DISTRIBUTED EXTENDED OBJECT TRACKING BASED ON DIFFUSION STRATEGY

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***Abstract*—In this work, we study the problem of ellipse extended object tracking with multiple measurements. We propose a distributed extended object tracking algorithm for heterogeneous networks based on the diffusion extended Kalman filter. We use a set of nodes with different parameters to estimate the kinematic state and extension of the extended object simultaneously. Simulation results verify that the proposed distributed approach could outperform the method without cooperation .**

***Index Terms*—object tracking, extended object, extended Kalman filter, diffusion strategy, distributed network**

I. Introduction

The object tracking is vital for many applications in areas such as navigation, robotics, etc [1], [2]. However, with the increasing sensor resolution capabilities, it becomes less valid to consider the object as a point. Recently, the extended object tracking has become attractive, which is potentially used in emerging fields such as autonomous driving and maritime surveillance [3]–[8].

For the extended object, not only the kinematic state such as position, velocity and acceleration, but also the extension including the orientation, size and shape of the object is to be estimated. And the kinematic state and the extension of the object are decoupled in [3], [5]. Several approaches using random matrix for the ellipse object tracking are proposed in [3]–[5]. As demonstrated in [9], an approach based on multiplicative noise could outperform other approaches in [4], [5] with the assumption of nearly static semi-axes.

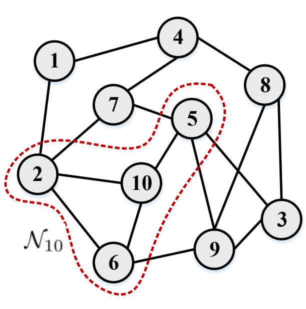

Fig. 1: topology

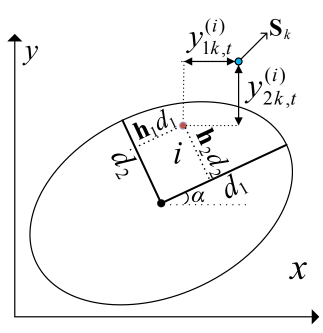

Fig. 2: measurement model

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To improve the tracking performance, we consider herein using multiple nodes to obtain more measurements. Over the past few years, various distributed algorithms in wireless sensor networks have been developed [10]–[12], where the scalability and robustness are desirable. Several algorithms for point object tracking based on distributed networks have been proposed in [13]–[17]. However, the aforementioned distributed methods for point target tracking can not be directly applied to extended object tracking.

In this work, we will fill this gap. We propose a distributed algorithm for the ellipse extended object tracking based on the diffusion strategy with heterogeneous networks. We do not impose the restriction that all nodes obtain the same estimation. We utilize a set of nodes (see Fig. 1) with different parameters and each node can obtain multiple measurements, where the measurement model and ellipse parameterization is illustrated in Fig. 2. We simultaneously process data from neighbors of each node to estimate the kinematic state and the extension of the object. Our solution is appropriate for the problem where new measurements are being taken in real time.

We compare the proposed algorithm with the extended object tracking Kalman filter and centralized method with the algorithm proposed in [9]. The simulation results verify that the proposed distributed method could significantly improve the estimate performance compared to the method without cooperation and the performance of that is close to centralized method with cooperation at each node.

II. Problem formulation

A. Dynamic Model

We consider the single ellipse extended object as in [3], [9]. The state translation equation is given by

$$\begin{aligned} \mathbf{r}_{t+1} &= \mathbf{A}_t^{\mathbf{r}} \mathbf{r}_t + \mathbf{w}_t^{\mathbf{r}}, \\ \mathbf{p}_{t+1} &= \mathbf{A}_t^{\mathbf{p}} \mathbf{p}_t + \mathbf{w}_t^{\mathbf{p}}. \end{aligned} \tag{1}$$

The parameters of extended object to be estimated consist of the kinematic state $\mathbf{r}_t = \left[\mathbf{x}_t^T \ \dot{\mathbf{x}}_t^T\right]^T$ and shape parameters (extension) $\mathbf{p}_t = [\alpha \ d_1 \ d_2]^T$ at time t, where $\mathbf{x}_t \in \mathbb{R}^2$ specifies the center of the object, $\dot{\mathbf{x}}_t$ denotes the derivative of $\mathbf{x}_t$, α, d_1 and d_2 denote the orientation, semi-major and semi-minor axes of the object, respectively. $\mathbf{A}_t^{\mathbf{r}}$ and $\mathbf{A}_t^{\mathbf{p}}$ denote the process matrices of the kinematic state and extension, respectively. $\mathbf{w}_t^{\mathbf{r}}$ and $\mathbf{w}_t^{\mathbf{p}}$ are the zero-mean Gaussian white noise vectors with the covariances $\mathbf{C}^{\mathbf{rw}}$ and $\mathbf{C}^{\mathbf{pw}}$, respectively.

