Reducing the Bias in DRSS-Based Localization: An Instrumental Variable Approach

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***Abstract*—This paper proposes a closed-form solution with reduced bias for differential received signal strength (DRSS) localization. During the linearization of DRSS measurement equations, the measurement noise is injected into the measurement data matrix, resulting in a correlation between the measurement noise and measurement data matrix. Existing closed-form solutions do not consider this correlation, which causes biased estimation results. The solution proposed here aims to eliminate the bias by introducing instrument variables (IV), whose role is to mitigate the correlation arising from linearization. Simulation results demonstrate the improved performance of the IV-based estimator over some existing closed-form solutions, in the form of root-mean-squared errors that are close to the Cramér-Rao lower bound, and significantly reduced bias, over a wide range of noise levels.**

***Index Terms*—Differential received signal strength, localization, instrumental variable, best linear unbiased estimator**

I. Introduction

Wireless Sensor Networks (WSNs) are regarded as an important technology in the 21st century in the field of sensing and telecommunications due to their high-fidelity sensing capabilities, ease of expansion and scalability [1], [2]. A large number of inexpensive, versatile sensors can be networked through wireless links realizing a wide variety of applications. WSNs are already becoming widely accepted as a standard technology in areas such as critical infrastructure, physical security, environmental science and manufacturing. For instance, in defence they are applied to large-scale acoustic surveillance and ground target detection [1], [3].

For sensor data to be useful, they need to be tagged with location information, hence localization is an important function of WSNs. Common localization methods employ sensing based on angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), received signal strength (RSS) and differential RSS (DRSS). Compared to AOA, TOA and TDOA, the RSS-based and DRSS-based localization techniques do not require additional hardware or precise clock synchronization for the sensors, so they facilitate economical and low-complexity implementations [4]–[6]. However, RSS-based localization methods strongly rely on accurate information of the transmitter and environmental parameters such as transmitter power and propagation loss factors. In many practical scenarios, even if the environmental parameters can be precisely determined, the transmitter parameters are not always available or accurately known because the manual operation of getting the transmitter parameters is time consuming if not infeasible [4]. In comparison, using DRSS eliminates or reduces the need for knowledge of the transmitter and environmental parameters, and consequently is more practical [4], [6].

DRSS-based localization involves solving a nonlinear and nonconvex optimization problem. One of the most popular solutions is based on the maximum likelihood estimator (MLE). The MLE is asymptotically efficient and unbiased. However, the nonlinear and nonconvex property of the MLE cost function makes it only possible to get the local minimum and a large estimation error occurs if it converges to a wrong local minimum point. As a result, a good initial point is required to reach the global minimum [7]–[9]. In addition, when the measurement noise is high, the MLE suffers from the threshold effect, which causes divergence issues [10]. In [11] many iterative algorithms for the MLE solution are compared, including steepest descent, Gauss-Newton, Levenberg-Marquardt (LM), and trust region (TR), and the TR method is recommended. However, when the number of unknown parameters is large, the search time becomes impractical [7]. Semidefinite Programming (SDP) can be used to convert a nonconvex MLE problem into a convex optimization problem using relaxation methods, which leads to a simple and efficient MLE implementation that guarantees the convergence to the global minimum solution [7], [12]. However, the SDP relaxation method can only provide a suboptimal solution and cannot offer the best possible performance in all situations [13]. An SDP solution of the DRSS optimization problem was proposed in [6]. It was observed that when the measurement noise is close to zero, the SDP method could not even give as good a result as weighted least squares [13]. Pseudolinear estimation is another popular method of source localization (see, e.g., [14] and the references therein). Thanks to its closed-form solution, the advantage of this approach is that it does not require iterative computation or initialisation, and has smaller computational complexity than the SDP method and the MLE [13]. However, pseudolinear estimation techniques have severe bias problems due to the correlation between the measurement matrix and the noise vector, which severely degrades the estimation result [15]. As variants of pseudolinear estimation, the work in [6] and [16] developed the advanced best linear unbiased estimator (A-BLUE) and 2-Step Weighted Least Squares (2-Step WLS) algorithm, respectively, for DRSS

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localization. Even though these solutions have smaller bias and more accurate estimation results than the originally unconstrained LS solution, the bias problem is still not avoided.

