (4), (3) can be expanded to:

$$\begin{aligned} c[n] &= \left\langle \sum_{m\in\mathbb{Z}} d[m]a(t - mT), s(t - nT) \right\rangle \\ &= \sum_{m\in\mathbb{Z}} d[m] \langle a(t - mT), s(t - nT)\rangle \\ &= \sum_{m\in\mathbb{Z}} d[m] r_{sa}[n - m]. \end{aligned} \tag{5}$$

The sampled cross-correlation sequence with only a few nonzero entries around $n = 0$ is particularly possible in B-spline spaces [6], [23], leading to an efficient implementation of the filters we introduce in the sequel.

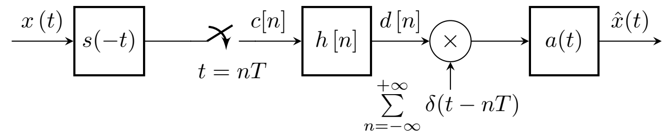


Fig. 1. Sampling and reconstruction of a signal in a shift-invariant space.

Let us denote with $\varphi_{SA}(e^{j\omega})$ the discrete-time Fourier transform (DTFT) of the sampled cross-correlation sequence $r_{sa}[n]$. To be able to reconstruct $x(t)$ from the samples $c[n]$, $\varphi_{SA}(e^{j\omega})$ has to satisfy a rather mild requirement [7]:

$$\left|\varphi_{SA}(e^{j\omega})\right| > \beta,$$

for some constant $\beta>0$. The expansion coefficients are recovered by discrete-time filtering of the samples $c[n]$ with a correction filter [5]–[7]:

$$H(e^{j\omega}) = \frac{1}{\varphi_{SA}(e^{j\omega})}, \tag{6}$$

since $C(e^{j\omega}) = D(e^{j\omega})\varphi_{SA}(e^{j\omega})$. The reconstruction $\hat{x}(t)$ of $x(t)$ is obtained by modulating the recovered expansion coefficients $d[n]$ by an impulse train $\sum_{n\in\mathbb{Z}} \delta(t - nT)$ prior to filtering with the corresponding continuous-time filter $a(t)$.

III. A Measurement Model for CS of Signals in SI Spaces

The random demodulator [19] is one of the first systems that attempted to extend CS paradigm to the analog domain. By upgrading the core idea of the RD, a number of hardware realizations [20], [21] were proposed. In the RD, an input signal is demodulated by a high-rate chipping sequence prior to integration and following by a low-rate sampling.

We propose a system consisting of a bank of the RDs for CS of analog signals in SI spaces. The pseudorandom chipping sequences are piecewise constant signals that can be seen as functions in the B_0-spline space for which (2) is satisfied. A generator of the B_0-spline space for $T = 1$ is the box function illustrated in Fig. 2. This type of function is very practical because it is easy to implement and still brings the perfect reconstruction in the conventional SI setting. A mixing function in the i-th channel of the proposed system can be denoted as:

$$z_i(t) = \sum_{n=1}^{N} \phi_i[n] \sum_{q\in\mathbb{Z}} s(t - nT - qNT), \tag{7}$$

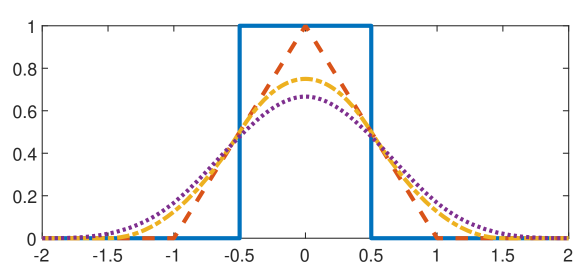


Fig. 2. The centered B_p-spline basis functions of degrees $p = 0$ to 3. As p increases, the basis functions flatten out and their support expands.