B. Measurement Model

In a distributed network $\mathcal{N}$ with N nodes, we say that two nodes are connected if they can communicate directly with each other. Each node is connected to itself. The neighborhood of node k, denoted by $\mathcal{N}_k$, consists of a set of nodes which are connected to it (i.e., $l \in \mathcal{N}_k$, see Fig. 1). The number of neighbors (degree) of node k is denoted by n_k.

We assume that the scattering sources are evenly distributed on the surface of the object [6], [18], [19]. We further assume herein that the fluctuant number of the scattering sources obtained at each node is independently drawn from the Poisson distribution with possibly different means. We denote the number of scattering sources obtained at each node k at time t by $m_{k,t}$.

Since the locations of scattering sources are not explicitly estimated, we consider modeling the target extent with the multiplicative noise proposed in [20]–[22]. We denote the position of node k as $\mathbf{o}_k = [o_{k1}\ o_{k2}]^T$. The measurement from scattering source i at node k is given by

$$\mathbf{y}_{k,t}^{(i)} = \mathbf{H}_{k,t}\mathbf{r}_t + \mathbf{S}_t\mathbf{h}_{k,t}^{(i)} - \mathbf{o}_k + \mathbf{v}_{k,t}^{(i)},\ i = 1, 2, ..., m_{k,t}, \quad (2)$$

where $\mathbf{y}_{k,t}^{(i)} = \left[\mathbf{y}_{1k,t}^{(i)} \quad \mathbf{y}_{2k,t}^{(i)}\right]^T$ denotes the position relationship between the scattering source i and node k at time t, see Fig.2. $\mathbf{H}_{k,t}$ is the measurement matrix at node k and picks the object position out of the kinematic state at time t as [9], with

$$\mathbf{H}_{k,t} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (3)$$

The matrix $\mathbf{S}_t$ specifies the orientation and the axis lengths of the ellipse extended object,

$$\mathbf{S}_t \triangleq \begin{bmatrix} \mathbf{S}_{1t} \\ \mathbf{S}_{2t} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}, \quad (4)$$

where $\mathbf{S}_{1t}, \mathbf{S}_{2t} \in \mathbb{R}^{1\times 2}$. The multiplicative noise vector $\mathbf{h}_{k,t}^{(i)}$ at each node k and at time t is give by

$$\mathbf{h}_{k,t}^{(i)} = \left[h_{1k,t}^{(i)}\ h_{2k,t}^{(i)}\right]^T, \quad (5)$$

which specifies the spread of the scattering sources on the object. Following [6], [9], we approximate the multiple noise $\mathbf{h}_{k,t}^{(i)}$ as the zero-mean Gaussian distribution with the covariance $\mathbf{C}^{\mathbf{h}} = \frac{1}{4}\mathbf{I}_2$ in order to match an elliptic uniform spatial distribution. The entries of $\mathbf{h}_{k,t}^{(i)}$ are assumed to be independent of $\mathbf{r}_t$, $\mathbf{S}_t$ and $\mathbf{v}_{k,t}^{(i)}$ [20]. In addition, we further assume that the multiple noises $\mathbf{h}_{k,t}^{(i)}$ at each node k are independent of those of other nodes. $\mathbf{S}_t\mathbf{h}_{k,t}^{(i)}$ denotes the relative position between the scattering source i and the center of ellipse object observed at node k. The additive measurement noise $\mathbf{v}_{k,t}^{(i)}$ at node k is temporally white Gaussian with zero-mean and the covariance matrix $\mathbf{C}_k^{\mathbf{v}}$, and is spatially uncorrelated with $\mathbf{v}_{l,t}^{(i)}$ at node l, $k \neq l$. We assume that the measurements $\mathbf{y}_{k,t}^{(i)}$ obtained from different scattering sources of the same node k are uncorrelated.

In distributed network, each node k only exchanges its information with its direct neighbors. We collect the measurements from all $m_{k,t}$ scattering sources at each node k at time t, and define the measurement as

$$\mathbf{Y}_{k,t} \triangleq \left[\mathbf{y}_{k,t}^{(1)} \quad \mathbf{y}_{k,t}^{(2)} \quad \cdots \quad \mathbf{y}_{k,t}^{(m_{k,t})}\right]. \quad (6)$$

Let us continue to assume that the collection $\mathbf{Y}_{k,t}$ of measurements at each node k is uncorrelated with that at node l in the network, with $k \neq l$. We collect the measurements of all the neighbors of node k

$$\mathbf{Z}_{k,t} \triangleq \left[\mathbf{Y}_{l_1,t} \quad \mathbf{Y}_{l_2,t} \quad \cdots \quad \mathbf{Y}_{l_{n_k},t}\right] \in \mathbb{R}^{2\times b_{k,t}}, \quad (7)$$

where $l_1, l_2, \cdots, l_{n_k}$ denote the indices of the neighbors of node k, and the aggregate number of scattering sources of the neighborhood of node k is defined as

$$b_{k,t} \triangleq \sum_{l\in\mathcal{N}_k} m_{l,t}. \quad (8)$$

Obviously, the jth column of $\mathbf{Z}_{k,t}$, $\mathbf{z}_{k,t}^{(j)}$ and $j = 1, 2, ..., b_{k,t}$, corresponds to one of the measurements of scattering sources at one of the neighbors of node k.

