Channel Hardening, Favorable Equalization and Propagation in Wideband Massive MIMO

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***Abstract*—This paper analyzes the channel hardening and favorable propagation behavior of frequency-selective massive MIMO channels. To this purpose the concept of *favorable equalization* is introduced to characterize the property of the channel to become frequency flat as the number of antennas grows when proper pre-filtering is adopted. It is shown that classic OFDM-based massive MIMO and time-reversal schemes, usually considered and analyzed as different technologies, are particular cases of the same framework. Their generalization leads to the concept of *massive waveforming*, which allows the creation of parallel wideband AWGN-like links between the base station and the users.**

I. Introduction

Massive multiple-input multiple-output (MIMO) is one of the most promising technologies proposed to boost the multi-user capacity of cellular networks. The key idea behind massive MIMO is to considerably increase the number M of antennas in such a way results from large-number theory can be successfully exploited, especially in rich multipath propagation conditions. Specifically, with M sufficiently large, the equivalent MIMO channel starts showing a deterministic behavior (*channel hardening*) and the vector-valued channels to the users tend to become mutually orthogonal (*favorable propagation*) [1]. A rich literature is available regarding the various aspects associated to massive MIMO networks design, such as the issue of pilot contamination and the possibility to adopt it in cell-free networks [2].

Fewer works investigate in detail the behavior of channel hardening and favorable propagation as a function of M and network configuration. For instance, [3] first proposes a distance-from-favorable-propagation measure defined as the gap between the sum-capacity and the maximum capacity obtained under favorable propagation. Secondly, it analyzes the rate of convergence of the channel to the favorable condition for two extreme scenarios: independent, identically distributed (i.i.d.) Rayleigh fading and uniform line-of-sight (LOS), showing a convergence rate of M.

The spatial filtering capability of massive MIMO in concentrated and distributed antenna scenarios is analyzed in [4]. Sufficient conditions for interference suppression in large M regime are provided based on the evaluation of the average signal-to-interference ratio using the classic Clarke's channel model.

In [5], authors compare channel hardening and favorable propagation in cell-based and cell-free networks. The main result is that better channel hardening can be obtained with few base stations (BSs) equipped with many antennas than with many BSs employing few antennas, at the expense of decreased diversity. Authors highlight that, since both channel hardening and favorable propagation are local characteristics, i.e., related only to short-term channel variations, when analyzing the system at network level (accounting for large-scale channel variations), the cumulative distribution function (CDF) of channel hardening and favorable propagation appears to be a more appropriate performance measure.

In all previous works channel hardening and favorable propagation have been addressed by considering narrowband propagation by having in mind single subcarrier (or resource block) in orthogonal frequency division multiplexing (OFDM) transmission. In most cases the Rayleigh statistical characterization has been used by assumption supposing a very rich scattering environment.

Frequency-selective massive MIMO is a less investigated topic. Early works, e.g., [6], investigated the possibility to employ a single-carrier transmission in large-scale antenna systems under wideband uncorrelated Rayleigh fading showing that the achievable sum-rate is near optimal at low signal-to-noise ratio and large M.

In this direction, our paper analyzes more in detail the channel hardening and favorable propagation in frequency-selective massive MIMO channels under mild assumptions about channel characteristics. Specifically, we investigate the CDF of channel hardening and favorable propagation as performance measures of massive MIMO-based networks. In addition, we introduce the new concept of *favorable equalization* as an indicator of how much the channel exhibits frequency flatness.

Simple expressions are derived for the widely used Clarke's propagation model, from which some useful insights on the interplay between number of antennas and bandwidth can be obtained. Specifically, it is shown how the channel hardening rate is proportional to M and system bandwidth W, whereas favorable propagation and equalization rates are proportional to M, but not to W.

Our framework generalizes the analysis of massive MIMO channel conditions in a wideband context having the classic narrowband massive MIMO and time-reversal (TR) schemes as particular cases. In fact, while TR is a single-antenna technique which pre-filters the signal in the time-domain such that all multipath components sum up coherently at receiver location [7], massive MIMO pre-filters the signal in the space-domain to achieve the same result. We will discuss in Sec. VII how such generalization leads to the concept of *massive waveforming*, i.e., space-time matching pre-filtering, which allows

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the creation of parallel wideband additive white Gaussian noise (AWGN)-like links between the BS and the users' device, the latter consisting in a simple receiver requiring neither equalizers nor OFDM demodulators.