Our main contribution is to develop a reduced-bias closed-form solution for DRSS-based localization, that uses the method of the *instrumental variables* (IV). The basic principle of the IV method is eliminating or reducing the correlation between the noise and the data matrix, by replacing the linearized data matrix with the IV matrix in the normal equations [17], [18]. The advantage of the IV method is that it can achieve asymptotically optimal performance similar to the MLE at a significantly reduced computational complexity.

The paper is organized as follows. Section II provides the formal problem definition. Section III briefly discusses existing closed-form solutions based on the linearization of DRSS measurements. Section IV proposes our IV-based algorithm, supported by simulation results given in Section V. The concluding remarks are made in Section VI.

II. Problem definition

Given N sensors distributed in a two-dimensional (2D) plane, whose locations are *a priori* known, we are concerned with the problem of localizing a target node of unknown location in the same 2D plane. Suppose the target node has unknown location $\boldsymbol{x} = [x, y]^\intercal$, where $^\intercal$ denotes matrix transpose; and the N known sensors — called *anchors* — have known locations $\boldsymbol{s}_i = [x_i, y_i]^\intercal$, $i = 1, \ldots, N$. The distance between the ith sensor and the target node is then

$$\|\boldsymbol{d}_i\|_2 = \|\boldsymbol{x} - \boldsymbol{s}_i\|_2 = \sqrt{(x - x_i)^2 + (y - y_i)^2}, \tag{1}$$

where $\|\cdot\|_2$ denotes the Euclidean norm. When the target node broadcasts a signal to the N anchors, each known sensor records the strength of the signal in the form of a received signal strength indicator (RSSI). Differential received signal strength (DRSS) refers to the differences among these N RSSIs. The principle of DRSS-based localization is to determine the location of the target node, $\boldsymbol{x}$, based on the relationship between the DRSS values and the distances of the N anchors from the target node, $\|\boldsymbol{d}_i\|_2$.

While there are $\binom{N}{2} = \frac{N(N-1)}{2}$ DRSS values in a network of N anchors, there are $N-1$ basic/independent values, and $\frac{(N-1)(N-2)}{2}$ redundant values [4]. To obtain $N-1$ independent values, we choose one sensor as the reference sensor, denoted by $\boldsymbol{s}_1$. If $P_{1,j} \triangleq P_j - P_1$ is the DRSS in dBW between s_1 and the jth known sensor, where $j \in \{2, \ldots, N\}$, then the $N-1$ DRSS values can be expressed as [6]

$$\begin{aligned} P_{1,j} &= G_j - G_1 - 10\gamma \log_{10}\left(\frac{\|\boldsymbol{d}_j\|_2}{\|\boldsymbol{d}_1\|_2}\right) + n_{1,j}, \\ &= G_j - G_1 - 10\gamma \log_{10}\left(\frac{\|\boldsymbol{x} - \boldsymbol{s}_j\|_2}{\|\boldsymbol{x} - \boldsymbol{s}_1\|_2}\right) + n_{1,j}, \end{aligned} \tag{2}$$

where

- γ is the path loss exponent, which measures the rate of signal strength decay with distance and is assumed *a priori* known;
- G_j and G_1 are the sensors' antenna gains and are assumed to be equal;
- $n_{1,j} = n_j - n_1$ is the difference between the measurement noises on P_j and P_1.

Assuming the received signal strength measurements in dB are subject to independent, identically distributed (i.i.d.) zero-mean additive Gaussian noise (log-normal shadowing), i.e., $n_i \sim N(0, \sigma_n^2)$, $i \in \{1, \ldots, N\}$, where σ_n^2 is the RSS noise variance, the covariance matrix of $[n_{1,2}, \cdots, n_{1,N}]^\intercal$ is given by

$$\boldsymbol{\Sigma} = \sigma_n^2 \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \cdots & 1 & 2 \end{bmatrix}, \tag{3}$$

and the target location can be obtained by solving the set of equations from (2). According to [4], the minimum set of sensors in theory is four, i.e., $N \geq 4$.