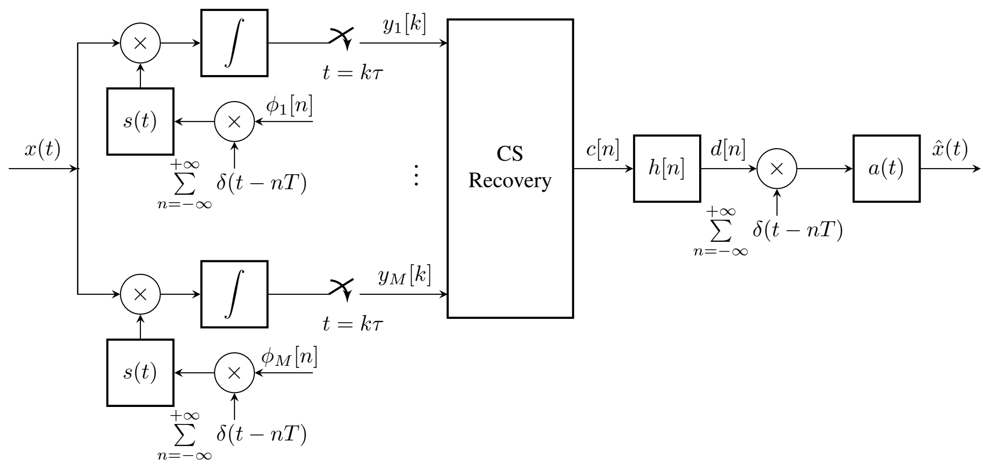


Fig. 3. Compressive sensing of an analog signal in a shift-invariant space with B_0-spline sampling kernel $s(t)$ and a discrete-time correction filter $h[n]$.

where $s(t)$ is the generator of the B_0-spline subspace and $\phi_i[n]$ are expansion coefficients of the mixing function. Without loss of generality, we set the mixing function to be periodic with period NT and the coefficients $\phi_i[n]$ are cyclically repeated.

An input signal is assumed to lie in an appropriate SI subspace $\mathcal{A}$ spanned by the shifts of a generator $a(t)$, as in (1). The signal is mixed by a set of mixing functions $\{z_i(t)\}_{i=1}^M$ before being integrated and subsequently sampled with period $\tau = NT$, leading to a system with sampling rate $M/(NT)$, where $M < N$. The acquisition procedure is illustrated on the left hand side of the scheme in Fig. 3.

A single measurement in the i-th channel is given by:

$$y_i[k] = \int_{k\tau}^{(k+1)\tau} x(t)z_i(t)dt, \tag{8}$$

where k denotes an integration interval. In this paper, generators of the functions are right-shifted by $T/2$ so that an integration interval covers exactly N basis functions $\{s(t-nT)\}$. By applying (1) and (7), (8) is expanded to:

$$y_i[k] = \sum_{n=1}^{N}\phi_i[n]\sum_{m\in\mathbb{Z}}d[m]\int_{k\tau}^{(k+1)\tau}s(t-nT-k\tau)a(t-mT)dt. \tag{9}$$

Since the integration interval covers the whole support of a corresponding B_0-spline basis function, the integration results in (9) for given n and k are equal to entries in the sampled cross-correlation sequence in (4). Thus, similarly to (5), equation (9) can be rewritten as:

$$y_i[k] = \sum_{n=1}^{N}\phi_i[n]\sum_{m\in\mathbb{Z}}d[m]r_{sa}[n+k\tau-m],$$

which, by applying (5), results in:

$$y_i[k] = \sum_{n=1}^{N}\phi_i[n]c_k[n].$$

Here, $c_k[n]$ are samples of the conventional SI setting in the k-th interval. The proposed measurement model discretizes an inherently infinite-dimensional model in an exact way. Note that $\{c_k[n]\}_{n=1}^N$ is finite and exact even for infinite-dimensional sequence $r_{sa}[n] \in \mathbb{R}^\infty$.

After finding the mathematical model of a measurement for a single channel, we can write the measurement procedure of the whole system in a matrix form:

$$\begin{bmatrix} y_1[k] \\ \vdots \\ y_M[k] \end{bmatrix} = \begin{bmatrix} \phi_1[1] & \cdots & \phi_1[N] \\ \vdots & \vdots & \vdots \\ \phi_M[1] & \cdots & \phi_M[N] \end{bmatrix} \cdot \begin{bmatrix} c_k[1] \\ \vdots \\ c_k[N] \end{bmatrix} + \mathbf{e}_k$$

or

$$\mathbf{y}_k = \mathbf{\Phi}\mathbf{c}_k + \mathbf{e}_k, \tag{10}$$

where $\mathbf{\Phi}$ is an $M \times N$ measurement matrix, $\mathbf{e}_k \in \mathbb{R}^M$ is an unknown error term, and $\mathbf{c}_k$ and $\mathbf{y}_k$ are vectors of the SI samples and CS measurements, respectively.