III. Distributed Extended Object Tracking

We now develop a fully distributed extended object tracking algorithm based on the extended Kalman filter and the diffusion strategy [13], [15], [17]. In this framework, each node could simultaneously estimate both the kinematic state and extension of the same object and those nodes in the network should obtain estimates that are close to the global solution which is calculated by the fusion center with measurements from all nodes.

We consider herein the adaptive-then-combination (ATC) strategy [10], [11], [15], [17], [23] and another strategy is Combine-then-Adapt (CTA) diffusion algorithm [24]. We first obtain the intermediate estimates $\hat{\boldsymbol{\psi}}_{k,t}$ and $\hat{\boldsymbol{\phi}}_{k,t}$ of the kinematic state and extension and their corresponding covariance matrices of intermediate estimation errors $\mathbf{C}_{k,t}^{\mathbf{r}}$ and $\mathbf{C}_{k,t}^{\mathbf{p}}$ in adaption step at each node k by sequentially incorporating the information from its neighbors. The information to be exchanged with the neighbors at each node in this step includes the measurement matrix of each node $\mathbf{H}_{k,t}$, the measurements $\mathbf{Y}_{k,t}$, multiple and additive noise terms, i.e., $\mathbf{h}_{k,t}^{(i)}$ and $\mathbf{v}_{k,t}^{(i)}$, from all scattering sources of each node. Then, we obtain the estimates of the kinematic state $\hat{\mathbf{r}}_{k,t}$ and extension $\hat{\mathbf{p}}_{k,t}$ by combining the corresponding intermediate estimates from the neighbors of each node k, i.e., $\hat{\boldsymbol{\psi}}_{l,t}$ and $\hat{\boldsymbol{\phi}}_{l,t}$, for $l \in \mathcal{N}_k$.

A. Localized Adaption

According to the irrelevance between the kinematic state and the extension [9], [21], we can adapt the intermediate kinematic state $\hat{\boldsymbol{\psi}}_{k,t}$, the extension $\hat{\boldsymbol{\phi}}_{k,t}$ and their corresponding covariance matrices of the intermediate estimation errors $\mathbf{C}_{k,t}^{\mathbf{r}}$ and $\mathbf{C}_{k,t}^{\mathbf{p}}$, respectively. Due to the nonlinearity of measurements in (2), we utilize the methodology of extended Kalman filter to adapt the estimate of kinematic state and the corresponding covariance matrix of intermediate estimation error. In addition to, we construct the corresponding pseudo-measurement $\boldsymbol{\eta}_{k,t}^{(j)}$ of each true measurement $\mathbf{z}_{k,t}^{(j)}$ to adapt the extension in Sec. III-A2, due to the lack of measurement information about the extension.

1) Kinematic State Estimation: Without loss of generality, we assume the measurement $\mathbf{z}_{k,t}^{(j)}$ is obtained at node l, $l \in \mathcal{N}_k$, i.e., $\mathbf{z}_{k,t}^{(j)} = \mathbf{y}_{l,t}^{(i)}$. We now approximate the measurement $\mathbf{z}_{k,t}^{(j)}$ using the Taylor's second-order expansion at the point $\left(\hat{\boldsymbol{\psi}}_{k,t}, \hat{\boldsymbol{\phi}}_{k,t}\right)$ which incorporates the measurement $\mathbf{z}_{k,t}^{(j-1)}$ at each node k. While $j = 1$, we approximate the measurement $\mathbf{z}_{k,t}^{(1)}$ at the point $\left(\hat{\boldsymbol{\psi}}_{k,t}, \hat{\boldsymbol{\phi}}_{k,t}\right)$ which is obtained by the time updates of $\hat{\mathbf{r}}_{k,t-1}$ and $\hat{\mathbf{p}}_{k,t-1}$. The measurement $\mathbf{z}_{k,t}^{(j)}$ can be written as