III. Existing Closed-Form Solutions

The nonlinear measurement equation (2) can be linearized into the form [6]

$$\boldsymbol{p} = \boldsymbol{\Psi}\boldsymbol{\phi} + \boldsymbol{\epsilon}, \tag{4}$$

where

$$\boldsymbol{p} = \begin{bmatrix} \|\boldsymbol{s}_1\|_2^2 - \|\boldsymbol{s}_2\|_2^2 P'_{1,2} \\ \|\boldsymbol{s}_1\|_2^2 - \|\boldsymbol{s}_3\|_2^2 P'_{1,3} \\ \vdots \\ \|\boldsymbol{s}_1\|_2^2 - \|\boldsymbol{s}_N\|_2^2 P'_{1,N} \end{bmatrix}, \qquad \boldsymbol{\phi} = \begin{bmatrix} \boldsymbol{x} \\ r \end{bmatrix},$$

$$\boldsymbol{\Psi} = \begin{bmatrix} 2\boldsymbol{s}_1^\intercal - 2P'_{1,2}\boldsymbol{s}_2^\intercal & P'_{1,2} - 1 \\ 2\boldsymbol{s}_1^\intercal - 2P'_{1,3}\boldsymbol{s}_3^\intercal & P'_{1,3} - 1 \\ \vdots & \vdots \\ 2\boldsymbol{s}_1^\intercal - 2P'_{1,N}\boldsymbol{s}_N^\intercal & P'_{1,N} - 1 \end{bmatrix}, \quad \boldsymbol{\epsilon} = \begin{bmatrix} -n'_{1,2}\|\boldsymbol{d}_1\|_2^2 \\ -n'_{1,3}\|\boldsymbol{d}_1\|_2^2 \\ \vdots \\ -n'_{1,N}\|\boldsymbol{d}_1\|_2^2 \end{bmatrix},$$

$P'_{1,i} \triangleq 10^{\frac{P_{1,i}}{5\gamma}}$, $n'_{1,i} \triangleq 10^{\frac{n_{1,i}}{5\gamma}} - 1$, and $r \triangleq \|\boldsymbol{x}\|_2^2$ is the auxiliary variable.

Since $n'_{1,i}$ is in exponential form, it is difficult to calculate the covariance of $\boldsymbol{\epsilon}$. When the shadowing effect is sufficiently small, $n'_{1,i}$ can be approximated by its first-order Taylor series expansion, which is $\frac{n_{1,i}}{5\gamma}\ln(10)$. Then the covariance of $\boldsymbol{\epsilon}$ is given by

$$\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} \approx \|\boldsymbol{d}_1\|_2^4 \left(\frac{\ln(10)}{5\gamma}\right)^2 \sigma_n^2 \boldsymbol{\Sigma} \tag{5}$$

As $\|\boldsymbol{d}_1\|_2^4 \left(\frac{\ln(10)}{5\gamma}\right)^2$ is a constant and the scaling of $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}$ does not influence the solution to (4), $\boldsymbol{\Sigma}$ can be used instead as the weighting matrix for solving (4).

A. 2-Step Weighted Least Squares (2-Step WLS)

Lin et al. [16] provided a closed-form solution that consists of two steps/stages. The first step uses the method of linear least squares to estimate the target position $\boldsymbol{x}_0$ and auxiliary variable r, which are given by

$$\hat{\boldsymbol{\phi}} = (\boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi})^{-1} \boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{p}, \tag{6}$$

where $\hat{\boldsymbol{\phi}} = \begin{bmatrix} \hat{\boldsymbol{x}}_0^\intercal & \hat{r} \end{bmatrix}^\intercal$, $\hat{\boldsymbol{x}}_0$ is the initial location estimate, and $\hat{r}$ is the initial estimate of the auxiliary variable. Note $\hat{r} \neq \|\hat{\boldsymbol{x}}_0\|_2^2$ because of estimation errors.

