Aircraft Acoustic Signal Modeled as Oscillatory Almost-Cyclostationary Process

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***Abstract*—The acoustic almost-cyclostationary signal emitted by a moving aircraft is modeled as an oscillatory almost-cyclostationary process when it is received by a stationary listener. Its autocorrelation function is constituted by the superposition of angle-modulated sinewaves, where the angle modulation is consequence of the time-varying delay due to the relative motion between aircraft and listener. Conditions under which the source almost-cyclostationary signal can be recovered by the received signal by time de-warping are established. Thus, cyclic features of the source signal carrying information on aircraft parameters are estimated by classical cyclic spectral analysis.**

***Index Terms*—oscillatory almost-cyclostationary process, Doppler, time-warping, aircraft acoustic signal**

I. Introduction

Aircraft parameter estimates can be obtained from the acoustic signal emitted by the aircraft [3]. Such estimates are suitable to be exploited for classification if the aircraft characteristics are unknown or for diagnostic and monitoring if these characteristics are known.

The acoustic signal emitted by an aircraft is originated by rotation of rotors and, hence, can be modeled as cyclostationary or, more generally, as almost-cyclostationary (ACS) [1], [4], [12, Sec. 10.6]. That is, statistical functions of the signal, such as mean and autocorrelation, are periodic or almost-periodic functions of time. Such a model has been widely exploited in communications [4], [6], [12, Chap. 9], [14].

The ACS acoustic signal can be collected when the engines are working and the aircraft is stationary in front of a fixed listener. Then, cyclic features such as cycle frequencies, cyclic autocorrelation functions, and cyclic spectra can be consistently estimated by well established techniques for cyclic spectral analysis [4], [12, Chap. 5]. From these cyclic features, estimates of parameters of interest of the aircraft can be obtained [8].

If the aircraft is moving with general motion law with respect to a stationary listener, then the Doppler effect in the received signal is suitably modeled as a time-varying delay in the source signal [9, Sec. 7.1]. Such a time-varying delay modifies the almost-cyclostationarity into a more general kind of nonstationarity [5], [11]. In such a case, classical cyclic spectral analysis techniques cannot be directly exploited.

In this paper, it is shown that in the case of general motion law between aircraft and listener, the received signal can be modeled as oscillatory almost cyclostationary process, a new class of nonstationary processes introduced in [10, Sec. 6], [12, Chap. 14] that generalizes the class of the ACS processes. Moreover, conditions are established under which cyclic features of the acoustic source signal can be recovered.

If the time-varying delay is slowly varying, the autocorrelation function of the received signal is shown to be constituted by the superposition of angle-modulated sinewaves. Starting from this model, a procedure is carried out to estimate the time-varying delay describing the Doppler effect. Such an estimate is then adopted for de-warping the received signal in order to recover an estimate of the ACS source signal. Thus, conventional cyclic spectral analysis is made on this ACS signal.

The signal models and the proposed estimation procedure are tested on the acoustic signal emitted by a B2 Spirit aircraft in two different operating situations. In the first case, the aircraft is slowly maneuvering on the land. Thus, the time-varying delay is due to micro-Doppler effects which are consequence of aircraft vibration, moving parts, and rotation movements. Consequently, the time-varying delay is slowly varying in a large observation interval allowing one to an effective recovery of the source ACS signal and an accurate estimate of its cyclic features. In the second case, the aircraft is taking-off. The time-varying delay accounts for both relative motion between aircraft and listener and micro-Doppler effects. In such a case, it is shown that the time-varying delay can be assumed slowly varying only during a small time interval which leads to a not accurate recovery of the source ACS signal.

The paper is organized as follows. In Section II, oscillatory almost-cyclostationary processes are reviewed. In Section III the model for the aircraft acoustic signal received by a fixed listener is derived in both cases of aircraft slowly maneuvering on the land (Sec. III-1) and aircraft taking-off in front of the listener (Sec. III-2). Numerical results are presented in Section IV. Conclusions are drawn in Section V.

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II. Oscillatory Almost-Cyclostationary Processes

In this section, the second-order statistical characterization of the oscillatory almost-cyclostationary processes introduced in [10, Sec. 6], [12, Chap. 14] is briefly reviewed.