II. System model

Consider a downlink scenario with one BS having M antennas and K single-antenna users randomly deployed within a circular area of radius R. Each $T = 1/W$ seconds the BS transmits an information symbols vector $\mathbf{s}[n] = [s_1[n], s_2[n], \ldots, s_K[n]]^T \in \mathbb{C}^K$, where W is the system bandwidth, and $s_k[n]$ is the symbol transmitted to user k at discrete time n. Without loss of generality, symbols are taken i.i.d. and normalized so that $\mathbb{E}\left\{|s_k[n]|^2\right\} = 1$.

We model the frequency-selective MIMO equivalent complex baseband channel as a tapped delay line so that the signal received by user k at time n can be written as

$$y_k[n] = \sum_{l=0}^{L-1} \mathbf{h}_k^\dagger[l]\, \mathbf{x}[n-l] + w_k[n], \tag{1}$$

where $\mathbf{x}[n] = [x_1[n], x_2[n], \ldots, x_M[n]]^T \in \mathbb{C}^M$ is the transmitted vector at time n, $\{w_k[n]\}$ are i.i.d. zero mean complex Gaussian random variables (RVs) representing the AWGN, and $\mathbf{h}_k[l]$ is the lth vector-valued tap of the channel between the BS and user k. Note that if the maximum channel propagation delay is τ_m, then $\mathbf{h}_k[l] = 0$, for $l \geq L$, where $L = \lceil W\,\tau_\text{m} \rceil$. The symbol $\dagger$ denotes the conjugate transpose.

We consider the following general multipath channel model

$$\mathbf{h}_k[l] = \sum_{p=1}^{P_k} \sqrt{\beta_{k,p}}\, \mathbf{a}(\theta_{k,p})\, \gamma(l - \tau_{k,p}\, W)\, e^{\jmath \phi_{k,p}}\,, \tag{2}$$

where P_k denotes the number of physical paths, $\beta_{k,p}$ and $\theta_{k,p}$ the gain and the angle of departure of the pth path, respectively. $\phi_{k,p}$ are modeled as i.i.d. RVs with uniform distribution in $[0, 2\pi)$, responsible for the short-term (fast) channel variations, and $\gamma(x) = \operatorname{sinc}(x)$. We do not make further assumptions on the channel. The vector $\mathbf{a}(\theta)$ is the steering vector defined as

$$\mathbf{a}(\theta) = \left[1, e^{-\jmath 2\pi \frac{D}{\lambda}\sin\theta}, e^{-\jmath 2\pi \frac{2D}{\lambda}\sin\theta}, \ldots, e^{-\jmath 2\pi \frac{(M-1)D}{\lambda}\sin\theta}\right]^T, \tag{3}$$

with λ being the carrier wavelength and D the antenna array inter-element distance we set to $D = \lambda/2$. We collect the slow-varying parameters of the channel, which depend on the scenario (e.g., position of users, scatterers, etc,), in the set $\Theta = \left\{\left\{\beta_{k,p}\right\}, \left\{\theta_{k,p}\right\}, \left\{\tau_{k,p}\right\}\right\}$.

The transmitted vector is the result of a pre-coding filtering operation

$$\mathbf{x}[n] = \sum_{l=-L+1}^{L-1} \mathbf{F}[l]\, \mathbf{s}[n-l]\,. \tag{4}$$

Supposing the channel state information (CSI) is available at the BS, e.g., exploiting channel reprocity [6], in this paper the following matching pre-filter is adopted:[1]

$$\mathbf{F}[l] = b\, \mathbf{H}^\dagger[-l] \tag{5}$$

for $-L+1 \leq l \leq L-1$, where $\mathbf{H}[l] = [\mathbf{h}_1[l], \mathbf{h}_2[l], \ldots \mathbf{h}_K[l]]^\dagger$. The filter (5) can be seen under different perspectives, i.e., as a multi-antenna TR precoding or, equivalently, as a space-time matching pre-filter leading to the final effect of adding coherently all path components at each user location. The coefficient b can be chosen so that the total transmitted power is restricted to a maximum desired value. Without loss of generality, we set $b = 1$ as it does not affect the following analysis. Note that when W is small (narrowband) then $L = 1$ and the scheme reduces to the classic OFDM-based massive MIMO with W being the sub-carrier (or resource block) bandwidth. On the other extreme, when $L > 1$ (wideband) and $M = 1$, the scheme reduces to TR [7]. When both L and M are much larger than one, we obtain more degrees of flexibility in processing the signal as discussed in Sec. VII.

