Complex FIR Digital Filter Sharpening With Three-Path Structures

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***Abstract*—The Kaiser & Hamming two-path sharpening structure has long been the primary processing arrangement for using a filter's coefficient set to improve its own performance. Here we study the three-path structure version they proposed, and also introduce a new such structure that makes use of prototype pairing. This expansion, in which we make use of conjugate-reversal of the coefficient set, brings about greatly increased sharpening applicability. Our new structure joins two recently-introduced two-path structures in comparison exercises here. Three of these four offer the option of linear-phase sharpened resultants and comprise what we believe to be the most versatile FIR sharpeners available for effective flattening of two-level gain plateaus in multiband filtering scenarios. All four are compared in complex-valued impulse response example settings, and we arrive at clear ranking of their merits.**

Keywords—filter sharpening, complex FIR filters, frequency response, sharpening polynomials.

I. Introduction

Digital filter sharpening offers a very useful, easily-applied, "quick-look" facility for exploring improvements for any existing filter that concentrates gain at two levels: zero and unity [1]. Flat-top targets with interlaced passbands and stopbands are of interest, alongside the traditional mainstays: lowpass, highpass, bandstop, bandpass filters. The goal of sharpening is to reduce peak errors in passbands and to increase peak attenuation in stopbands, thereby compacting transition bands. Other considerations – such as specification of resulting impulse response symmetry patterns – cannot in general be guaranteed in sharpening processes.

Sharpening involves repeated usage of only the original prototype's coefficient set and has earned its place as "a valued and natural addition to the DSP person's toolbox" [2]. A sharpener's first step toward "partial compensation" of any gain shortcomings of the given prototype is to cascade a repeated instance of that filter. Of course the stopbands' deviations from zero become smaller but the passbands' deviations from unity become larger. The "Multi-Path Section" must next reduce the ripple levels of the passbands without compromising the stopband enhancements achieved by the cascaded prototype.

Kaiser and Hamming in [1] introduced and emphasized a two-path structure for doing this which has long been the focus of attention for sharpening activity in the DSP community. They also devised a family of higher-order structures to allow for greater sharpening potency. It is their Three-path version in Fig. 1 which we choose for starting off the discussion in this paper.

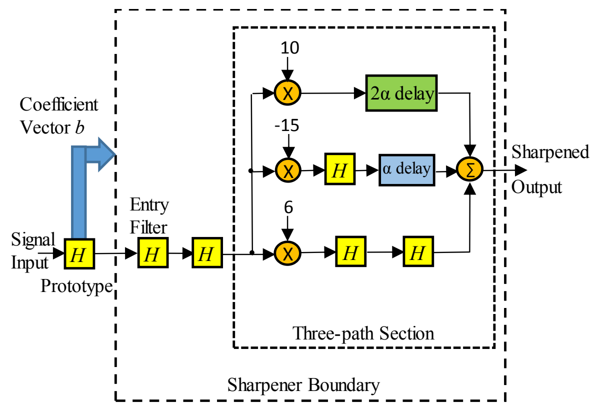

Fig.1 Kaiser & Hamming ("K&H3") Three-path Sharpener

We designate this structure the "K&H3" sharpener. (The widely-used simpler "K&H" standard two-path version can be viewed in [3] and an example of K&H3 usage can be seen in [4]). Its capability comes at a cost of increased computational burden, amounting to **5*N*–4 equivalent coefficients for any *N*-coefficient filter** explored.

It is our main objective in this paper to also introduce a new alternative three-path sharpener, and to exercise both in comparison with a couple of two-path structures, paying particular attention to usability issues and to any benefit computationally heavier structures may bring.

We exercise all four methods in examples with FIR filters having complex-valued impulse responses. This scenario is perhaps especially advantageous for application of sharpening in view of the relative shortage of complex filter design tools - as contrasted with the mature range of real-filter design facilities available [5], [6].