Let $x(t)$ be a second-order harmonizable ACS process [12, Chap. 1] with (conjugate) autocorrelation function

$$\mathrm{E}\left\{x(t+\tau)\,x^{(*)}(t)\right\} = \sum_{\alpha\in A} R_{\boldsymbol{x}}^{\alpha}(\tau)\,e^{j2\pi\alpha t} \tag{2.1}$$

where $(*)$ denotes optional complex conjugation, A is the countable set of the (possibly incommensurate) cycle frequencies, and the Fourier coefficient

$$R_{\boldsymbol{x}}^{\alpha}(\tau) \triangleq \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\mathrm{E}\left\{x(t+\tau)\,x^{(*)}(t)\right\}\,e^{-j2\pi\alpha t}\,\mathrm{d}t \tag{2.2}$$

are the (conjugate) cyclic autocorrelation functions. If $Z(f)$ is the integrated complex spectrum of the ACS process $x(t)$, that is, $x(t)$ admits the Cramér representation

$$x(t) = \int_{\mathbb{R}} e^{j2\pi ft}\,\mathrm{d}Z(f) \tag{2.3}$$

then

$$\mathrm{E}\left\{\mathrm{d}Z(f_1)\,\mathrm{d}Z^{(*)}(f_2)\right\} = \sum_{\alpha\in A}\delta(f_2-_{(-)}(\alpha-f_1))\mathrm{d}\mu_{\boldsymbol{x}}^{\alpha}(f_1)\mathrm{d}f_2 \tag{2.4}$$

where $_{(-)}$ is an optional minus sign linked to $(*)$ and $\mu_{\boldsymbol{x}}^{\alpha}(f)$, $\alpha\in A$, is a family of complex measures. If $\mu_{\boldsymbol{x}}^{\alpha}(f)$ does not contain singular component [2], [7, p. 197], then $\mathrm{d}\mu_{\boldsymbol{x}}^{\alpha}(f) = S_{\boldsymbol{x}}^{\alpha}(f)\,\mathrm{d}f$, where $S_{\boldsymbol{x}}^{\alpha}(f)$ is the (conjugate) cyclic spectrum, that is, the Fourier transform of the (conjugate) cyclic autocorrelation function $R_{\boldsymbol{x}}^{\alpha}(\tau)$.

A second-order harmonizable stochastic process $y(t)$ is said to be *oscillatory almost-cyclostationary (OACS)* with respect to the family of oscillatory functions $\{A_t(f)\,e^{j2\pi ft}\}$ if it admits the representation [10, Sec. 6], [12, Chap. 14]

$$y(t) = \int_{\mathbb{R}} A_t(f)\,e^{j2\pi ft}\,\mathrm{d}Z(f) \tag{2.5}$$

where $Z(f)$ satisfies (2.4) and the modulating functions $A_t(f)$, as functions of t, are in general low-pass functions. The process $x(t)$ is referred to as the *underlying ACS process* for the OACS process $y(t)$.

If $Z(f)$ is an orthogonal-increment process (the set A contains only the element $\alpha = 0$), then $y(t)$ is an oscillatory process in the sense of Priestley [9, Sec. 1.1.5], [13].

The (conjugate) autocorrelation of $y(t)$ is given by [10, Sec. 6], [12, Chap. 14]

$$\mathrm{E}\left\{y(t+\tau)\,y^{(*)}(t)\right\} = \sum_{\alpha\in A}\rho_{\boldsymbol{y}}^{\alpha}(t,\tau)\,e^{j2\pi\alpha t} \tag{2.6}$$

where the functions

$$\rho_{\boldsymbol{y}}^{\alpha}(t,\tau) \triangleq \int_{\mathbb{R}} A_{t+\tau}(f)\,A_t^{(*)}(_{(-)}(\alpha-f))\,e^{j2\pi f\tau}\,\mathrm{d}\mu_{\boldsymbol{x}}^{\alpha}(f) \tag{2.7}$$

are referred to as *evolutionary (conjugate) cyclic autocorrelation functions*.