Substituting (5) in (4) and (1), the signal received by the generic user k is

$$\begin{aligned} y_k[n] =& \sum_{q=-L+1}^{L-1} \sum_{r=1}^{K} g_{k,r}[q]\, s_r[n-q] + w_k[n] \\ =& g_{k,k}[0]\, s_k[n] + \sum_{q=-L+1, q\neq 0}^{L-1} g_{k,k}[q]\, s_k[n-q] \\ &+ \sum_{q=-L+1}^{L-1} \sum_{r=1, r\neq k}^{K} g_{k,r}[q]\, s_r[n-q] + w_k[n], \end{aligned} \tag{6}$$

where we have defined

$$\begin{aligned} g_{k,r}[q] =& \sum_{l=0}^{L-1} \mathbf{h}_k[l]^\dagger\, \mathbf{h}_r[l-q] \\ =& \sum_{p=1}^{P_k} \sum_{i=1}^{P_r} \sqrt{\beta_{k,p}}\, \sqrt{\beta_{r,i}}\, \mathbf{a}(\theta_{k,p})^\dagger\, \mathbf{a}(\theta_{r,i})\, e^{-\jmath\phi_{k,p}} e^{\jmath\phi_{r,i}} \\ & \cdot \sum_{l=0}^{L-1} \gamma(l - W\,\tau_{k,p})\, \gamma(l - q - W\,\tau_{r,i})\,. \end{aligned} \tag{7}$$

The four terms put in evidence in (6) are, respectively, the useful, inter-symbol interference (ISI), multi-user interference (MUI), and the thermal noise components.

III. Channel Hardening

The key channel property exploited in massive MIMO systems is the (asymptotic) *channel hardening*, i.e., when the equivalent channel between the BS and the user tends to become deterministic when letting the number M of antennas grow to infinity. More precisely, channel hardening is a local

[1] The precoding filter is not casual for notation convenience. In practical implementations the filter will introduce a delay of L time instants.

property of a particular location/configuration (scenario Θ) and appears when

$$\frac{g_{k,k}[0]}{\mathbb{E}\left\{g_{k,k}[0]|\Theta\right\}} \to 1 \quad \text{as } M \to \infty, \tag{8}$$

where the convergence has to be intended in probability. The denominator in (8) is

$$\mathbb{E}\left\{g_{k,k}[0]|\Theta\right\} = \sum_{p=1}^{P_k} \beta_{k,p} \sum_{l=0}^{L-1} |\gamma(l - W\,\tau_{k,p})|^2 . \tag{9}$$

Since the converge in mean square sense implies convergence in probability, then (8) is satisfied when

$$\mathsf{CH} = \frac{\mathrm{Var}\left(g_{k,k}[0]|\Theta\right)}{\left(\mathbb{E}\left\{g_{k,k}[0]|\Theta\right\}\right)^2} \to 0 \quad \text{as } M \to \infty \tag{10}$$

where

$$\mathrm{Var}\left(g_{k,k}[0]|\Theta\right) = \sum_{p=1}^{P_k} \sum_{i=1, i\neq p}^{P_k} \beta_{k,p}\, \beta_{k,i} |g_a(\theta_{k,p}, \theta_{k,i})|^2 \cdot |\gamma(W\,(\tau_{k,p} - \tau_{k,i}))|^2 , \tag{11}$$

having defined $g_a(\theta_1, \theta_2) = \mathbf{a}^{\dagger}(\theta_1)\, \mathbf{a}(\theta_2)$. We consider the following upper bound for $|g_a(\theta_1, \theta_2)| \leq C_a(\theta_1, \theta_2)$, with

$$C_a(\theta_1, \theta_2) = \begin{cases} 1 & |\sin(\theta_1) - \sin(\theta_2)| \leq \frac{1}{M} \\ \min\left(1, \frac{1}{M(\sin(\theta_1) - \sin(\theta_2))}\right) & \text{otherwise} \end{cases} \tag{12}$$

