Unbiased FIR Filtering under Bernoulli-Distributed Binary Randomly Delayed and Missing Data

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***Abstract*—This paper develops an unbiased finite impulse response (UFIR) filtering algorithm for networked systems where uncertain delays and packet dropouts can happen due to measurement failures and unreliable communication. The binary Bernoulli distribution with known delay probability is used to model the randomly arrived measures. A novel representation of the stochastic model is presented for FIR-type filter structures. To avoid packet dropouts and improve the estimation accuracy when a message arrives with no data, a predictive algorithm is used. An advantage of the UFIR filtering approach is demonstrated by comparing the mean square errors with the Kalman and H_∞ filters under the same conditions. Experimental verifications are provided based on GPS vehicle tracking.**

***Index Terms*—delayed data, missing data, unbiased FIR filter**

I. Introduction

Nowadays, many systems employ wireless communications to transmit data from smart sensors to a central station (CS), where information is processed. A common concern is to assure the effectiveness of data transmission. Unfortunately, factors such as the limited bandwidth, unreliable communication channels, atmospheric disturbances, noise, etc. often degenerate the effectiveness of the transmitted data and cause errors associated with delayed and missing data.

Delay measurements and packets dropout are two potential problems that degrade the effectiveness and performance of engineering techniques providing estimation and control. The research of state estimation problems in network systems with data loss or fading measurement has been increased in recent decades due to wide applications and the necessity to reduce errors caused by this phenomena in estimation algorithms. Accordingly, diverse estimation strategies have been proposed based on different approaches to reduce communication failures, deterministic or stochastic. In [1]–[5] several estimation algorithms have been designed assuming that data delays are constant and always known. However, it serves well for time-stamped data and is not totally true otherwise. In some real applications such as object tracking, navigation, manufacturing process, and space exploration [6], data are subject to random variations. That hinder the application of traditional estimation theories, therefore special solutions are required [7]–[9]. To address appropriately such issues, the measurement failures should be assumed to be random with some proper stochastic model and known conditional probabilities [10]–[12].

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The Bernoulli distribution is most widely used to describe random delays [1], [6], [13], [14]. In [15], [16], the Kalman filter (KF) is modified to solve the problem with intermittent information, where a stochastic model is used to obtain a minimum mean square error (MSE). A flaw is that the KF performance strongly depends on the noise statistics, initial values, and correct interpretation of delays and missing data [6], [17]. The robust H_∞ filter is able to improve the KF performance [18], [19], but it may diverge if a tuning factor is not set properly. The least-squares filter [20] also suffers of extra errors in uncertain environments. One can employ the iterative unbiased finite impulse response (UFIR) filter, which advantage is that it does not require any information about noise and initial values [21], [22]. This makes the UFIR filter a more robust option for systems with delays.

In this paper, we develop the UFIR state estimator for WSN with uncertain measurements perturbed by random delays, packet dropouts, or arbitrary parameter matrices. The binary Bernoulli distribution is used to model arrived measurements at each discrete time instant, correct failed data, and adjust delayed data. The UFIR filter is tested experimentally by the Global Positioning System (GPS)-based tracking problem. It is shown that the UFIR filter in many cases outperforms the KF and H_∞ filter.

II. Model and Problem Formulation

The system dynamics and observations with binary-delayed and missing data can be modeled as

$$x_n = Fx_{n-1} + w_n\,, \tag{1}$$

$$\tilde{y}_n = HFx_{n-1}\,, \tag{2}$$

$$y_n = \kappa\left[\gamma Hx_n + (1-\gamma)\,Hx_{n-1}\right] + (1-\kappa)\tilde{y}_n + v_n\,, \tag{3}$$

where $x_n \in \mathbb{R}^K$, $y_n \in \mathbb{R}^M$, and $F \in \mathbb{R}^{K\times K}$ and $H \in \mathbb{R}^{K\times M}$ are known matrices. Here, $w_n \sim \mathcal{N}(0,Q) \in \mathbb{R}^K$ and $v_n \sim \mathcal{N}(0,R) \in \mathbb{R}^K$ are white Gaussian uncorrelated noise vectors with zero mean and the covariances $Q = E\left\{w_n w_n^T\right\}$ and $R = E\left\{v_n v_n^T\right\}$. The measurement delays and missing data are modeled using two scalar coefficients, γ and κ. The delayed state x_{n-1} and undelayed state x_n are combined in the observation equation using the binary Bernoulli distribution and the probability γ of the undelayed state occurrence is supposed to be known. When there are no lost data, an auxiliary data-sensor generates $\kappa = 1$ and the second term in (3) becomes identically zero. Otherwise, a sensor generates $\kappa = 0$ and the prediction (2) is used as an observation.