IV. Reconstruction of Signals in SI Spaces from CS Measurements

By assuming that the samples $\mathbf{c}_k$ are sparse in a certain transform basis $\mathbf{\Psi} \in \mathbb{R}^{N\times N}$, the ill-posed system of equations in (10) can be solved by the CS recovery techniques. The SI samples can be represented as $\mathbf{c}_k = \sum_{j=1}^N \alpha_k[j]\psi_j$, where $\{\psi_j\}_{j=1}^N$ are columns of $\mathbf{\Psi}$ and $\{\alpha_k[j]\}_{j=1}^N = \boldsymbol{\alpha}_k$ is a set of expansion coefficients with at most Q nonzero entries, where $Q \ll N$. Thus, the measurements in (10) can be written as:

$$\mathbf{y}_k = \mathbf{\Phi}\mathbf{\Psi}\boldsymbol{\alpha}_k + \mathbf{e}_k = \mathbf{\Theta}\boldsymbol{\alpha}_k + \mathbf{e}_k,$$

where $\mathbf{\Theta}$ is an $M \times N$ sensing matrix that satisfies CS properties [10]. The SI samples can be recovered by solving the *Quadratically-Constrained Basis Pursuit* (QCBP) optimization program:

$$\min_{\boldsymbol{\alpha}_k\in\mathbb{R}^N} \|\boldsymbol{\alpha}_k\|_1 \quad \text{s.t.} \quad \|\mathbf{\Theta}\boldsymbol{\alpha}_k - \mathbf{y}_k\|_2 \le \kappa, \tag{11}$$

where $\boldsymbol{\alpha}_k$ is a vector of coefficients in the transform domain $\mathbf{\Psi}$ and κ is the threshold parameter. Usually, an *a priori* estimate of error $\mathbf{e}_k$ defines κ such that $\|\mathbf{e}_k\|_2 \le \kappa$ [10]. Despite in

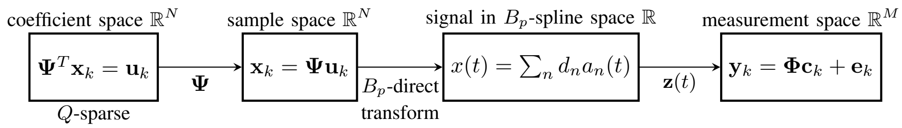


Fig. 4. The coefficient, sample, signal and measurement domains.

many applications an estimate of $\mathbf{e}_k$ is unlikely to be known, the QCBP shows to be quite robust even for $\|\mathbf{e}_k\|_2 > \kappa$ [24].

The right-hand side of the scheme in Fig. 3 shows the reconstruction procedure. Recovered $\boldsymbol{\alpha}_k$, and consequently $\mathbf{c}_k$, are subsequently used to reconstruct the expansion coefficients $d[n]$ of the signal in SI subspace $\mathcal{A}$. The procedure of obtaining $d[n]$ from SI samples $\mathbf{c}_k$ is the same as in conventional SI setting described in Section II. The recovered SI samples $\{\mathbf{c}_k\}_{k=1}^K$ are arranged into a sequence $c[n]$ and filtered by the correction filter $h[n]$ with the DTFT given in (6). In case when the generator $a(t)$ is a B-spline basis function, $H(e^{j\omega})$ can be determined analytically. The filter in (6) is then a non-causal infinite impulse response (IIR) filter. We may therefore apply forward filtering with a causal part and then backward filtering with an anti-causal part of the IIR filter [23]. The impulse response of the IIR filter defined by B-splines has a fast decay and alternatively can be approximated by finite impulse response filter, which allows for a continuous filtering of the samples with a small delay. The obtained coefficients $d[n]$ are modulated by an impulse train with period T and filtered by a corresponding filter $a(t)$ in order to get reconstruction $\hat{x}(t)$.