For ACS processes the (conjugate) autocorrelation function is the superposition of complex sinewaves (see (2.1)) whose frequencies are the (conjugate) cycle frequencies and whose complex amplitudes are the (conjugate) cyclic autocorrelation functions (that depend only on the lag parameter τ). For OACS processes, the (conjugate) autocorrelation function is the superposition of amplitude- and angle-modulated complex sinewaves (see (2.6)) whose frequencies are the (conjugate) cycle frequencies of the underlying ACS process (see (2.4)) and the amplitude and angle modulating functions are the magnitude and phase, respectively, of the evolutionary (conjugate) cyclic autocorrelation functions $\rho_{\boldsymbol{y}}^{\alpha}(t,\tau)$.

III. Model for the Aircraft Acoustic Signal

The acoustic signal arising from an aircraft and received by a stationary listener is analyzed in two different operating conditions. In the first case, the aircraft is maneuvering on the land. In the second case the aircraft is taking-off in front of the listener.

Let $x(t)$ be the real-valued acoustic signal generated by the aircraft. The sound propagation model is the same as that for electromagnetic waves. Thus, the (noisy free) received signal $y(t)$ recorded by a listener is modeled as [9, Sec. 7.1]

$$y(t) = A_0\,x(t-D(t)) \tag{3.1}$$

where A_0 is attenuation and $D(t)$ is a time-varying delay that accounts for both relative motion between aircraft (transmitter) and listener (receiver) and micro-Doppler effects due to aircraft vibration, moving parts, and rotation movements.

The signal $x(t)$ is originated by rotation of rotors and, hence, can be modeled as cyclostationary or, more generally, as almost-cyclostationary [1], [4], [12, Sec. 10.6], that is, with autocorrelation function (2.1). Therefore, the autocorrelation function of the real-valued signal $y(t)$ in (3.1) is given by

$$\begin{aligned}
&\mathrm{E}\{y(t+\tau)\,y(t)\}\\
&= A_0^2\,\mathrm{E}\{x(t+\tau-D(t+\tau))\,x(t-D(t))\}\\
&= A_0^2\sum_{\alpha\in A} R_{\boldsymbol{x}}^{\alpha}(\tau+D(t)-D(t+\tau))\,e^{-j2\pi\alpha D(t)}\,e^{j2\pi\alpha t}\,.
\end{aligned} \tag{3.2}$$

That is, $y(t)$ is an OACS process with evolutionary (conjugate) cyclic autocorrelation functions

$$\rho_{\boldsymbol{y}}^{\alpha}(t,\tau) = A_0^2\,R_{\boldsymbol{x}}^{\alpha}(\tau+D(t)-D(t+\tau))\,e^{-j2\pi\alpha D(t)}\,. \tag{3.3}$$

The statistical characterization of the process $y(t)$ in the most general case cannot be easily obtained. However, if the time warping introduced by the time-varying delay $D(t)$ is not excessive, then the second-order characterization of $y(t)$ can be obtained by estimating $D(t)$ and then characterizing the underlying ACS process $x(t)$.

Let $[t_*, t_*+T]$ be the observation interval and let us consider the first-order Taylor series expansion with Lagrange residual term

$$D(t+\tau) - D(t) = \dot{D}(\bar{t}_\tau)\,\tau \tag{3.4}$$

with $\dot{D}(t)$ first-order derivative of $D(t)$ and $\bar{t}_\tau \in (\min(t_*, t_*+\tau), \max(t_*, t_*+\tau))$. Under the assumption that the time-varying delay is slowly varying in $[t_*, t_*+T]$, that is,

$$\sup_{t\in[t_*,t_*+T]} \left|\dot{D}(t)\right| \ll 1 \tag{3.5}$$

we have that for $\forall t \in [t_*, t_*+T]$ the time-varying delay can be neglected in the arguments of $R_{\boldsymbol{x}}^{\alpha}(\cdot)$ in (3.2) and the autocorrelation function of $y(t)$ can be written as

$$\begin{aligned} &\mathrm{E}\left\{y(t+\tau)\, y(t)\right\} \\ &= A_0^2 \sum_{\alpha\in A} R_{\boldsymbol{x}}^{\alpha}(\tau)\, e^{-j2\pi\alpha D(t)}\, e^{j2\pi\alpha t}\,. \end{aligned} \tag{3.6}$$