To apply an estimator, model (1)–(3) is modified to have no delay by representing the delayed state via (1) as

$$x_{n-1} = F^{-1}x_n - F^{-1}w_n \tag{4}$$

and then transform the observation equation (3) for $\kappa = 1$ to

$$y_n = \bar{H}_n x_n + \bar{v}_n \,, \tag{5}$$

where

$$\bar{H}_n = \gamma H + (1-\gamma)\, HF^{-1} \,, \tag{6}$$

$$\bar{v}_n = v_n - (1-\gamma)\, HF^{-1}w_n \,, \tag{7}$$

and the covariance $\bar{R} = E\left\{\bar{v}_n\bar{v}_n^T\right\}$ of noise $\bar{v}_n$ is given by

$$\bar{R}_n = R + (1-\gamma)^2 HF^{-1}QF^{-1^T}H^T \,. \tag{8}$$

Note that since factor γ is time-varying, matrices $\bar{H}_n$ and $\bar{R}_n$ are also time-varying.

Given model (1) and (5), we can now modify the UFIR filtering algorithm and use the KF and H_∞ filter.

III. UFIR Filtering Algorithm for Binary Randomly Delayed and Missing Data

To apply the UFIR filter, model (1) and (5) must be extended on a horizon $[m, n]$, from $m = n - N + 1$ to n, as [23]:

$$x_{m,n} = A_N x_m + B_N w_{m,n} \,, \tag{9}$$

$$y_{m,n} = C_{m,n}x_m + G_{m,n}w_{m,n} + v_{m,n} \,, \tag{10}$$

where the extended state vector $x_{m,n}$, observation vector $y_{m,n}$, and matrices are given by

$$x_{m,n} = \left[x_m^T \, x_{m+1}^T \, \dots \, x_n^T\right]^T , \tag{11}$$

$$y_{m,n} = \left[y_m^T \, y_{m+1}^T \, \dots \, y_n^T\right]^T , \tag{12}$$

$$A_N = \left[I \; F^T \, \dots \, F^{N-1^T}\right]^T , \tag{13}$$

$$B_N = \begin{bmatrix} I & 0 & \dots & 0 & 0 \\ F & I & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F^{N-2} & F^{N-3} & \dots & I & 0 \\ F^{N-1} & F^{N-2} & \dots & F & I \end{bmatrix} , \tag{14}$$

$$C_{m,n} = \begin{bmatrix} \bar{H}_m \\ \bar{H}_{m+1}F \\ \bar{H}_{m+1}F^2 \\ \vdots \\ \bar{H}_nF^{n-1} \end{bmatrix} , \tag{15}$$

$$G_{m,n} = \begin{bmatrix} \bar{H}_m & 0 & 0 & \dots & 0 \\ \bar{H}_{m+1}F & \bar{H}_{m+1} & 0 & \dots & 0 \\ \bar{H}_{m+2}F^2 & \bar{H}_{m+2}F & \bar{H}_{m+2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \bar{H}_nF^{N-1} & \bar{H}_nF^{N-2} & \bar{H}_nF^{N-3} & \dots & \bar{H}_n \end{bmatrix} , \tag{16}$$

$$v_{m,n} = \begin{bmatrix} v_m - H\bar{B}_nw_{p_m,m} \\ v_{m+1} - H\bar{B}_nw_{p_{m+1},m+1} \\ \vdots \\ v_{n-1} - H\bar{B}_nw_{p_{n-1},n-1} \\ v_n - H\bar{B}_nw_{p_n,n} \end{bmatrix} , \tag{17}$$

where $\bar{B}_n = (1-\gamma)F^{-1}$.