V. Experimental Results

We validate the performance of the proposed system based on simulations conducted on synthetically sparse signals and real-world signals. A row from a standard test image was taken to represent the samples $\mathbf{x}$ of an input signal. The samples were divided into $K = 5$ intervals with $N = 64$ samples in each of the intervals. To synthetically induce the sparsity, we applied the discrete cosine transform (DCT) to $\mathbf{x}_k \in \mathbb{R}^N$ and set $N - Q$, where $Q = \{6, 10, 13\}$, coefficients in a vector of transform coefficients $\mathbf{u}_k$ to zero. We assume that the analog signal $x(t)$, whose samples are $\mathbf{x}$, lies in a B-spline space spanned by shifts of a generator $a(t)$. For simplicity, we set the period of the functions to $T = 1$. The samples $\mathbf{x}$ are filtered by a direct B-spline transform to obtain expansion coefficients $d[n]$. We compare results of the experiments with the signal chosen to lie in different B_p-spline spaces, $p = 0, \ldots, 3$ (see Fig. 2). By definition, the signal $x(t)$ is equal to the samples $\mathbf{x}$ at time instants n for all B_p-spline representations.

Mixing functions $\{z_i(t)\}_{i=1}^M$, where $M < N$, lie in B_0-spline subspace. Its expansion coefficients $\{\phi_i[n]\}_{i=1}^M$ have random Bernoulli distribution for noiseless acquisition of ideal Q-sparse signals and the Walsh-Hadamard transform (WHT) distribution in other settings. In the latter case, due to high correlation between the low-order frequency components in WHT and DCT bases, we use the multilevel random subsampling scheme [14]. The sampling rate in a single channel is N times lower than the signal information rate $1/T$. Fig. 4 summarizes the process of obtaining measurements $\mathbf{y}_k$ and the relationship between various domains in the simulations. The first two stages in the diagram are omitted when the simulations are conducted on real-world signals.

TABLE I
Z-Transforms of Correction Filters for B_p-Spline Generators

p	0	1	2	3
$H(z)$	1	$\frac{8}{z+6+z^{-1}}$	$\frac{6}{z+4+z^{-1}}$	$\frac{384}{z^2+76z+230+76z^{-1}+z^{-2}}$

SI samples $\mathbf{c}_k$ are recovered from measurements $\mathbf{y}_k$ by solving the optimization program (11). The sensing matrix $\boldsymbol{\Theta}$ is a product of the measurement matrix $\boldsymbol{\Phi}$ and the DCT sparsity matrix $\boldsymbol{\Psi}$. Recovered set of SI samples $\{\mathbf{c}_k\}_{k=1}^K$ are arranged into a sequence and filtered by a correction filter $h[n]$. Z-transform representations of the correction filters for different generators $a(t)$ are given in Table I. All correction filters for B_p-spline generators are symmetric and stable, i.e. the poles are reciprocal and do not lie on the unit circle.

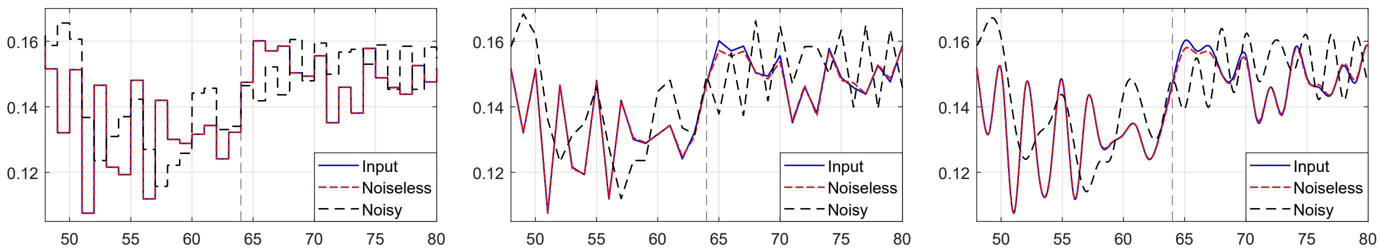
Table II shows the reconstruction quality of the proposed framework for different B-spline representations of the signal and different measurement settings. The reconstruction quality is measured in terms of signal-to-noise ratio (SNR) with omitting the DC value in the calculus. We run 200 simulations for every setting and calculated mean values of SNRs. The best values in the columns are made bold. We validated reconstruction quality for different standard deviations σ_n of additive white Gaussian noise, namely 0%, 5% and 10% of the measurement vector $\mathbf{y}_k$'s standard deviation σ_y. The number of measurements M was 3 times the sparsity for Q-sparse signals. When the generator is B_0-spline, SI samples are equal to expansion coefficients and to samples, i.e. $\mathbf{c}_k$=$\mathbf{d}_k$=$\mathbf{x}_k$, due to the orthonormality of the basis functions. Thus, the SI samples $\mathbf{c}_k$ are ideally Q-sparse in the DCT domain and the reconstruction quality is the highest. However, the equality $\mathbf{c}_k$=$\mathbf{x}_k$ does not hold for other choices of the generator. Nevertheless, the framework has shown to be robust for a variety of settings. Furthermore, we conducted simulations on the original image row that can be seen as asymptotically sparse [14]. The reconstruction quality was assessed for different number of measurements M. For noisy measurements of Q-sparse signals and all measurement setups for asymptotically sparse signals, B_1-spline representation yields the best reconstruction results. Segments of the signals and their reconstructions for different settings are shown in Fig. 5.