$$\mathbf{z}_{k,t}^{(j)} \approx \mathbf{H}_{l,t}\mathbf{r}_t + \hat{\mathbf{S}}_{k,t}^{(j-1)}\mathbf{h}_{l,t}^{(i)} + \begin{bmatrix} \left(\mathbf{h}_{l,t}^{(i)}\right)^T \hat{\mathbf{J}}_{1k,t}^{(j-1)} \\ \left(\mathbf{h}_{l,t}^{(i)}\right)^T \hat{\mathbf{J}}_{2k,t}^{(j-1)} \end{bmatrix} \left(\mathbf{p}_t - \hat{\mathbf{p}}_{k,t}^{(j-1)}\right) - \mathbf{o}_k + \mathbf{v}_{l,t}^{(i)}, \tag{9}$$

where $\hat{\mathbf{S}}_{k,t}^{(j-1)}$ denotes the estimated matrix incorporating the measurement $\mathbf{z}_{k,t}^{(j-1)}$ at each node k. The Jacobian matrices $\hat{\mathbf{J}}_{1k,t}^{(j-1)}$ and $\hat{\mathbf{J}}_{2k,t}^{(j-1)}$ are respectively given by

$$\hat{\mathbf{J}}_{1k,t}^{(j-1)} = \left.\frac{\partial \mathbf{S}_{1t}}{\partial \mathbf{p}_t}\right|_{\mathbf{p}_t = \hat{\mathbf{p}}_{k,t}^{(j-1)}}, \quad \hat{\mathbf{J}}_{2k,t}^{(j-1)} = \left.\frac{\partial \mathbf{S}_{2t}}{\partial \mathbf{p}_t}\right|_{\mathbf{p}_t = \hat{\mathbf{p}}_{k,t}^{(j-1)}}. \tag{10}$$

The cross-covariance $\mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)}$ between the kinematic state estimate $\hat{\boldsymbol{\psi}}_{k,t}$ and the measurement $\mathbf{z}_{k,t}^{(j)}$ obtained from the neighbors of each node k can be calculated as

$$\mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)} = \mathbf{C}_{k,t}^{\mathbf{r}}\mathbf{H}_{l,t} \tag{11}$$

The covariance matrix $\mathbf{C}_{k,t}^{\mathbf{z}(j)}$ of the measurement $\mathbf{z}_{k,t}^{(j)}$ can be calculated as

$$\mathbf{C}_{k,t}^{\mathbf{z}(j)} = \mathbf{H}_{l,t}\mathbf{C}_{k,t}^{\mathbf{r}}\mathbf{H}_{l,t}^T + \hat{\mathbf{S}}_{k,t}^{(j-1)}\mathbf{C}^{\mathbf{h}}\left(\hat{\mathbf{S}}_{k,t}^{(j-1)}\right)^T + \mathbf{C}^I + \mathbf{C}_l^{\mathbf{v}}, \tag{12}$$

where

$$\left[\mathbf{C}^I\right]_{mn} = \mathrm{tr}\left\{\mathbf{C}_{k,t}^{\mathbf{p}}\left(\hat{\mathbf{J}}_{nk,t}^{(j-1)}\right)^T \mathbf{C}^{\mathbf{h}}\hat{\mathbf{J}}_{mk,t}^{(j-1)}\right\}, m, n \in \{1, 2\}, \tag{13}$$

where $(\cdot)^T$ and $\mathrm{tr}\{\cdot\}$ respectively denote the transposition and the trace of a matrix. We designate the (m,n)th entry of a matrix with the notation $[\cdot]_{mn}$. The update of $\mathbf{C}_{k,t}^{\mathbf{p}}$ is given in (24) in the following subsection.

According to the standard Kalman filter update equation, the update of kinematic state estimate can be written as

$$\hat{\boldsymbol{\psi}}_{k,t} \leftarrow \hat{\boldsymbol{\psi}}_{k,t} + \mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)}\left(\mathbf{C}_{k,t}^{\mathbf{z}(j)}\right)^{-1}\left(\mathbf{z}_{k,t}^{(j)} - \bar{\mathbf{z}}_{k,t}^{(j)}\right), \tag{14}$$

where "$\leftarrow$" denotes sequential or non-concurrent assignment. The prediction $\bar{\mathbf{z}}_{k,t}^{(j)}$ of measurement $\mathbf{z}_{k,t}^{(j)}$ is given by

$$\bar{\mathbf{z}}_{k,t}^{(j)} = \mathbf{H}_{l,t}\hat{\boldsymbol{\psi}}_{k,t} - \mathbf{o}_l. \tag{15}$$

And the covariance of the estimation error of the kinematic state as

$$\mathbf{C}_{k,t}^{\mathbf{r}} \leftarrow \mathbf{C}_{k,t}^{\mathbf{r}} - \mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)}\left(\mathbf{C}_{k,t}^{\mathbf{z}(j)}\right)^{-1}\left(\mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)}\right)^T. \tag{16}$$

TABLE I: Distributed Extended Object Tracking Kalman filter (DEOKF)

Consider the updating process for one span at the node k.
Initialize with $\hat{\mathbf{r}}_{k,0}$, $\hat{\mathbf{p}}_{k,0}$, $\mathbf{C}_{k,0}^{\mathbf{r}}$, $\mathbf{C}_{k,0}^{\mathbf{p}}$ for all nodes.
for $t = 0, 1, \cdots$.