1) Batch UFIR Filter: In a batch form, the UFIR Filter can now be designed if we satisfy on $[m, n]$ the unbiasedness condition $E\{x_n\} = E\{\hat{x}_n\}$ [23]. Then the batch UFIR estimate $\hat{x}_n \triangleq \hat{x}_{n|n}$ appears as

$$\hat{x}_n = \mathcal{H}_{m,n}y_{m,n} \,, \tag{18}$$

where the UFIR filter gain $\mathcal{H}_{m,n}$ is given by

$$\mathcal{H}_{m,n} = (\mathcal{C}_{m,n}^T\mathcal{C}_{m,n})^{-1}\mathcal{C}_{m,n}^T \tag{19}$$

with a matrix $\mathcal{C}_{m,n}$ specified as

$$\mathcal{C}_{m,n} = \begin{bmatrix} HF^{-N+1-k_m} \\ \vdots \\ HF^{-1-k_{n-1}} \\ HF^{-k_n} \end{bmatrix} , \tag{20}$$

where the time-varying delay index k_n can be either 0 or 1. The batch UFIR filtering estimate can also be written as

$$\hat{x}_n = G_n\mathcal{C}_{m,n}^Ty_{m,n} \,, \tag{21}$$

where $y_{m,n}$ is a vector of real data and $G_n = (\mathcal{C}_{m,n}^T\mathcal{C}_{m,n})^{-1}$ is the *generalized noise power gain* (GNPG).

2) Iterative UFIR Filtering Algorithm: An iterative computation of (21) can be provided using recursions if we introduce an auxiliary time index $s = m + K - 1$ and provide computations from $s + 1$ to n using the following equations,

$$G_l = [\bar{H}_l^T\bar{H}_l + (FG_{l-1}F^T)^{-1}]^{-1} \,, \tag{22}$$

$$\hat{x}_l = F\hat{x}_{l-1} + G_l\bar{H}_l^T\left(y_l - \bar{H}_lF\hat{x}_{l-1}\right) \,, \tag{23}$$

where $\bar{H}_n = HF^{-1}$. A pseudo code of the iterative UFIR filtering algorithm for measurements with binary-delayed and missing data is listed as Algorithm 1.

Provided model (1) and (5), the KF and H_∞ filter algorithms [24] can be applied straightforwardly.

IV. Experimental Verification

In this section, we consider an experimental example based on tracking of a vehicle trajectory using a GPS reading systems. Measurements were conducted in the Cook County of Illinois and are available from the University of Illinois at Chicago [25]. We consider these data as a true trajectory and suppose that they were sent to a central station (CS)

Algorithm 1: Iterative UFIR Filtering Algorithm for Binary-Delayed and Missing Data

Data: y_n, N, κ_n
Result: $\hat{x}_n$
begin
  for $n = N-1 : \infty$ **do**
    $m = n - N + 1, \quad s = m + K - 1;$
    if $\kappa_n = 0$ **then**
      $y_n = HF\hat{x}_{n-1};$
    end if
    $\bar{H}_n = HF^{-k_n};$
    $G_s = (\mathcal{C}_{m,s}^T \mathcal{C}_{m,s})^{-1};$
    $\tilde{x}_s = G_s \mathcal{C}_{m,s}^T y_{m,s};$
    for $l = s+1 : n$ **do**
      $G_l = [\bar{H}_l^T \bar{H}_l + (FG_{l-1}F^T)^{-1}]^{-1};$
      $K_l^{\mathrm{UF}} = G_l \bar{H}_l^T;$
      $\tilde{x}_l = F\tilde{x}_{l-1} + K_l^{\mathrm{UF}}(y_l - \bar{H}_l F \tilde{x}_{l-1});$
    end for
    $\hat{x}_n = \tilde{x}_n;$
  end for
end
† Data y_0, y_1,..., y_{N-1} must be available.

by a wireless sensors network for further processing. We also assume that data arrive at the station with or without a one-step time delay and that the random delay appearances are distributed with the binary Bernoulli law. The measured vehicle trajectory in the north-east coordinates x, km and y, km is shown in Fig. 1.