II. The New CHY3 Three-path Sharpening Structure

We start by clarifying our notation. All our prototype filters are characterized by impulse response sequences $h(k)$ indexed with k=0,1,..(N–1) and transfer function

$$H(e^{j2\pi v}) = \sum_{k=0}^{N-1} h(k)\, e^{-j2\pi k v} \qquad (1)$$

where normalized frequency ν, in its primary period, ranges from –0.5 to +0.5, and "gain" is the absolute value of (1). We speak of filter coefficient vector b, composed of that same $h(k)$ impulse response sequence. And, linear-phase filters have group delay

$$\tau_h(\nu) = (N-1)/2 = \alpha \,. \qquad (2)$$

Now in Fig. 2 we introduce our new "CYH3" sharpener:

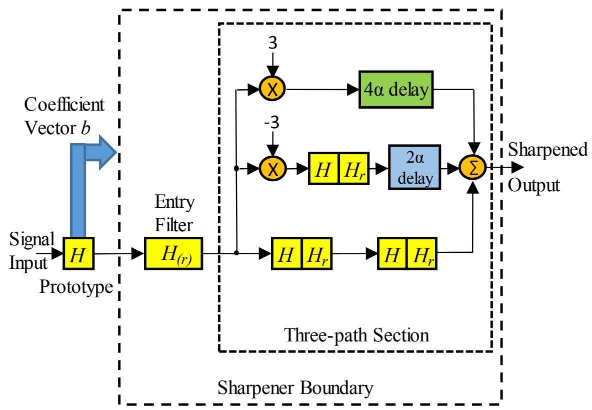

Fig. 2 The CYH3 Three-Path Conjugate-Reversal FIR Sharpener

Phase plays a pivotal role in sharpening and we have embedded shorthand information about that in the block symbols given in Fig. 2:

H : The original prototype filter supplying all coefficient data to the process, as well as passing on its signal output; three such duplicates are also located inside the Three-path Section.

H_r : A duplicate which has undergone conjugate-reversal of its "b" coefficient vector.

$H_{(r)}$: An "Entry Filter" duplicate which is software-switchable between H and H_r states.

Of course the pairing in the Three-path Section locks in matched filter couplings, yielding a linear-phase construct. Since 2α is integer-valued for any prototype, all Three-path branches are perfectly delay-aligned. If we wish to deliver linear-phase overall, then we also switch the Entry Filter to the conjugate-reversal state [in MATLAB, via the single simple command conj(fliplr(b))]. Otherwise our style of sharpening delivers a mixed-phase resultant. For CYH3 **the total FIR equivalent filter length amounts to 6*N*-5**.

Another sharpener with "cost" **6*N*-5** that we use is **"*harris2*"**, which is a recently-modified "*harris*" sharpener, originally advanced for use with IIR filters having even-symmetric transfer function numerators. Prior to [3] *harris* seems to have been the only IIR sharpener with published performance results [2]. The *harris2* variant incorporates conjugate-reversal modifications (as per Fig. 2) in its own distinctive "super-prototype" two-path structure, upgrading that to all-filter capability.

The fourth candidate we use here is called our "CYH", reported in detail in [3], [7], and [8]. Table I provides a handy summary of the sharpening methods under study here.

To give a feel for the sort of multiband, two-level band-flattening action that is sought through sharpening, Fig. 3 shows an erratic, crude gain target prototype (in black). Focusing just near frequency –0.3 we see *harris2* suffering a major gain **down**swing (while CYH dips only modestly) and meanwhile CYH3 and K&H3 have manifested **up**swings in harmony with the prototype. Only the unity-level plateaus in the vicinity of frequency 0.11 and the nulls (most notably due to *harris2*) near d.c. and minus Nyquist frequency exhibit much flatness.

TABLE I. FEATURES OF FOUR FIR SHARPENERS

Sharpener	Number Equiv. Coeffs.	Corridor Low Edge	Corridor High Edge	Limitations on Prototypes
CYH	4*N*–3	0.618	1.2056	none
K&H3	5*N*–4	0.5	1.2638	*N*-odd even-symmetric *h*
harris2	6*N*–5	0.89	1.0708	none
CYH3	6*N*–5	0.3894	1.2888	none

The defining Sharpening Polynomials orchestrate all this detail, and the tradeoffs in flattening effectiveness are predictable once the User makes the choice of which sharpening scheme to employ. No other parameters need to be adjusted, and no additional decisions face the User.