VI. Conclusion

We proposed a novel framework for sub-Nyquist sampling of analog signals that lie in a shift-invariant subspace. The

TABLE II
RECONSTRUCTION QUALITY IN TERMS OF SNR IN DECIBELS

	Q-sparse signals									Asymptotically sparse signals											
	Q=6 (10%)			Q=10 (15%)			Q=13 (20%)			M=6 (10%)			M=13 (20%)			M=19 (30%)			M=26 (40%)		
σ_n	0	$\frac{\sigma_y}{20}$	$\frac{\sigma_y}{10}$	0	$\frac{\sigma_y}{20}$	$\frac{\sigma_y}{10}$	0	$\frac{\sigma_y}{20}$	$\frac{\sigma_y}{10}$	0	$\frac{\sigma_y}{20}$	$\frac{\sigma_y}{10}$	0	$\frac{\sigma_y}{20}$	$\frac{\sigma_y}{10}$	0	$\frac{\sigma_y}{20}$	$\frac{\sigma_y}{10}$	0	$\frac{\sigma_y}{20}$	$\frac{\sigma_y}{10}$
B_0	**39.48**	13.74	8.89	**62.20**	12.54	8.56	**64.73**	12.52	8.62	7.61	6.33	4.50	9.84	8.31	6.11	10.86	9.00	6.69	11.60	9.61	7.22
B_1	24.14	**14.02**	**8.93**	27.20	**13.29**	**8.71**	27.90	**13.30**	**8.75**	**8.50**	**6.91**	**4.82**	**11.60**	**9.32**	**6.60**	**13.06**	**10.16**	**7.25**	**14.33**	**10.95**	**7.82**
B_2	28.93	13.89	8.88	38.35	12.92	8.57	43.08	12.90	8.58	7.96	6.56	4.63	10.52	8.71	6.29	11.73	9.47	6.90	12.72	10.16	7.43
B_3	26.93	13.87	8.88	33.37	12.88	8.55	32.10	12.83	8.54	7.95	6.55	4.63	10.53	8.70	6.29	11.72	9.47	6.90	12.71	10.16	7.43



(a) Signal lies in the B_0-spline space. SNR is 64.11 dB for reconstruction from noiseless and 10.12 dB from noisy measurements.

(b) Signal lies in the B_1-spline space. SNR is 34.53 dB for reconstruction from noiseless and 10.45 dB from noisy measurements.

(c) Signal lies in the B_3-spline space. SNR is 38.73 dB for reconstruction from noiseless and 10.01 dB from noisy measurements.

Fig. 5. Same segments of reconstructions for different generators $a(t)$. Reconstructions from noiseless and noisy measurements are shown in each of the subfigures. The input signal is Q-sparse with $Q = 10$. Gaussian noise with standard deviation of 10% of $\mathbf{y}_k$'s standard deviation is added to $\mathbf{y}_k$. Random Bernoulli measurement matrix is used in noiseless and WHT measurement matrix in noisy setups. The subsampling ratio M/N is $30/64$ in all simulations.

signal is successfully recovered by a proposed combination of solving the compressive sensing ℓ_1 optimization program and the discrete-time correction filtering that is characteristic for the shift-invariant setting. We have shown that the proposed framework can efficiently be implemented in B-spline spaces providing robust reconstructions. The results could directly be extended to higher dimensions that way making a powerful tool for block-based compressive imaging.

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