The second step improves the estimation accuracy by considering the constraint relationship between $\boldsymbol{x}_0$ and r, and yields

$$\hat{\boldsymbol{\phi}}_2 = (\boldsymbol{\Psi}_2^\intercal \boldsymbol{\Sigma}_2^{-1} \boldsymbol{\Psi}_2)^{-1} \boldsymbol{\Psi}_2^\intercal \boldsymbol{\Sigma}_2^{-1} \boldsymbol{p}_2, \tag{7}$$

where

$$\boldsymbol{p}_2 = \begin{bmatrix} \hat{x}_0^2 \\ \hat{y}_0^2 \\ \hat{r} \end{bmatrix}, \quad \boldsymbol{\Psi}_2 = \begin{bmatrix} \mathbf{I}_{2\times 2} \\ \mathbf{1}_{1\times 2} \end{bmatrix},$$
$$\boldsymbol{\Sigma}_2 = \operatorname{diag}(2x, 2y, 1)(\boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi})^{-1} \operatorname{diag}(2x, 2y, 1).$$

Above, $\hat{x}_0$ and $\hat{y}_0$ are the initial 2-dimensional location estimates. Note that $\boldsymbol{\Sigma}_2$ is a function of the true target location (x, y), but since this is unknown, $\boldsymbol{\Sigma}_2$ is approximated using the initial estimates $\hat{x}_0$ and $\hat{y}_0$.

The final location estimate is calculated as

$$\hat{\boldsymbol{x}} = \left[\operatorname{sign}(\hat{x}_0)\sqrt{\hat{\boldsymbol{\phi}}_2(1)} \quad \operatorname{sign}(\hat{y}_0)\sqrt{\hat{\boldsymbol{\phi}}_2(2)} \right]^\intercal, \tag{8}$$

where $\operatorname{sign}(\cdot)$ is the signum function.

B. A-BLUE

Hu and Leus [6] provided a closed-form solution called the A-BLUE method. Firstly, (4) is reformulated as the whitened model:

$$\begin{aligned} \boldsymbol{\Sigma_\epsilon}^{-\frac{1}{2}} \boldsymbol{p} &= \boldsymbol{\Sigma_\epsilon}^{-\frac{1}{2}} \boldsymbol{\Psi\phi} + \boldsymbol{\Sigma_\epsilon}^{-\frac{1}{2}} \boldsymbol{\epsilon}, \\ \Rightarrow (\boldsymbol{\Gamma\Gamma}^\intercal)^{-1/2} \boldsymbol{p} &= (\boldsymbol{\Gamma\Gamma}^\intercal)^{-1/2} \boldsymbol{\Psi\phi} + (\boldsymbol{\Gamma\Gamma}^\intercal)^{-1/2} \boldsymbol{\epsilon}, \\ \Rightarrow \boldsymbol{p}_w &= \boldsymbol{\Psi}_w \boldsymbol{\phi} + \boldsymbol{\epsilon}_w, \end{aligned} \tag{9}$$

where

$$\boldsymbol{\Gamma} \approx \begin{bmatrix} -\mathbf{1}_{(N-1)\times 1} & \mathbf{I}_{N-1} \end{bmatrix}_{N-1\times N},$$
$$\boldsymbol{\Sigma_\epsilon} \approx \|\boldsymbol{d}_1\|_2^4 \left(\frac{\ln(10)}{5\gamma} \right)^2 \sigma_n^2 \boldsymbol{\Gamma\Gamma}^\intercal, \quad \boldsymbol{\Psi}_w = (\boldsymbol{\Gamma\Gamma}^\intercal)^{-1/2} \boldsymbol{\Psi},$$
$$\boldsymbol{p}_w = (\boldsymbol{\Gamma\Gamma}^\intercal)^{-1/2} \boldsymbol{p}, \quad \boldsymbol{\epsilon}_w = (\boldsymbol{\Gamma\Gamma}^\intercal)^{-1/2} \boldsymbol{\epsilon}.$$