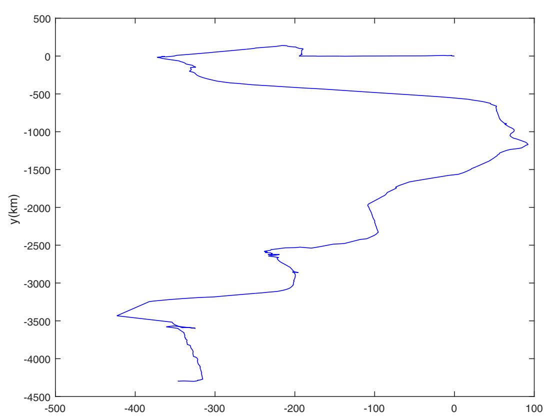

Fig. 1. A vehicle trajectory measured using a GPS reading system [25].

The vehicle dynamics is represented with the state vector $x_n = \begin{bmatrix} x_{1n} & x_{2n} & x_{3n} & x_{4n} \end{bmatrix}^T$, where x_{1n} , m is the x_n coordinate, x_{2n} is the velocity in m/s in the coordinate x_n, x_{3n} , m is the y_n coordinate, and x_{4n} is the velocity in m/s in the coordinate y_n. Accordingly, the system matrix F and the observation matrix H are specified as [26]

$$F = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

To apply the KF and H_∞ filter, it need specifying the noise statistics and error matrices. Because no information is provided in [25] about noise, we do it using a general knowledge under the following assumptions. The vehicle moves with an average speed of $10\,\mathrm{m/s}$ or $36\,\mathrm{m/s}$. Accepting that the vehicle speed standard deviation in the second state along each of the coordinates is about 20%, we assign $\sigma_{2w} = 2\,\mathrm{m/s}$ and ignore noise in the first state, $\sigma_{1w} = 0\,\mathrm{m}$. The GPS service produces navigation errors of less than 15 meters with the probability of 95% in the 2σ sense and we thus assign $\sigma_v = 3.75\,\mathrm{m}$. Accordingly, the noise covariance matrices become

$$Q = \sigma_{w2}^2 \begin{bmatrix} \tau^2/4 & \tau/2 & 0 & 0 \\ \tau/2 & 1 & 0 & 0 \\ 0 & 0 & \tau^2/4 & \tau/2 \\ 0 & 0 & \tau/2 & 1 \end{bmatrix}, \; R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}.$$

To run the H_∞ filter, we will use Q and R as the error matrices.

A. Tuning Factors

It is known that the UFIR and H_∞ filters require tuning factors to produce the best estimates. The UFIR filter needs N_{opt} to minimize the MSE in the estimate. By minimizing the trace of the error covariance for the ground truth provided by the GPS reading system, we find $N_{\mathrm{opt}} = 5$. The H_∞ filter requires a tuning factor θ to improve the performance of the KF filter. Experimentally, we find this factor to be $\theta = 1.86 \times 10^{-3}$. Note that both N_{opt} and θ are measured under the condition that a system has no delay and missing data. Otherwise, these values must be time-varying that is not feasible.

B. Delayed and Missing Data

We think of a situation as ideal when data arrive at an estimator with no delay. Otherwise, we learn effect of the probability factor γ on the estimation accuracy in the Kalman, UFIR and H_∞ filters. In the binary Bernoulli model, γ takes values of "0" or "1" in accordance with the probability of the delay in the transmission channel.

As an example, in Fig. 2 we show the short-scale estimated vehicle coordinates in the north direction for $\gamma = 0.8$. As can be seen, the difference between the estimates is clearly seen when the vehicle rapidly changes the trajectory, as at about 700th hour of the movement. We watch here for typical filter responses [21]: the UFIR filter produces a larger excursion but a shorter transient due to the FIR, while the KF produces a shorter excursion and a longer transient due to the IIR. As expected, the H_∞ filter improves a bit the KF performance that is better seen in Fig. 3. Beyond the transient range, the estimates produced by the filters are consistent.