Adaptive update:

1: $\mathbf{Z}_{k,t} \triangleq \left[\mathbf{Y}_{l_1,t} \quad \mathbf{Y}_{l_2,t} \quad \cdots \quad \mathbf{Y}_{l_{n_k},t}\right]$

for $l \in \mathcal{N}_k$ and $j = 1, 2, \cdots, b_{k,t}$, repeat

2: $\mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)} = \mathbf{C}_{k,t}^{\mathbf{r}}\mathbf{H}_{l,t}^T$,

3: calculate the covariance $\mathbf{C}_{k,t}^{\mathbf{z}(j)}$ as (12),

4: $\hat{\boldsymbol{\psi}}_{k,t} \leftarrow \hat{\boldsymbol{\psi}}_{k,t} + \mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)}\left(\mathbf{C}_{k,t}^{\mathbf{z}(j)}\right)^{-1}\left(\mathbf{z}_{k,t}^{(j)} - \bar{\mathbf{z}}_{k,t}^{(j)}\right)$,

5: $\mathbf{C}_{k,t}^{\mathbf{r}} \leftarrow \mathbf{C}_{k,t}^{\mathbf{r}} - \mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)}\left(\mathbf{C}_{k,t}^{\mathbf{z}(j)}\right)^{-1}\left(\mathbf{C}_{k,t}^{\boldsymbol{\psi}\mathbf{z}(j)}\right)^T$,

6: $\boldsymbol{\eta}_{k,t}^{(j)} = \mathbf{F}\left(\left(\mathbf{z}_{k,t}^{(j)} - \bar{\mathbf{z}}_{k,t}^{(j)}\right) \otimes \left(\mathbf{z}_{k,t}^{(j)} - \bar{\mathbf{z}}_{k,t}^{(j)}\right)\right)$,

7: $\bar{\boldsymbol{\eta}}_{k,t}^{(j)} = \mathbf{F}\mathrm{vec}\left(\mathbf{C}_{k,t}^{\mathbf{z}(j)}\right)$,

8: $\mathbf{C}_{k,t}^{\boldsymbol{\eta}(j)} = \mathbf{F}\left(\mathbf{C}_{k,t}^{\mathbf{z}(j)} \otimes \mathbf{C}_{k,t}^{\mathbf{z}(j)}\right)\left(\mathbf{F} + \tilde{\mathbf{F}}\right)^T$,

9: $\mathbf{C}_{k,t}^{\boldsymbol{\phi}\boldsymbol{\eta}(j)} = \mathbf{C}_{k,t}^{\mathbf{p}}\left(\hat{\mathbf{M}}_{k,t}^{(j-1)}\right)^T$,

10: $\hat{\boldsymbol{\phi}}_{k,t} \leftarrow \hat{\boldsymbol{\phi}}_{k,t} + \mathbf{C}_{k,t}^{\boldsymbol{\phi}\boldsymbol{\eta}(j)}\left(\mathbf{C}_{k,t}^{\boldsymbol{\eta}(j)}\right)^{-1}\left(\boldsymbol{\eta}_{k,t}^{(j)} - \bar{\boldsymbol{\eta}}_{k,t}^{(j)}\right)$,

11: $\mathbf{C}_{k,t}^{\mathbf{p}} \leftarrow \mathbf{C}_{k,t}^{\mathbf{p}} - \mathbf{C}_{k,t}^{\boldsymbol{\phi}\boldsymbol{\eta}(j)}\left(\mathbf{C}_{k,t}^{\boldsymbol{\eta}(j)}\right)^{-1}\left(\mathbf{C}_{k,t}^{\boldsymbol{\phi}\boldsymbol{\eta}(j)}\right)^T$,

end

Combine update:

12: $\hat{\mathbf{r}}_{k,t} = \sum_{l \in \mathcal{N}_k} c_{k,l}\hat{\boldsymbol{\psi}}_{l,t}$, $\quad \hat{\mathbf{p}}_{k,t} = \sum_{l \in \mathcal{N}_k} c_{k,l}\hat{\boldsymbol{\phi}}_{l,t}$,

13: $\hat{\boldsymbol{\psi}}_{k,t+1} \leftarrow \mathbf{A}_t^{\mathbf{r}}\hat{\mathbf{r}}_{k,t}$, $\quad \hat{\boldsymbol{\phi}}_{k,t+1} \leftarrow \mathbf{A}_t^{\mathbf{p}}\hat{\mathbf{p}}_{k,t}$,