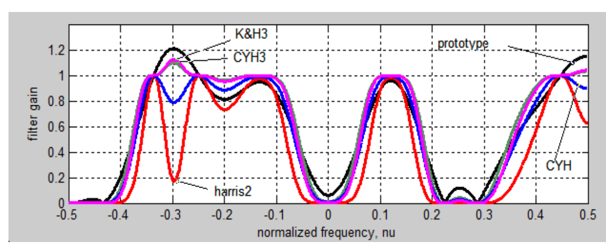

Fig. 3 Crude *N*=25 FIR complex multiband prototype filter subjected to processing by four sharpeners

III. INFLUENCE OF THE SHARPENING POLYNOMIALS

Set out in [9] is a family of "sharpening polynomials" (for paired prototypes) that complements the family furnished in [1]. Since these sorts of polynomials underpin sharpening strategies and structures, [9] opened up an alternative way of sharpening, using (in the parenthetical terms) only even powers of transfer function magnitude. This is used in all here except K&H3, and it is odd-power usage that underlies the applicability limitations afflicting K&H3:

$$\text{CYH: } P_{CYH} = |H|^2\,(2 - |H|^2) \qquad (3)$$

$$\text{K\&H3: } P_{K\&H3} = |H|^3\,(10 - 15\,|H| + 6\,|H|^2) \qquad (4)$$

$$\textit{harris2}\text{: } P_{harris2} = |H|^4\,(3 - 2\,|H|^2) \qquad (5)$$

$$\text{CYH3: } P_{CYH3} = |H|^2\,(3 - 3\,|H|^2 + |H|^4) \qquad (6)$$

(Here we suppress mention of frequency considerations by defining the symbol $|H| = |H(e^{j2\pi v})|$).

Although we have encountered only four, there is a broad variety of such polynomials which can be brought into service; the authors of [10] introduce two in addition to K&H, in [11] Chebyshev polynomials are utilized, piecewise-linear polynomials are studied in [12], and so forth.

Such polynomials act on filter transfer functions to flatten gain values in the region of zero and unity, so have the common requirement of equalling unity at argument value one, and zero at the origin. Also there must be a leading multiplier term with $|H|^2$ (representing the prototype and the Entry Filter) as a factor for all the structures we consider "standard" in this paper. Also we take the opportunity to use the nomenclature "Magnitude Change Function" ("MCF") when referring to the type of sharpening polynomial we use [3], [8]. Fig. 4 shows MCFs (3) - (6) and much can be learned from them.

Since all curves lie above that for *harris2*, any passband ripple excursion permitted by the remaining three will nestle closer to the unity gain plateau than *harris2* will allow, making them always more potent for passband flattening. But the argument for stopband attenuation reverses the story: *harris2* will inevitably excel there.

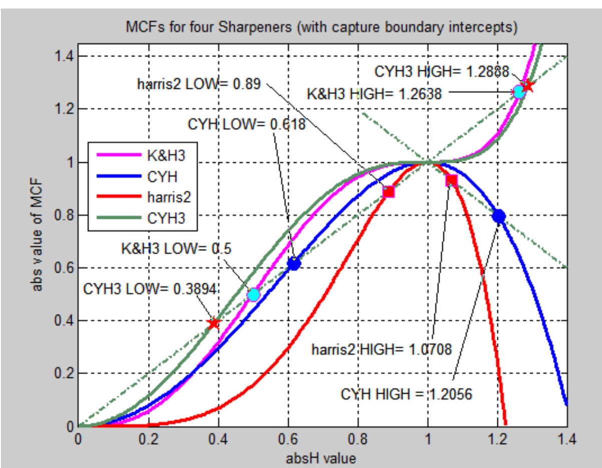

Fig. 4. Magnitude Change Functions for four sharpeners

Important MCF aspects are the four specially-labelled coordinate pairs where the dotted lines delineating linear ramp crossovers occur. We interpret these as critical points where gain values will be driven either toward intended or unintended gain plateaus. All this brings home strongly the message that **sharpening is a nonlinear operation**. It can be overdriven, if the crudity of the prototype is too extreme. The truth of this assertion stands out starkly: "…sharpening makes good filters better, but bad filters worse" [13].