The solution to (9) is given by

$$\hat{\boldsymbol{\phi}}_{u-blue} = (\boldsymbol{\Psi}_w^\intercal \boldsymbol{\Psi}_w)^{-1} \boldsymbol{\Psi}_w^\intercal \boldsymbol{p}_w, \tag{10}$$

where $\hat{\boldsymbol{\phi}}_{u-blue} = \begin{bmatrix} \hat{\boldsymbol{x}}_{u-blue}^\intercal & \hat{r} \end{bmatrix}^\intercal$, $\hat{\boldsymbol{x}}_{u-blue}$ is the initial location estimate, and $\hat{\boldsymbol{r}}$ is the initial estimate of the auxiliary variable. In [6] this solution was named the *unconstrained best linear unbiased estimator* (U-BLUE).

To keep the constraint among the entries of $\hat{\boldsymbol{\phi}}_{u-blue}$, in [6] the *advanced best linear unbiased estimator* (A-BLUE) was proposed:

$$\hat{\boldsymbol{x}}_{a-blue} = \hat{\boldsymbol{x}}_{u-blue} - (\boldsymbol{\Psi}_{2w}^\intercal \boldsymbol{\Psi}_w^\intercal \boldsymbol{\Psi}_w \boldsymbol{\Psi}_{2w})^{-1} \boldsymbol{\Psi}_{2w}^\intercal \boldsymbol{\Psi}_w^\intercal \boldsymbol{\Psi}_w \boldsymbol{p}_2, \tag{11}$$

where

$$\boldsymbol{\Psi}_{2w} = \begin{bmatrix} \mathbf{I}_2 \\ 2\hat{\boldsymbol{x}}_{u-blue}^\intercal \end{bmatrix}, \quad \boldsymbol{p}_2 = \begin{bmatrix} \mathbf{0}_{2\times 1} \\ \|\hat{\boldsymbol{x}}_{u-blue}\|_2^2 - \hat{r} \end{bmatrix}. \tag{12}$$

IV. Proposed Solution for Reducing Bias Using Instrumental Variables

The weighted least squares solution in (6) is biased because the matrix $\boldsymbol{\Psi}$ is injected with measurement noise $n_{1,j}$ during the linearization process, which leads to a correlation between the data matrix and measurement noise vector [19].

An estimator is unbiased if $\mathbb{E}\{\hat{\boldsymbol{\phi}}\} = \boldsymbol{\phi}$, or equivalently $\mathbb{E}\{\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\} = 0$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator [20]. Using (6) as the estimator for $\boldsymbol{\phi}$ in (4), we have

$$\mathbb{E}\{\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\} = \mathbb{E}\{(\boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi})^{-1} \boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}\}. \tag{13}$$

Under a mild condition with sufficiently large number of measurements and/or small measurement noise, using Slutsky's theorem [21], (13) can be approximated as

$$\mathbb{E}\{\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\} \approx \mathbb{E}\{\boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi}\}^{-1} \mathbb{E}\{\boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}\}. \tag{14}$$

If $\boldsymbol{\Psi}$ is statistically independent of $\boldsymbol{\epsilon}$ (thus $\mathbb{E}\{\boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}\} = \mathbb{E}\{\boldsymbol{\Psi}\}^\intercal \boldsymbol{\Sigma}^{-1} \mathbb{E}\{\boldsymbol{\epsilon}\}$) and the noise is zero mean, i.e., $\mathbb{E}\{\boldsymbol{\epsilon}\} = \mathbf{0}$, we have $\mathbb{E}\{\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\} \approx \mathbf{0}$ and the estimator (6) is approximately unbiased.