A suitable second-order statistical characterization of the process $y(t)$ can be made starting from (3.6). In fact, provided that at least one angle modulated sinewave, say at cycle frequency α_0, in (3.6) has the power spectrum that does not significantly overlap the power spectra of the other sinewaves, thus, this modulated sinewave can be extracted by band-pass filtering the lag product $y(t+\tau)y(t)$ around the frequency α_0. Specifically, defining

$$\ell^{\alpha_0}(t,\tau) \triangleq y(t+\tau)\, y(t)\, e^{-j2\pi\alpha_0 t} \tag{3.7}$$

the estimated time-varying delay $\widehat{D}(t)$ is obtained by angle demodulation of

$$z^{\alpha_0}(t,\tau) \triangleq \ell^{\alpha_0}(t,\tau) \otimes h_W(t) \tag{3.8a}$$

$$\simeq A_0^2\, R_{\boldsymbol{x}}^{\alpha_0}(\tau)\, e^{-j2\pi\alpha_0 D(t)} \tag{3.8b}$$

where $h_W(t)$ is a low-pass filter with bandwidth W whose purpose is to leave practically unaltered the sinewave at frequency α_0 that has been down-converted (see (3.7)) and to filter out all the other sinewaves in the second-order lag product.

An estimate of the underlying ACS signal $\widehat{x}(t)$ can be obtained by the de-warping procedure

$$\widehat{x}(t) = y(t+\widehat{D}(t))\,. \tag{3.9}$$

In fact, under condition (3.5) and small approximation error $|D(t)-\widehat{D}(t)| \ll 1/B_x$, with B_x bandwidth of $x(t)$, we have that $t+\widehat{D}(t)$ is an approximate inverse of $t-\widehat{D}(t)$ for the purpose of time de-warping [11].

If the cycle frequency α_0 is only roughly known, then, an approximate value, say $\widetilde{\alpha}_0$, can be used in (3.8a) in place of α_0. In such a case, the estimated $D(t)$ also contains a linear term with t that compensates the error in the knowledge of α_0 (see (3.6)). The slope of this linear term is the error in the knowledge of α_0. Compensating such an error, a more accurate estimate $\widehat{\alpha}_0$ of the nominal cycle frequency α_0 is obtained.

Two cases are considered: aircraft slowly maneuvering on the land and aircraft taking-off in front of a stationary listener.

1) Aircraft Maneuvering on the Land: The time-varying delay is only due to the micro-Doppler effects. In such a case, the slowly varying condition (3.5) is generally satisfied in wide observation intervals. In these intervals, the autocorrelation of $y(t)$ has the simplified expression (3.6).

2) Aircraft Taking-Off in Front of the Listener: The aircraft has velocity vector with both direction and modulus constant within the observation interval. Without any lack of generality, the aircraft and listener positions are modeled as $\boldsymbol{P}_{\text{air}}(t) = v\,(t-t_0)\,\boldsymbol{i}_1$ and $\boldsymbol{P}_{\text{lis}} = R_0\,\boldsymbol{i}_2$ respectively, with $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ orthonormal vectors [9, Sec. 7.2.2] (Fig. 1).

In the absence of micro-Doppler, the time-varying delay $D(t)$ is only due to the relative motion with constant velocity vector and can be expressed as [9, Sec. 7.2.2]

$$D(t) = k_1\,(t-t_0) + \sqrt{k_2 + k_3\,(t-t_0)^2} \tag{3.10}$$

with $k_1 \triangleq -v^2/(c^2-v^2)$, $k_2 \triangleq R_0^2/(c^2-v^2)$, and $k_3 \triangleq c^2v^2/(c^2-v^2)^2$ where $c \simeq 1225\,\text{km h}^{-1} \simeq 340\,\text{m s}^{-1}$ is the sound propagation speed in the air. In general, v has the same order of magnitude of c. For example, $v = 400\,\text{km h}^{-1} \simeq 111\,\text{m s}^{-1}$.