Thanks to the intercepts given in Fig. 4, we are in a position to gauge whether or not a given prototype will be receptive to worthwhile sharpening. We proceed to sharpening only if the prototype fits within one of the <u>Passband Acceptance Corridors</u> shown in Fig. 5:

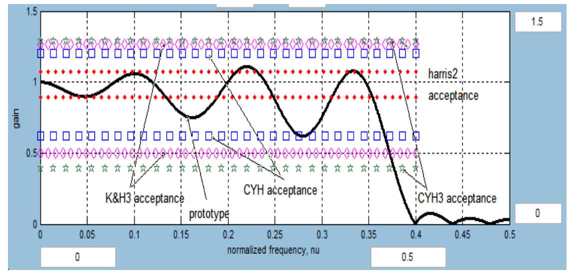

Fig. 5 All four Passband Acceptance Corridors

Once inside an Acceptance Corridor, improvement is certain, achieving ripple compression in both stop and pass bands. Visual inspection rarely does justice to just how gigantic and **disproportionate** the taming of peaks can be, in relative terms, for filters already of good quality.

IV. Complex FIR Sharpening Examples

Example 1:

In this Example we want to avoid the passband distortion effects seen earlier in Fig. 2, so we ensure that our prototype can be engaged by all our sharpeners. Thus, we want an *N*-odd linear-phase prototype comfortably inside the Passband Acceptance Corridors of all four sharpeners. Since we will be designing an equiripple filter then our most severe corridor limit of 1.0708 is set by the *harris2* sharpener.

Generating our *N*=19 prototype by the MATLAB call **[b, err] = cfirpm(18, [–0.5 –0.25 –0.2 0.35 0.4 0.5]*2, @lowpass)**, we find a peak err value of 0.0698, which gives a satisfactorily low starting point. Fig. 6 displays the sharpening results.

We see, as expected, the most impressive stopband attenuation - by far - from *harris2*, while at the same time the least desirable passband behavior. The best passband flattening is turned in jointly by K&H3 and CYH3 (with CYH3 being marginally the better stopband performer of those two), while a solid compromise position in both pass- and stopbands is occupied by CYH.

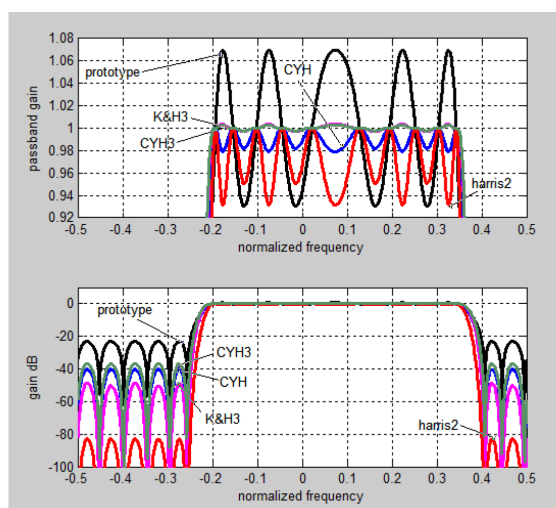

Fig. 6 Effects of 4 sharpeners on an *N*=19 complex lowpass filter (peak passband ripple = 0.0698; stopband attenuation: 23.11 dB)

Table II gives relevant measurements, with "PPIR" (Peak Passband Improvement Ratio) signifying the before-to-after ratio of that performance quality factor and "Additional Attenuation" indicating how many dB of stopband attenuation have been picked up through sharpening:

TABLE II. EXAMPLE 1 QUALITY IMPROVEMENTS

Sharpener	Peak Passband Error	PPIR	Stopband Attenuation (dB)	Additional Attenuation (dB)
CYH	0.0209	3.34	40.22	17.11
K&H3	0.0038	18.37	48.54	25.43
harris2	0.0688	1.01	82.92	59.81
CYH3	0.0031	22.52	36.78	13.67

To examine these relative positionings as the initial prototype quality is made better, we simply sweep a range of prototype N values. The results are given in Fig. 7 where we see staggering improvements in the passband by both CYH3 and K&H3. Naturally, *harris2* convincingly dominates the stopband arena.