However, $\boldsymbol{\Psi}$ is *in fact not independent* of $\boldsymbol{\epsilon}$ because $\boldsymbol{\Psi}$ is constructed from noisy DRSS measurements (see (4)). For this reason, there exists a correlation between $\boldsymbol{\Psi}$ and $\boldsymbol{\epsilon}$, thus leading to $\mathbb{E}\{\boldsymbol{\Psi}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}\} \neq \mathbf{0}$. As a result, we have

$$\mathbb{E}\{\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\} \neq \mathbf{0}, \tag{15}$$

and the estimator (6) is biased. Consequently, this makes the 2-Step WLS estimator biased. Similarly, the A-BLUE method can be shown to be biased due to the correlation between $\boldsymbol{\Psi}_w$ and $\boldsymbol{\epsilon}_w$.

To overcome this bias problem, we now propose a new method by introducing an IV matrix $\mathbf{F}$ into the 2-Step WLS method, where $\mathbf{F}$ is strongly correlated with $\boldsymbol{\Psi}$ and approximately uncorrelated with $\boldsymbol{\epsilon}$, as desired. Specifically, the weighted least-squares solution in (6) is modified as

$$\hat{\boldsymbol{\phi}}^{IV} = (\mathbf{F}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi})^{-1} \mathbf{F}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{p}. \tag{16}$$

Provided that the IV matrix $\mathbf{F}$ is selected such that $\mathbb{E}\left\{\frac{\mathbf{F}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi}}{N-1}\right\}$ is nonsingular, and $\mathbb{E}\left\{\frac{\mathbf{F}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}}{N-1}\right\} = 0$ as $N \to \infty$, we have

$$\mathbb{E}\left\{\hat{\boldsymbol{\phi}}^{IV} - \boldsymbol{\phi}\right\} = \mathbb{E}\left\{\frac{\mathbf{F}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\Psi}}{N-1}\right\}^{-1} \mathbb{E}\left\{\frac{\mathbf{F}^\intercal \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}}{N-1}\right\} = \mathbf{0} \tag{17}$$

as $N \to \infty$, i.e., $\hat{\boldsymbol{\phi}}^{IV}$ in (16) becomes asymptotically unbiased [17], [18]. Letting $\hat{\boldsymbol{\phi}}^{IV} = [\hat{x}^{IV} \quad \hat{y}^{IV} \quad \hat{r}^{IV}]^\intercal$, the final location estimate is obtained by replacing $\hat{x}_0$, $\hat{y}_0$, $\hat{r}$ in (7) with $\hat{x}^{IV}$, $\hat{y}^{IV}$, $\hat{r}^{IV}$, and performing the rest of the 2-Step WLS from (7) onwards with $\boldsymbol{\Sigma}_2 =$

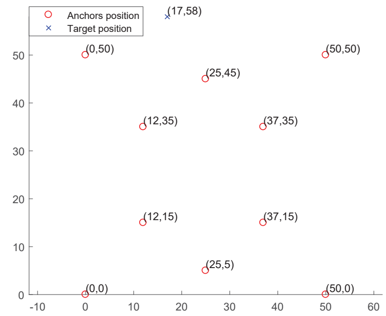

Fig. 1: Sensor coordinates in a 50 m $\times$ 50 m area.

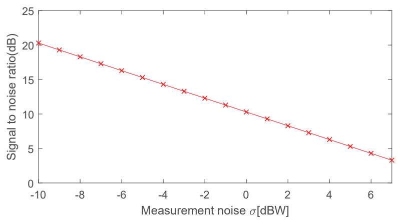

Fig. 2: Average signal-to-noise ratio versus σ when $\gamma = 4$ and $N = 10$.

$\mathrm{diag}(2x, 2y, 1)(\mathbf{F}^\intercal \mathbf{\Sigma}^{-1}\mathbf{\Psi})^{-1}\mathbf{F}^\intercal\mathbf{\Sigma}^{-1}\mathbf{F}(\mathbf{\Psi}^\intercal\mathbf{\Sigma}^{-1}\mathbf{F})^{-1}\,\mathrm{diag}(2x, 2y, 1)$.