We have that

$$\frac{\mathrm{d}D(t)}{\mathrm{d}t} = k_1 + \frac{k_3\,(t-t_0)}{\sqrt{k_2+k_3\,(t-t_0)^2}}\,. \tag{3.11}$$

The validity of condition

$$\sup_{t\in(t_*,t_*+T)} \left|\frac{\mathrm{d}D(t)}{\mathrm{d}t}\right| \ll 1 \tag{3.12}$$

depends on the considered observation interval (t_*, t_*+T). Specifically, by letting $\mathrm{d}D(t)/\mathrm{d}t = 0$ one obtains

$$t - t_0 = \sqrt{\frac{k_1^2 k_2}{k_3^2 - k_1^2 k_3}} = \frac{R_0}{c} \tag{3.13}$$

which is the value of t around which condition (3.12) is verified. Increasing the data-record length T makes (3.12) not satisfied. In fact

$$\lim_{t\to+\infty} \frac{\mathrm{d}D(t)}{\mathrm{d}t} = k_1 + \sqrt{k_3} = \frac{v}{c+v} \tag{3.14}$$

with c and v having the same order of magnitude.

The general lack of validity of the slowly varying condition (3.12) for $D(t)$ is not influenced if also micro-Doppler effects in $D(t)$ are accounted for.

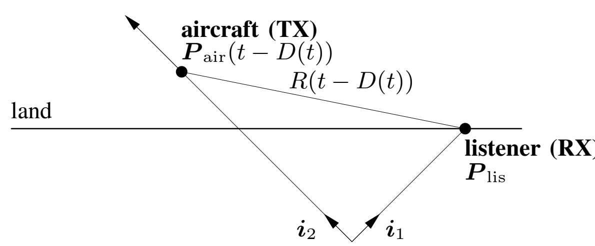

Fig. 1. Aircraft taking-off in front of a stationary listener.

IV. Numerical Results

The audio file of the acoustic signal generated by a B2 Spirit aircraft is extracted from `https://www.youtube.com/watch?v=3Mt6F57D9bo`. It is obtained with sampling frequency $f_s = 1/T_s = 44.1\text{kHz}$. Left and right channels are mixed together.

1) Aircraft Maneuvering on the Land: In this experiment, the time-varying delay $D(t)$ only accounts for micro-Doppler effects and condition (3.5) is verified in large observation intervals. The analyzed acoustic signal corresponds to samples from time 0:05 to time 0:09 in the audio file (only the first 360 samples are reported in Fig. 2). The data-record length is $T = 4$ s.

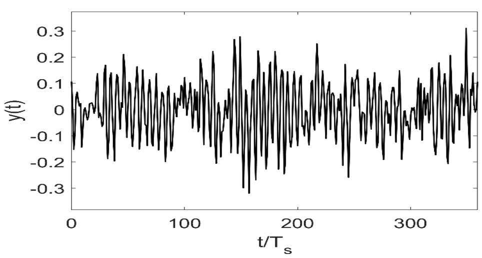

Fig. 2. Acoustic signal $y(t)$ generated by a B2 Spirit aircraft slowly maneuvering on the land as received by a stationary listener.

In Fig. 3 the function

$$\lambda_{\boldsymbol{y}}^{(T)}(\alpha) \triangleq \int_{-\tau_M}^{\tau_M} |R_{\boldsymbol{y}}^{(T)}(\alpha, \tau)|^2 \,\mathrm{d}\tau \tag{4.1}$$

where $(-\tau_M, \tau_M)$ is the set in which $R_{\boldsymbol{x}}^{\alpha}(\tau)$ is significantly nonzero, is reported as function of α. No sharp peaks are present except that at $\alpha = 0$. That is, the possible cyclic features of the underlying ACS signal $x(t)$ have been spread by the time-warping induced by the time-varying delay $D(t)$.