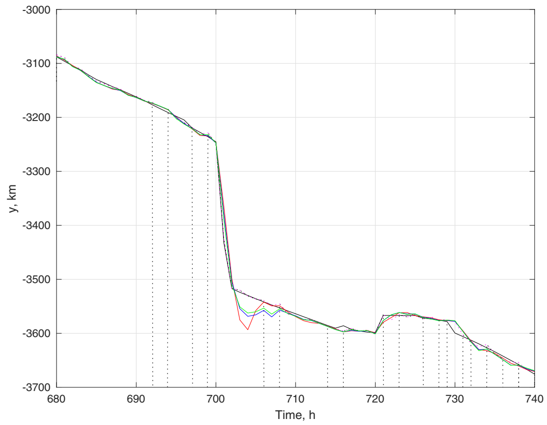

Fig. 2. Estimated vehicle coordinates in the north direction for $\gamma = 0.8$ provided by the UFIR filter ($N_{\rm opt} = 5$), KF, and H_∞ filter ($\Theta = 1.86 \times 10^{-3}$): (a) $0 \leqslant n \leqslant 850$ and (b) $680 \leqslant n \leqslant 740$.

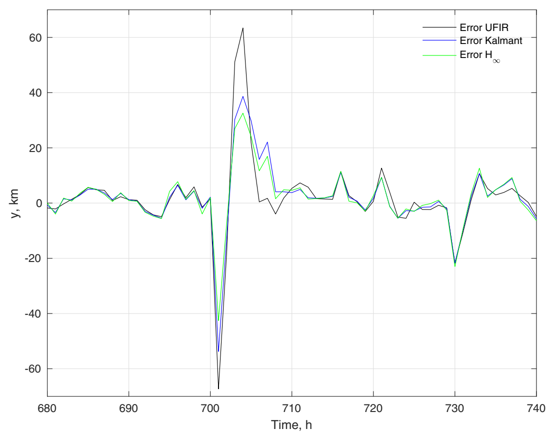

Fig. 3. Estimation errors in the north direction for $\gamma = 0.8$ produced by the UFIR filter ($N_{\rm opt} = 5$), KF, and H_∞ filter ($\Theta = 1.86 \times 10^{-3}$) in a time span of $680 \leqslant n \leqslant 740$.

The estimation root MSEs (RMSEs) caused by uncertainties in the delay probability factor γ with respect to the true factor 0.8 are listed in Table 1. It follows that when we set $\gamma =$ 0.8 to all algorithms, then the RMSE reaches a minimum in all filters. Otherwise, errors grow with a deviation in γ from the true values of 0.8. It is also seen that the KF produces minimal errors, because the noise covariances are set correctly. Although the H_∞ filter is tuned optimally to data with no latency, it does not produce a better estimate than the KF otherwise. That means that the H_∞ filter requires a proper tuning factor θ for each γ, which practically does not seem to be possible. Finally, as expected, the UFIR filter produces a bit more errors, but we notice that it does not require the noise statistics and initial values and is easy in tuning.

TABLE I
RMSEs Caused by Uncertainties in the Delay Factor γ with Respect to the True Factor of 0.8 Under $\beta = 1.0$.

γ	UFIR filter	KF	H_∞ filter
0.2	10.7920	10.7646	15.4474
0.5	9.2526	8.8149	9.9992
0.8	9.0603	8.7766	9.2172

C. Effect of Errors in the Noise Statistics

The system and measurement noise statistics are typically not well-known due to the practical inability to obtain their exact values. To evaluate the effect of errors in the noise covariances and error matrices on the estimation accuracy, we introduce the error factors $\alpha > 0$ and $\beta > 0$ and then substitute in the algorithms the noise covariances Q and R with $\alpha^2 Q$ and $\beta^2 R$. For $N_{\rm opt} = 5$, $\theta = 0.86 \times 10^{-4}$, and $\gamma = 0.5$, the RMSEs produced by the filters in the north direction (y-RMSE) for $\alpha = 1/\beta$ and β varying from 0.1 to 10.0 are sketched in Fig. 4. It is seen that all three filters have a good performance when $\beta = 1$ and that the UFIR filter is here a bit less accurate, as expected. A picture changes when $\beta \neq 1$. In this case errors in the KF and H_∞ filter grow rapidly and with $\beta < 0.9$ and $\beta > 2.2$ the UFIR filter becomes more accurate. Furthermore, the H_∞ filter goes to divergence when β falls below 0.2. Overall, we conclude that the UFIR filter is a robust estimator against errors in the noise statistics in the presence of delayed and missing data.