In addition we make the following approximation: $\gamma(x) = \mathrm{sinc}\,(x) \approx T(x) = 1$ for $|x| \leq 1/2$, zero otherwise, whose marginal impact on the performance will be assessed in the numerical results. As a consequence, (10) can be approximated as

$$\mathsf{CH} \simeq \frac{\sum_{p=1}^{P_k} \sum_{i=1, i\neq p}^{P_k} \beta_{k,p}\, \beta_{k,i}\, C_a^2(\theta_{k,p}, \theta_{k,i})\, T(W\,(\tau_{k,p} - \tau_{k,i}))}{\sum_{p=1}^{P_k} \sum_{i=1}^{P_k} \beta_{k,p}\, \beta_{k,i}} . \tag{13}$$

Since channel hardening is a local measure, i.e., $\mathsf{CH} = \mathsf{CH}(\Theta)$, when analyzing large networks it is more meaningful to introduce the following *channel hardening measure* [5]:

$$P_{\mathsf{CH}}(\eta) = \mathrm{P}\left\{\mathsf{CH} < \eta\right\} , \tag{14}$$

with respect to different scenarios Θ. $P_{\mathsf{CH}}(\eta)$ is the CDF of the RV CH and provides the percentage of randomly located users that experience CH smaller than a certain desirable threshold η (e.g., -20 dB). In other words, it represents the fading-free coverage of the massive MIMO network.

IV. Favorable Propagation

Asymptotic *favorable propagation* is the condition such that the equivalent channels between the BS and the users tend to be orthogonal as M grows, i.e., $\forall q$

$$\frac{g_{k,r}[q]}{\sqrt{\mathbb{E}\left\{g_{k,k}[0]|\Theta\right\} \mathbb{E}\left\{g_{r,r}[0]|\Theta\right\}}} \to 0 \quad \text{as } M \to \infty, \text{ for } r \neq k \tag{15}$$

Since $\mathbb{E}\left\{g_{k,r}[q]|\Theta\right\} = 0$ for $r \neq k$ and $\forall q$, the sufficient condition for favorable propagation for the generic user k is

$$\mathsf{FP} = \sum_{r=1, r\neq k}^{K} \frac{\mathrm{Var}\left(\sum_{q=-L+1}^{L-1} g_{k,r}[q]|\Theta\right)}{\mathbb{E}\left\{g_{k,k}[0]|\Theta\right\} \mathbb{E}\left\{g_{k,r}[0]|\Theta\right\}} \to 0 \quad \text{as } M \to \infty \tag{16}$$

After some math and using (7) it is

$$\mathrm{Var}\left(\sum_{q=-L+1}^{L-1} g_{k,r}[q]|\Theta\right) = \sum_{p=1}^{P_k} \sum_{i=1}^{P_r} \beta_{k,p}\, \beta_{r,i}\, |g_a(\theta_{k,p}, \theta_{r,i})|^2 \cdot \sum_{q=-L+1}^{L-1} |\gamma(W\,(\tau_{k,p} - \tau_{r,i}) - q)|^2$$

which, substituted in (16) and making the same approximations as for CH, gives

$$\mathsf{FP} \simeq \sum_{r=1, r\neq k}^{K} \frac{\sum_{p=1}^{P_k} \sum_{i=1}^{P_r} \beta_{k,p}\, \beta_{r,i} C_a^2(\theta_{k,p}, \theta_{r,i})\, E_{k,r}(p,i)}{\sum_{p=1}^{P_k} \sum_{i=1}^{P_r} \beta_{k,p}\, \beta_{r,i}} \tag{17}$$

with $E_{k,r}(p,i) = \sum_{q=-L+1}^{L-1} T(W\,(\tau_{k,p} - \tau_{r,i}) - q)$. When operating at network level, analogously to channel hardening, we consider the following *favorable propagation measure*:

$$P_{\mathsf{FP}}(\eta) = \mathrm{P}\left\{\mathsf{FP} < \eta\right\} \tag{18}$$

with respect to different scenarios Θ. It represents the interference-free coverage of the massive MIMO network.