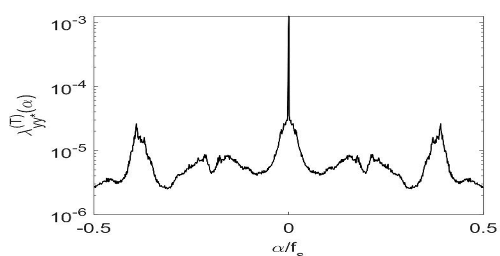

Fig. 3. OACS signal $y(t)$. Function $\lambda_{\boldsymbol{y}}^{(T)}(\alpha)$

A cycle frequency α_0 of $x(t)$ is roughly estimated as the location of a peak of the power spectral density (PSD) of the function $t \mapsto y(t+\tau)\, y(t)$. The value $\tau = 0$ is selected. The roughly estimated cycle frequency of $x(t)$ is $\widetilde{\alpha}_0 = 0.382\, f_s$ and the selected bandwidth for the low-pass filter $h_W(t)$ is $W = 0.02\, f_s$. In Fig. 4, the PSD of the function defined in (3.7) with α_0 replaced by $\widetilde{\alpha}_0$ is reported and the band of the ideal low-pass filter $h_W(t)$ is evidenced. The estimated time-varying delay (Fig. 5) is obtained by angle demodulation of the function defined in (3.8a) with α_0 replaced by $\widetilde{\alpha}_0$.

The de-warped signal (3.9) is an estimate of the underlying ACS signal $x(t)$. In Fig. 6 the function $\lambda_{\widehat{\boldsymbol{x}}}^{(T)}(\alpha)$, obtained by replacing y with $\widehat{x}$ in (4.1), is reported as function of α. Unlike Fig. 3, two sharp peaks are present at $\alpha = \pm\widehat{\alpha}_0$. That is, the almost-cyclostationary nature of the source signal $x(t)$ has been recovered. The real part of the estimated cyclic autocorrelation of $\widehat{x}(t)$ at the estimated cycle frequency $\widehat{\alpha}_0$ is reported in Fig. 7.

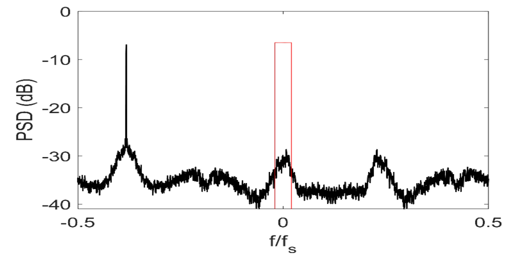

Fig. 4. OACS signal $y(t)$. PSD of the function $t \mapsto y(t+\tau)\, y(t)\, e^{-j2\pi\widetilde{\alpha}_0 t}$ for $\tau = 0$. The band of the ideal low-pass filter $h_W(t)$ is evidenced.

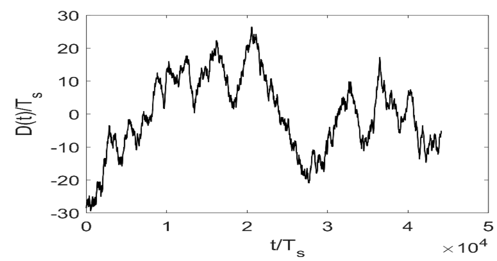

Fig. 5. OACS signal $y(t)$. Estimated $D(t)$.

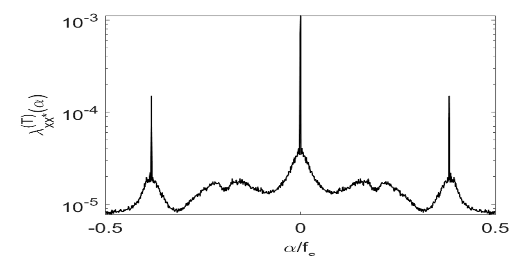

Fig. 6. Estimated underlying ACS signal $x(t)$. Function $\lambda_{\widehat{\boldsymbol{x}}}^{(T)}(\alpha)$

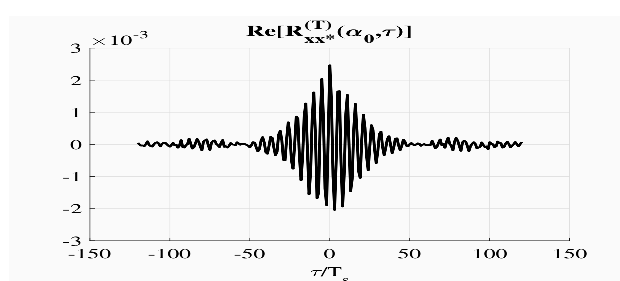

Fig. 7. Estimated underlying ACS signal $x(t)$. Real part of the estimate of the cyclic autocorrelation at cycle frequency $\widehat{\alpha}_0$.