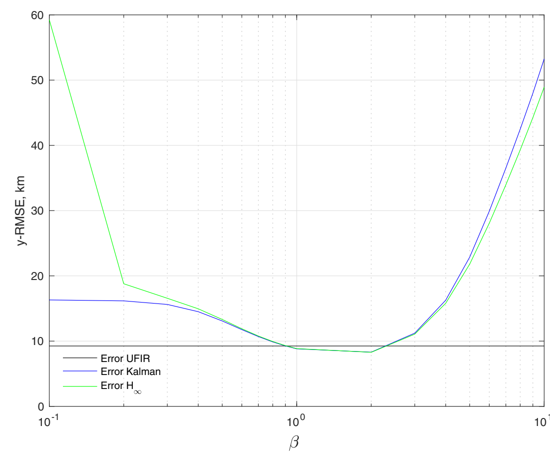

Fig. 4. Effect of errors in the noise standard deviation on the y-RMSE produced by the UFIR filter, KF, and H_∞ filter as functions of β.

We now wonder how robust the above filters are against errors in the delay factor γ in the presence of errors in the noise covariances, $\beta \neq 1$. The RMSEs produced by the filters under such conditions in the north direction (y-RMSE) are sketched in Fig. 5. In this test, data are transmitted with a delay probability of $\gamma = 0.8$, but this value is supposed to be not known exactly at an estimator and we vary it from 0 to 1.0 assuming moderate errors in the noise statistics caused by $\beta =$ 2. As can be seen, errors in all filter grow with a deviation of γ from the true value and even fairly small errors in the noise covariances caused by $\beta = 0.5$ make the UFIR filter more accurate. But the most impressive is that the H_∞ filter, which is tuned to transmissions with no delays and missing data, completely loses an ability to be an accurate estimator due to the divergence. We support these observations with Table

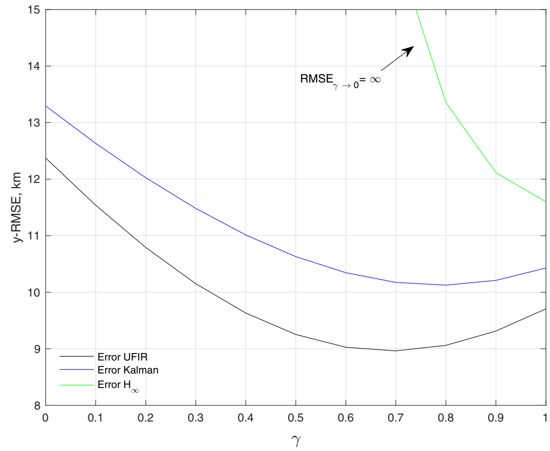

Fig. 5. Effect of errors in the delay factor γ with respect to the true value of $\gamma = 0.8$ on the y-RMSEs produced by the UFIR filer, KF and H_∞ filter under the moderate errors in the noise covariances caused by $\beta = 0.5$.

II, which lists the y-RMSEs for several values of γ and the true value of $\gamma = 0.8$. It is neatly seen that the UFIR filter

TABLE II
RMSEs Caused by Uncertainties in the Delay Factor γ with Respect to the True Factor of 0.8 Under $\beta = 0.5$

γ	UFIR filter	KF	H_∞ filter
0.2	10.7920	12.0264	Diverges
0.5	9.2526	10.6299	62.5527
0.8	9.0603	10.1256	13.3570

is a more accurate estimator under errors in γ and the noise covariances that is practically a most feasible situation.

V. Conclusions

The UFIR filter modified in this paper for measurements arrived at an estimator with the binary Bernoulli-distributed delays and missing data has demonstrated a better robustness than the KF and H_∞ filter under errors in the delay factor and noise covariances that is practically a most feasible situation. The result was achieved by transforming the state-space equations with delays to those without the delays and incorporating a predictive algorithm to build a bridge over missing data using the prior estimate.

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