V. Favorable Equalization

Here we introduce the new concept of (asymptotic) *favorable equalization* as the condition such that the ISI becomes negligible with respect to the useful component, i.e., the channel becomes frequency flat, as M grows. With reference to the generic user k, it is defined as follows

$$\frac{g_{k,k}[q]}{(\mathbb{E}\left\{g_{k,k}[0]|\Theta\right\})^2} \to 0 \quad \text{as } M \to \infty, \ \forall q \neq 0 . \tag{19}$$

Considering that $\mathbb{E}\left\{g_{k,k}[q]|\Theta\right\} = 0$ for $q \neq 0$, the sufficient condition for favorable equalization is

$$\mathsf{FE} = \frac{\mathrm{Var}\left(\sum_{q=-L+1, q\neq 0}^{L-1} g_{k,k}[q]|\Theta\right)}{\left(\mathbb{E}\left\{g_{k,k}[0]|\Theta\right\}\right)^2} \to 0 \quad \text{as } M \to \infty \tag{20}$$

The numerator of (20) is

$$\mathrm{Var}\left(\sum_{q=-L+1, q\neq 0}^{L-1} g_{k,k}[q] | \Theta\right) = \sum_{p=1}^{P_k} \sum_{i=1, i\neq p}^{P_k} \beta_{k,p}\, \beta_{k,i} |g_a(\theta_{k,p}, \theta_{k,i})|^2 \cdot \sum_{q=-L+1, q\neq 0}^{L-1} |\gamma(W(\tau_{k,p} - \tau_{k,i}) - q)|^2$$

which substituted in (20) (and making the same approximation as for CH) gives

$$\mathsf{FE} \simeq \frac{\sum_{p=1}^{P_k} \sum_{i=1, i\neq p}^{P_k} \beta_{k,p}\, \beta_{k,i}\, C_a^2(\theta_{k,p}, \theta_{k,i})\, D_k(p,i)}{\sum_{p=1}^{P_k} \sum_{i=1}^{P_k} \beta_{k,p}\, \beta_{k,i}}, \quad (21)$$

having defined $D_k(p,i) = \sum_{q=-L+1, q\neq 0}^{L-1} T(W(\tau_{k,p} - \tau_{k,i}) - q)$.

As for favorable propagation, we introduce the *favorable equalization measure*:

$$P_{\mathsf{FE}}(\eta) = \mathrm{P}\{\mathsf{FE} < \eta\} \quad (22)$$

with respect to different scenarios Θ. It represents the ISI-free coverage of the massive MIMO network.

VI. Results for Typical Propagation scenarios

We derive simple explicit relations for $P_{\mathsf{FP}}(\eta)$, $P_{\mathsf{CH}}(\eta)$, and $P_{\mathsf{FE}}(\eta)$ in remarkable propagation scenarios in order to get some insights about the impact of M and W on frequency-selective massive MIMO channel characteristics.

We consider a Clarke-based model in which a uniform set of $P_k = P$, $\forall k$, scatterers is supposed to be present around the transmitter at a distance of R_s meters and $\beta_{k,p} \approx \beta_{k,0}$, $p = 1, 2, \ldots P$ [4]. Note that in this case the channel delay spread is $\tau_d = 2 R_s/c$, being c speed of light. Under this model, equations (13), (17) and (21) simplify, respectively, to:

$$\mathsf{CH} \simeq \frac{2}{P^2} \sum_{p=1}^{P-1} \sum_{i=p+1}^{P} C_a^2(\theta_{k,p}, \theta_{k,i}) T(W(\tau_{k,p} - \tau_{k,i})) \quad (23)$$

$$\mathsf{FP} \simeq \frac{1}{P^2} \sum_{r=1, r\neq k}^{K} \sum_{p=1}^{P} \sum_{i=1}^{P} C_a^2(\theta_{k,p}, \theta_{r,i}) \quad (24)$$

$$\mathsf{FE} \simeq \frac{2}{P^2} \sum_{p=1}^{P-1} \sum_{i=p+1}^{P} C_a^2(\theta_{k,p}, \theta_{k,i})\, D_k(p,i)\,. \quad (25)$$

We start analyzing the favorable propagation measure. According to the Clarke's model, $\theta_{k,p}$ and $\theta_{k,i}$ are independent and uniformly distributed in $[0, 2\pi)$, therefore, for large M, $C_a(\theta_{k,p}, \theta_{k,i})$ tends to be distributed as a Bernoulli RV with parameter $p_x = \sum_{n=1}^{M} p_n^2$, where p_n represents the probability that $\theta_{k,p}$ falls in the nth cosine direction of width $2/M$ (see (12)), i.e,

$$p_n = \mathrm{P}\left\{\theta_{k,p} \in \left[-1 + \frac{2(n-1)}{M}, -1 + \frac{2n}{M}\right]\right\} = \frac{1}{\pi}\left(\arcsin\left(-1 + \frac{2n}{M}\right) - \arcsin\left(-1 + \frac{2(n-1)}{M}\right)\right) \quad (26)$$

and p_x is the probability that that both $\theta_{k,p}$ and $\theta_{k,i}$ fall in the same cosine direction of width $2/M$, thus providing a contribution different from zero in (24).