14: $\mathbf{C}_{k,t+1}^{\mathbf{r}} \leftarrow \mathbf{A}_t^{\mathbf{r}}\mathbf{C}_{k,t}^{\mathbf{r}}\left(\mathbf{A}_t^{\mathbf{r}}\right)^T + \mathbf{C}^{\mathbf{rw}}$,
$\mathbf{C}_{k,t+1}^{\mathbf{p}} \leftarrow \mathbf{A}_t^{\mathbf{p}}\mathbf{C}_{k,t}^{\mathbf{p}}\left(\mathbf{A}_t^{\mathbf{p}}\right)^T + \mathbf{C}^{\mathbf{pw}}$.

end

2) The Extension Estimation: Following [25], we construct the pseudo-measurement irrelevant to the actual measurement with the Kronecker product

$$\boldsymbol{\eta}_{k,t}^{(j)} = \mathbf{F}\left(\left(\mathbf{z}_{k,t}^{(j)} - \bar{\mathbf{z}}_{k,t}^{(j)}\right) \otimes \left(\mathbf{z}_{k,t}^{(j)} - \bar{\mathbf{z}}_{k,t}^{(j)}\right)\right), \tag{17}$$

where $\mathbf{F}$ and $\tilde{\mathbf{F}}$ are used to simplify the presentation

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; \tilde{\mathbf{F}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{18}$$

with $\otimes$ denoting the Kronecker product. The prediction of the pseudo-measurement is written as

$$\bar{\boldsymbol{\eta}}_{k,t}^{(j)} = \mathbf{F}\mathrm{vec}\left(\mathbf{C}_{k,t}^{\mathbf{z}(j)}\right), \tag{19}$$

where $\mathrm{vec}(\cdot)$ denotes vectorizing of a matrix. The update of pseudo-measurement covariance $\mathbf{C}_{k,t}^{\boldsymbol{\eta}(j)}$ can be written as

$$\mathbf{C}_{k,t}^{\boldsymbol{\eta}(j)} = \mathbf{F}\left(\mathbf{C}_{k,t}^{\mathbf{z}(j)} \otimes \mathbf{C}_{k,t}^{\mathbf{z}(j)}\right)\left(\mathbf{F} + \tilde{\mathbf{F}}\right)^T. \tag{20}$$

Refer to [9], the cross-covariance matrix $\mathbf{C}_{k,t}^{\boldsymbol{\phi}\boldsymbol{\eta}(j)}$ between the extension estimate $\hat{\boldsymbol{\phi}}_{k,t}$ and the jth pseudo-measurement $\boldsymbol{\eta}_{k,t}^{(j)}$ is given by

$$\mathbf{C}_{k,t}^{\boldsymbol{\phi}\boldsymbol{\eta}(j)} = \mathbf{C}_{k,t}^{\mathbf{p}}\left(\hat{\mathbf{M}}_{k,t}^{(j-1)}\right)^T, \tag{21}$$

where

$$\hat{\mathbf{M}}_{k,t}^{(j-1)} = \begin{bmatrix} 2\hat{\mathbf{S}}_{1k,t}^{(j-1)}\mathbf{C}^{\mathbf{h}}\hat{\mathbf{J}}_{1k,t}^{(j-1)} \\ 2\hat{\mathbf{S}}_{2k,t}^{(j-1)}\mathbf{C}^{\mathbf{h}}\hat{\mathbf{J}}_{2k,t}^{(j-1)} \\ \hat{\mathbf{S}}_{1k,t}^{(j-1)}\mathbf{C}^{\mathbf{h}}\hat{\mathbf{J}}_{2k,t}^{(j-1)} + \hat{\mathbf{S}}_{2k,t}^{(j-1)}\mathbf{C}^{\mathbf{h}}\hat{\mathbf{J}}_{1k,t}^{(j-1)} \end{bmatrix}. \quad (22)$$

Similar to (14) and (16), the update of extension estimate $\hat{\boldsymbol{\phi}}_{k,t}$ is given by

$$\hat{\boldsymbol{\phi}}_{k,t} \leftarrow \hat{\boldsymbol{\phi}}_{k,t} + \mathbf{C}_{k,t}^{\boldsymbol{\phi\eta}(j)}\left(\mathbf{C}_{k,t}^{\boldsymbol{\eta}(j)}\right)^{-1}\left(\boldsymbol{\eta}_{k,t}^{(j)} - \overline{\boldsymbol{\eta}}_{k,t}^{(j)}\right), \quad (23)$$