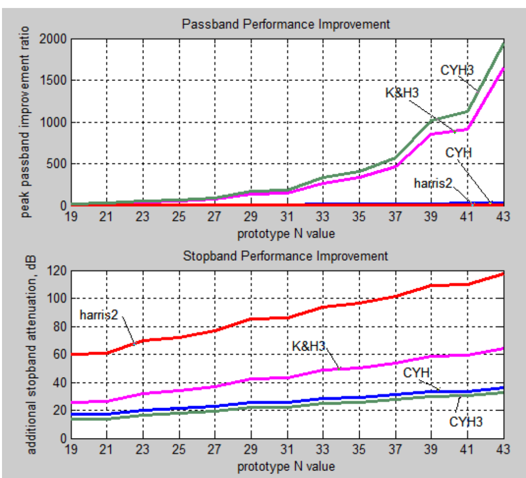

Fig. 7 Passband and stopband relative improvement versus increasing complex lowpass prototype length

Example 2:

Next we investigate complex Hilbert filter sharpening. It is very interesting to note that K&H3 (and K&H before it) are not capable (without modification) of sharpening these, even if they have odd length (which is itself not Hilbert-favorable!) and despite the fact that they possess linear phase. This is due to the fact that their sharpening polynomial does not cater for an imaginary quantity once the prototype's transfer function has been relieved of its linear phase. The authors of [1] were very careful to warn of this, specifying that they could only engage **even-symmetric nonrecursive** filter prototypes (thereby ruling out half the class of linear-phase FIR).

We must understand that being able to undertake sharpening on a given prototype is the vital first step in its improvement does not guarantee that the sharpened resultant will be delivered with any particular, well-controlled phase condition. Sharpening is concerned with flattening of gain bands, and any conformance of post-sharpening phase with any chosen target requires supplementary efforts that may not succeed.

Dealing with real-valued impulse responses having odd mid-array symmetry is one important area of difficulty, with CYH, CYH3 and *harris2* offering only even-symmetric resultant impulse responses when they are employed with linear-phase options activated. Thus a real-valued Hilbert filter will lose its 90-degree phase-shift nature when treated to such CYH, CYH3 or *harris2* sharpening, but will improve its gain shape markedly.

This might in some applications – such as noise suppression – give acceptable service (and is much preferable to use of K&H3, which results in a wildly-shaped gain plot). With this "impairment of functional nature" in mind, we can go on to create a complex Hilbert specimen for examination.

To construct our test prototype we first obtain a real-valued Hilbert filter via the MATLAB command

[h, err] = firpm(15, [0.05 0.49]*2, [1 1],'hilbfilt');

and noting that err = 0.0424 satisfies the acceptance corridor limits of all our sharpeners, we then frequency-shift the coefficient set such that d.c. is transported to $\nu = -0.1$, giving us a complex-valued impulse response. Since we had to disqualify K&H3, Fig. 8 shows only the three remaining sharpeners, with CYH3 easily outclassing all the others with respect to passband performance.