Define now X_j, $j = 1, 2, \ldots, J$, with $J = (K-1) P^2$, a set of i.i.d. Bernoulli RVs with parameter p_x. It follows that (24) is statistically equivalent to the following RV

$$\mathsf{FP} \simeq \frac{1}{P^2} \sum_{j=1}^{J} X_j = \frac{1}{P^2} Y \quad (27)$$

being Y a Binomial RV with parameters p_x and J. Therefore, the favorable propagation measure can be obtained as

$$P_{\mathsf{FP}}(\eta) \simeq \mathrm{CB}\left(\eta\, P^2; p_x, (K-1)\, P^2\right) \quad (28)$$

with $\mathrm{CB}(x; p, n)$ denoting the CDF of the Binomial RV with parameters p and n. Note that p_x decreases proportionally to M and the same does $\mathbb{E}\{\mathsf{FP}\} = p_x\, J/P^2 = p_x\, (K-1)$.

Regarding the evaluation of the channel hardening measure in (23), using similar arguments as for favorable propagation, it is

$$\mathsf{CH} \simeq \frac{2}{P^2} \sum_{j=1}^{J} X_j = \frac{2}{P^2} Y\,, \quad (29)$$

with $J = (P-1)(P-2)/2$. Now $p_x = \sum_{n=1}^{M} p_n^2$ is the probability that that both $\theta_{k,p}$ and $\theta_{k,i}$ fall in the same cosine direction of width $2/M$ and $\tau_{k,p}$, $\tau_{k,i}$ fall in the same time bin of width $2/W$, thus providing a contribution different from zero in (23), where

$$p_n = \frac{1}{\pi\, \tau_d\, W}\left(\arcsin\left(-1 + \frac{2n}{M}\right) - \arcsin\left(-1 + \frac{2(n-1)}{M}\right)\right)\,.$$

The channel hardening CDF results

$$P_{\mathsf{CH}}(\eta) \simeq \mathrm{CB}\left(\eta\, P^2/2; p_x, (P-1)(P-2)/2\right)\,. \quad (30)$$

The mean value of CH decreases proportionally to $M\, \tau_d\, W$.

Favorable equalization is the same as channel hardening with p_n given by

$$p_n = \left(\frac{\tau_d\, W - 1}{\pi \tau_d\, W}\right)\left(\arcsin\left(-1 + \frac{2n}{M}\right) - \arcsin\left(-1 + \frac{2(n-1)}{M}\right)\right)$$

where p_x now represents the probability that that both $\theta_{k,p}$ and $\theta_{k,i}$ fall in the same cosine direction of width $2/M$, and $\tau_{k,p}$, $\tau_{k,i}$ do not fall in the same time bin of width $2/W$, thus providing a contribution different from zero in (25). The mean value of FE decreases proportional to M.

Another remarkable propagation scenario is the non-fading LOS. In this case it is $P = 1$, therefore it follows that $\mathsf{CH} = \mathsf{FE} = 0$ and only the MUI affects the performance. The favorable propagation measure is still given by (28) with $P = 1$.

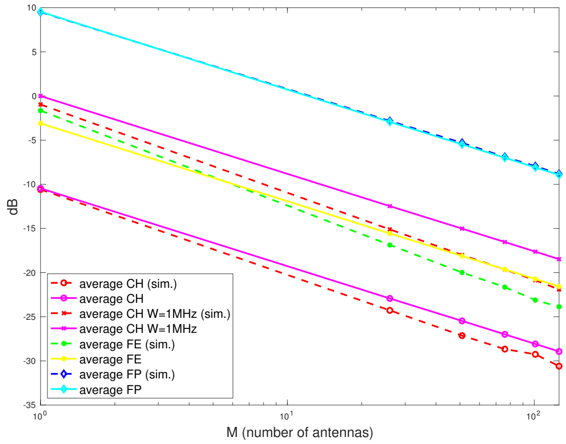

Fig. 1. Average CH, FE and FP as a function of the number of antennas.