And the covariance matrix $\mathbf{C}_{k,t}^{\mathbf{p}}$ of the estimation error of the extension is given by

$$\mathbf{C}_{k,t}^{\mathbf{p}} \leftarrow \mathbf{C}_{k,t}^{\mathbf{p}} - \mathbf{C}_{k,t}^{\boldsymbol{\phi\eta}(j)}\left(\mathbf{C}_{k,t}^{\boldsymbol{\eta}(j)}\right)^{-1}\left(\mathbf{C}_{k,t}^{\boldsymbol{\phi\eta}(j)}\right)^{T}. \quad (24)$$

B. Combination

The purpose of combination is to share and collaboratively improve the estimates of the kinematic state and the extension at each node k. To simplify, herein, we just consider the constant weight for combination. Some possible combination rules are given in [11] and some adaptive combiners are proposed in [24], [26]. We respectively combine the intermediate estimates from the neighborhood $\mathcal{N}_k$ of node k,

$$\hat{\mathbf{r}}_{k,t} = \sum_{l\in\mathcal{N}_k} c_{k,l}\hat{\boldsymbol{\psi}}_{l,t}, \quad \hat{\mathbf{p}}_{k,t} = \sum_{l\in\mathcal{N}_k} c_{k,l}\hat{\boldsymbol{\phi}}_{l,t}, \quad (25)$$

where the non-negative coefficients $\{c_{k,l}\}$ denote the weighting of the neighbors with respect to node k, satisfying $\sum\limits_{l\in\mathcal{N}_k} c_{k,l} = 1$ [11].

According to (1), the time updates of the kinematic state and extension at each node k can be respectively written as

$$\hat{\boldsymbol{\psi}}_{k,t+1} \leftarrow \mathbf{A}_t^{\mathbf{r}}\hat{\mathbf{r}}_{k,t}, \quad \hat{\boldsymbol{\phi}}_{k,t+1} \leftarrow \mathbf{A}_t^{\mathbf{p}}\hat{\mathbf{p}}_{k,t}. \quad (26)$$

We can respectively update the covariance matrices of the estimation error of the kinematic state and the extension as

$$\begin{aligned} \mathbf{C}_{k,t+1}^{\mathbf{r}} &\leftarrow \mathbf{A}_t^{\mathbf{r}}\mathbf{C}_{k,t}^{\mathbf{r}}\left(\mathbf{A}_t^{\mathbf{r}}\right)^T + \mathbf{C}^{\mathbf{rw}}, \\ \mathbf{C}_{k,t+1}^{\mathbf{p}} &\leftarrow \mathbf{A}_t^{\mathbf{p}}\mathbf{C}_{k,t}^{\mathbf{p}}\left(\mathbf{A}_t^{\mathbf{p}}\right)^T + \mathbf{C}^{\mathbf{pw}}. \end{aligned} \quad (27)$$

We summarize the proposed distributed algorithm based on the ATC strategy (DEOKF) at each node k in Table I. At time t, we sequentially iterate each scattering source from all the neighbors of each node k.

IV. SIMULATION EXAMPLES

We evaluate the performance of the proposed DEOKF, the extended object tracking Kalman filter (EOKF), and centralized method using the algorithm (CEOKF) proposed in [9]. We consider the same object example as in [9], where the elliptical object with the major and minor diameters of 340m and 80m moves at a constant speed of 50km/h and goes through a 45° and two 90° turns in sequence, and the sizes of the object are constant during the movement. We consider the distributed network of 10 nodes as illustrated in Fig. 1. The fluctuant number of scattering sources at each node k per time step follows the Poisson distribution with the mean depicted in Fig. 3, and with the mean 10 for the EOKF. For simplicity, we use the uniform coefficients for the combination at each node k:, i.e., $c_{k,l} = \frac{1}{n_k}$ [11]. We assume that the covariance of measurement noise at each node k is a diagonal matrix $\mathbf{C}_k^{\mathbf{v}} = \mathrm{diag}\{100, 50\}$. The process noise covariances matrix of the kinematic state and the extension are set to be $\mathbf{C}^{\mathbf{rw}} = \mathrm{diag}\{100, 100, 1, 1\}$ and $\mathbf{C}^{\mathbf{pw}} = \mathrm{diag}\{0.05, 0.001, 0.001\}$, respectively.