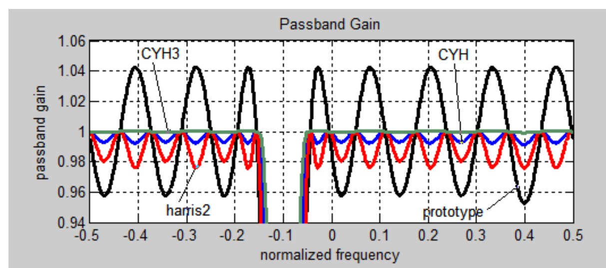

Fig. 8 Three sharpenings of N=16 equiripple Hilbert filter;
peak passband ripples: CYH3: 7.97e–4; CYH: 0.0086; *harris2*: 0.0242

Example 3:

In this Example we investigate the retention of group delay after sharpening. The initial design is an N= 35 complex bandstop filter, which possesses the visually distinctive group delay shape shown in black at the very bottom of Fig. 9. The question is, would there be any damage to the group delay information even if our collection of sharpeners disrupts the passband shaping brutally?

Excluding K&H3 from participation again, we see the familiar passband postures. All sharpeners succeed in preserving the group delay shaping perfectly. This is due to the setting of the option (available on each of these sharpeners) that kept all switchable prototype replicas toggled to their native phase conditions.

Therefore the group delay is a biased doubling for CYH and CYH3 while *harris2* has a biased quadrupling of initial prototype group delay. We easily find three simple equations that recover the original prototype group delay $\tau_{prot}(\nu)$:

$$\tau_{prot}(\nu) = \frac{1}{2}[\tau_{CYH}(\nu) - 2\alpha] \quad (7)$$

$$\tau_{prot}(\nu) = \frac{1}{4}[\tau_{harris\,2}(\nu) - 2\alpha] \quad (8)$$

$$\tau_{prot}(\nu) = \frac{1}{2}[\tau_{CYH3}(\nu) - 4\alpha]. \quad (9)$$

Applying these, every one of the various colored plots in the lower subplot of Fig. 9 falls perfectly on top of the black plot, providing a remarkable degree of coincidence. This (along with other more extreme experiments we have conducted) suggests that group delay is shielded from even the heavy gain shape distortion that sharpening outside designated passband acceptance corridors is sure to cause.

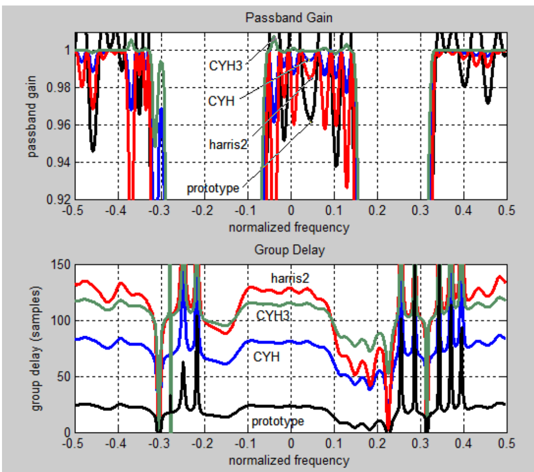

Fig. 9 N=35 Complex bandstop FIR filter treated with three sharpeners

V. Conclusion

Three sharpeners (the newly-introduced CYH3, CYH, and the recently upgraded *harris2*) have shown themselves able in principle to sharpen any FIR filter with complex-valued impulse response. The key proviso is that any candidate prototype's passband gain excursion limits fall within the amplitude interval (0.89, 1.0708). Should the passband bounds extend beyond (0.3894, 1.2888) then processing by any one of these sharpeners will lead to deterioration, rather than enhancement, of band-flattening.

We find it convenient to think about the effectiveness of this sharpening trio (ordered as above) with this string of adjectives: "smooth", "moderate", and "deep". By this we allude to CYH3's strength in flattening passbands, CYH's solidity as a balanced compromise selection, and the superb depths of stopbands achieved by *harris2*.

Although - in sharpening - group delay considerations are of secondary importance, all three methods offer two useful options. Linear phase can be delivered; alternatively, delivery is of group delay strongly reflecting (the shape of) the prototype's group delay. The original FIR filter's group delay can always be extracted.

We specifically highlighted three-path sharpeners in order to bring more dimensionality into play. In the case of the new CYH3 sharpener, this proved to be an important advance. K&H3, however, can be expected to perform very close to CYH3 (and at a slightly reduced cost of 5N-4), but only for a greatly restricted class of prototypes.

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