2) Aircraft Taking-Off in Front of the Listener: In this experiment, accounting for the results of Section III-2, condition (3.5) is verified only within a small observation interval. The analyzed acoustic signal corresponds to samples from time 1:26 to time 1:28 in the audio file (only the first 360 samples ar reported in Fig. 8). Thus, the data-record length is $T = 2\,\mathrm{s}$. Increasing the length of the observation interval or changing its staring point makes the slowly-varying condition (3.5) for $D(t)$ not satisfied. The roughly estimated cycle frequency of $x(t)$ is $\widetilde{\alpha}_0 = 0.0625\, f_s$ and the LPF bandwidth is $W = 0.006\, f_s$.

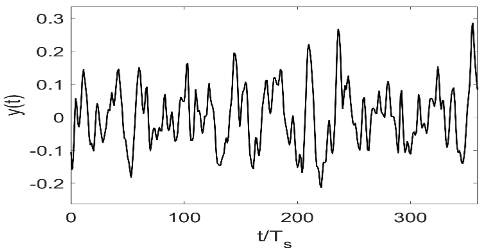

Fig. 8. Acoustic signal $y(t)$ generated by a taking-off B2 Spirit aircraft as received by a stationary listener.

In Fig. 9 the function $\lambda_{\boldsymbol{y}}^{(T)}(\alpha)$ is reported as function of α. As in the previous case (Sec. IV-1), no sharp peaks are present except that at $\alpha = 0$. That is, the possible cyclic features of the underlying ACS signal $x(t)$ have been spread by the time-warping induced by the time-varying delay $D(t)$.

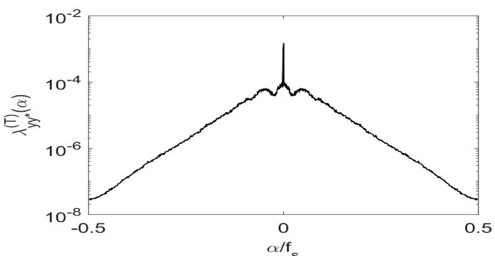

Fig. 9. OACS signal $y(t)$. Function $\lambda_{\boldsymbol{y}}^{(T)}(\alpha)$

The same procedure adopted for the previous case is carried out to estimate the time-varying delay $D(t)$ and for de-warping the available data $y(t)$ in order to recover an estimate of $x(t)$. In this case, however, the smaller data-record length leads to a not accurate estimate whose second-order lag-product does not exhibit spectral lines with significant strength (see Fig. 10 where the function $\lambda_{\widehat{\boldsymbol{x}}}^{(T)}(\alpha)$ is reported).

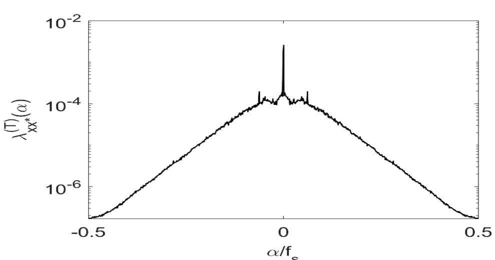

Fig. 10. Estimated underlying ACS signal $x(t)$. Function $\lambda_{\widehat{\boldsymbol{x}}}^{(T)}(\alpha)$

V. Conclusion

The acoustic signal emitted by a moving aircraft and received by a stationary listener is modeled as oscillatory almost-cyclostationary. Its autocorrelation function is constituted by the superposition of angle-modulated sinewaves whose frequencies are the cycle frequencies of the almost-cyclostationary source signal. Assuming that the time-varying delay introduced in the received signal by the relative motion is slowly varying, it is estimated and then exploited to de-warp the received signal in order to recover an estimate of the source signal. Thus, cyclic features of this signal are estimated by conventional cyclic spectral analysis. The acoustic signal emitted by a B2 Spirit aircraft in two operating situations is analyzed. If the aircraft is slowly maneuvering on the land, the time-varying delay is due to micro-Doppler effects and is slowly varying in a wide observation interval leading to an effective recovery of the source signal and an accurate estimate of its cyclic features. If the aircraft is taking-off, the time-varying delay can be assumed slowly varying only during a small time interval which leads to a not accurate recovery of the source signal.

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