Different from the point object tracking that we evaluate the tracking performance with the mean-squared error [13], we can evaluate that of the extended object by the Gaussian Wasserstein distance (GWD), which incorporates both the estimate errors of the kinematic state and the extension at each node k [9], [27]. All the results presented herein are obtained by averaging over $\mathcal{L} = 1000$ independent trials. The square of the GWD of each node k at time t and in the ℓth trial is defined as

$$\begin{aligned} \mathrm{GWD}_{k,t}^2(\ell) &= \left\|\mathbf{m}_t - \hat{\mathbf{m}}_{k,t}^{(\ell)}\right\|^2 \\ &\quad + \mathrm{tr}\left\{\mathbf{P}_t + \hat{\mathbf{P}}_{k,t}^{(\ell)} - 2\sqrt{\sqrt{\mathbf{P}_t}\hat{\mathbf{P}}_{k,t}^{(\ell)}\sqrt{\mathbf{P}_t}}\right\}, \end{aligned} \quad (28)$$

where $\|\cdot\|$ denotes the 2-norm. $\mathbf{m}_t$ and $\hat{\mathbf{m}}_{k,t}^{(\ell)}$ denote the centroid of the true object and estimated object at time t obtained at each node k in the ℓth trial, respectively. The symmetric and positively definite matrices $\mathbf{P}_t$ and $\hat{\mathbf{P}}_{k,t}^{(\ell)}$ describe the extension of the true object and the estimated object obtained at each node k in the ℓth trial , respectively [3],

$$\mathbf{P}_t = \mathbf{S}_t\mathbf{S}_t^T, \quad \hat{\mathbf{P}}_{k,t}^{(\ell)} = \hat{\mathbf{S}}_{k,t}^{(\ell)}\left(\hat{\mathbf{S}}_{k,t}^{(\ell)}\right)^T. \quad (29)$$

We define the network GWD (NGWD) to evaluate the tracking performance

$$\mathrm{NGWD}(t) = \sqrt{\frac{1}{\mathcal{L}}\frac{1}{N}\sum_{\ell=1}^{\mathcal{L}}\sum_{k=1}^{N}\mathrm{GWD}_{k,t}^2(\ell)}. \quad (30)$$

The trajectory, one typical run of the estimates of the kinematic state and extension as well as measurements at node 1 and 10 are depicted in Fig. 4. We observe that the estimates obtained with the DEOKF are quite close to the true trajectory. Fig. 5 illustrates that the DEOKF significantly outperforms the EOKF, and the performance of DEOKF is comparable to the CEOKF but with fewer scattering sources and lower computation cost at each node. We can achieve similar performance at node 1 with fewer scattering sources to that of other nodes via the distributed strategy. Moreover, according to Fig. 1 and Fig. 3, it is observed in Fig. 5 that the performance of node 1 is superior to that of node 10 since node 1 connects to node 4. It indicates that the proposed approach is promising to enhance the tracking performance of the network by increasing the number of scattering sources of individual node.

V. CONCLUSIONS

We propose a distributed extended object tracking method based on the extended Kalman filter and diffusion strategy for heterogeneous networks. We consider the nonlinear

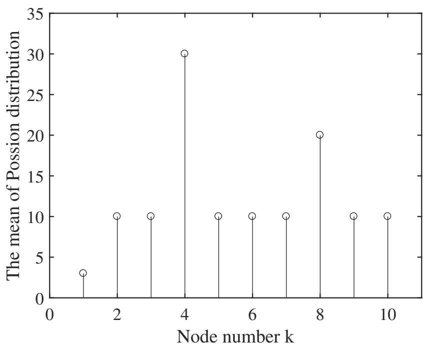

Fig. 3: The mean of fluctuant number of scattering sources at each node

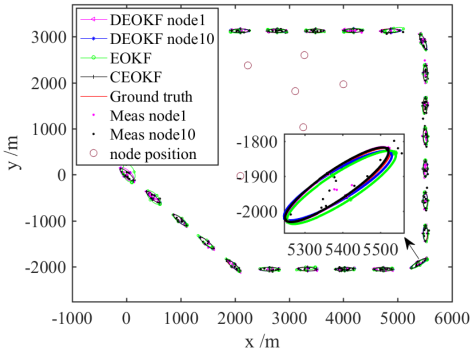

Fig. 4: The true, estimated trajectory and measurements

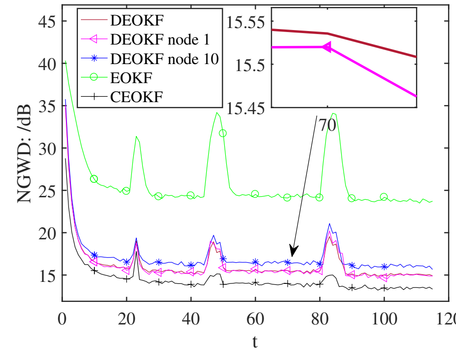

Fig. 5: The Gaussian Wasserstein distance

measurement model of the extension of the extended object. We approximate this nonlinear measurement model by the Taylor's second-order expansion and construct the pseudo-measurement. Each node with different parameters communicates directly with its neighbors by sharing the data and propagating the intermediate estimates. We evaluate the performance with the Gaussian Wasserstein distance which incorporates both the estimate errors of the kinematic state and extension. Our distributed approach could outperform both the extended object tracking method and the centralized counterpart with multiple nodes.

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