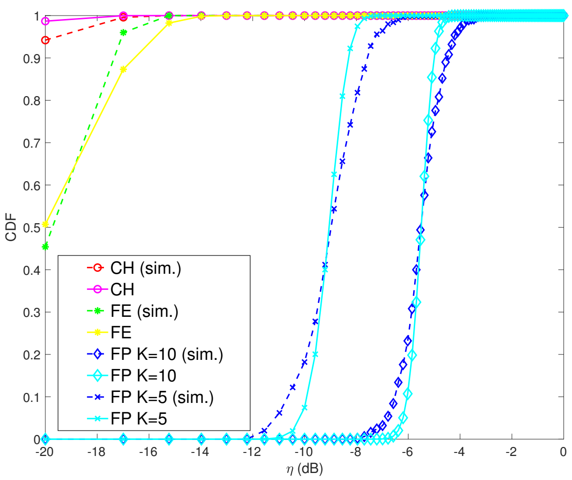

Fig. 2. CDF of CH, FP and FE with $M = 50$, $W = 20\,\text{MHz}$.

A. Numerical results

If not otherwise specified, in the numerical results we consider the following parameters: $W = 20\,\text{MHz}$, $R = 1000\,\text{meters}$, $R_s = 50\,\text{meters}$, $P = 100$, $K = 10$ users, Clarke's model, $\lambda = 1\,\text{cm}$. In Fig. 1 the average CH, FE, and FP as a function of M are reported. Approximated analytical results are compared with Monte Carlo simulations confirming that FP and FE decrease proportionally to M regardless the channel bandwidth W. On the contrary, channel bandwidth affects the behavior of CH, in particular it contributes to accelerate the rate of hardening. Similar curve for FP is obtained in the non-fading LOS channel.

The CDFs of CH, FE, and FP are shown in Fig. 2 for $M = 50$. The approximated analytical expressions (30), (28) are compared with Monte Carlo simulation results showing the capability of the former to capture the main behavior. Typically, the largest impact on coverage is determined by the MUI which is dominant as soon as $K > 1$. From this plot one can investigate the percentage of user locations experiencing the "massive channel", i.e., fading, ISI and MUI free.

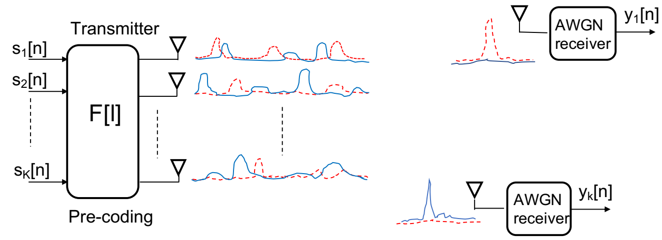

Fig. 3. Massive waveforming.

VII. Massive Waveforming and Conclusions

With reference to Fig. 3, the space-time matching pre-filter considered realizes a sort of *massive waveforming* scheme in which, for large M, coherent combination of waveforms is obtained in most of intended user locations (according to the CDF analysis above). It is important to notice that under channel hardening, favorable propagation and favorable equalization conditions, the link between the BS and the generic user becomes AWGN-like, i.e., $y_k[n] \simeq h_0\, s_k[n] + w_k[n]$, $\forall n$, regardless the bandwidth W, with $h_0 = \mathbb{E}\left\{g_{k,k}[0]|\Theta\right\}$ being a deterministic constant depending on scenario configuration Θ. As a consequence, K parallel high-speed links can be realized using simple AWGN receivers at user side with no CSI. This allows the adoption of extremely low-complexity and energy-efficient receivers as the complexity, in the order of $O(K\,M\,L)$ as worst-case estimate,[2] is entirely moved to the BS. What is interesting to underline is that at the receiver neither equalizers nor OFDM-like schemes are needed (hence no cyclic prefix overhead price is paid). Future investigation will address the impact of imperfect CSI on wideband massive MIMO channel characteristics.

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[2]Since $\mathbf{F}[l]$ is likely sparse, smarter processing schemes can be adopted to reduce the complexity.