The optimal IV construction is to use the noise-free version of $\mathbf{\Psi}$ which is not available since it is a function of the unknown true target position $\boldsymbol{x}$. Following the approach in [22], we construct and utilize a suboptimal IV matrix by approximating the noise-free version of $\mathbf{\Psi}$ using the initial location estimate $\hat{\boldsymbol{x}}_0$ obtained from (6):

$$\mathbf{F} = \begin{bmatrix} 2\boldsymbol{s}_1^\intercal - 2P_{1,2}^{IV'}\boldsymbol{s}_2^\intercal & P_{1,2}^{IV'} - 1 \\ 2\boldsymbol{s}_1^\intercal - 2P_{1,3}^{IV'}\boldsymbol{s}_3^\intercal & P_{1,3}^{IV'} - 1 \\ \vdots & \vdots \\ 2\boldsymbol{s}_1^\intercal - 2P_{1,N}^{IV'}\boldsymbol{s}_N^\intercal & P_{1,N}^{IV'} - 1 \end{bmatrix}, \tag{18}$$

where $P_{1,i}^{IV'} = 10^{\frac{P_{1,i}^{IV}}{5\gamma}}$ and $P_{1,i}^{IV} = -10\gamma\log_{10}\left(\frac{\|\hat{\boldsymbol{x}}_0 - \boldsymbol{s}_i\|_2}{\|\hat{\boldsymbol{x}}_0 - \boldsymbol{s}_1\|_2}\right)$.

Although developed based on the framework of asymptotically unbiased IV estimation, the proposed method is capable of significantly removing estimation bias even for a finite and small number of anchors N, as demonstrated in Section V.

V. Simulation Results

MATLAB-based Monte Carlo simulations were carried out to compare the proposed IV-based estimator with the MLE, 2-Step WLS [16], U-BLUE and A-BLUE [6], in terms of the root-mean-squared error (RMSE) and bias.

Each Monte Carlo simulation consists of 50,000 runs, with the sensor noises randomized in each run. Network topology is fixed, consisting of ten anchors and one target sensor deployed in a 50 m $\times$ 50 m area (see Fig. 1). The path loss exponent is assumed known and set to $\gamma = 4$. The indoor acceptable range of noise is set at $\sigma \in [-10, 7]$ dBW following [23]. The average signal-to-noise (SNR) ratio corresponding to σ is given by $\mathrm{SNR}_{\mathrm{ave}} = \sum_{i=2}^{N}\frac{P_{1,i}}{(N-1)\sigma_n^2}$. Fig. 2 shows that the simulated average SNR ratios are realistic.

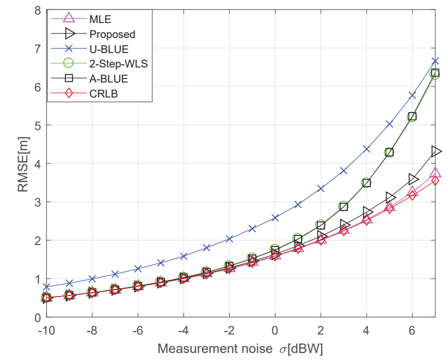

Fig. 3: RMSE versus σ when $\gamma = 4$ and $N = 10$.

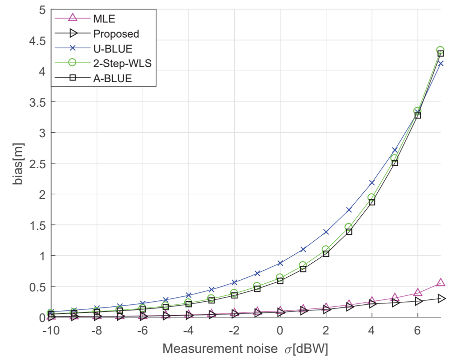

Fig. 4: Bias versus σ when $\gamma = 4$ and $N = 10$.

Fig. 3 shows the RMSE versus noise. It is seen that the U-BLUE method cannot attain the CRLB (see [16] for the derivation of the CRLB for DRSS localization). This is because the U-BLUE method ignores the relation between the location estimate and the auxiliary variable. The 2-Step WLS and A-BLUE are able to approach the CRLB when noise is below 0 dBW, but both estimators start to deviate significantly from the CRLB when the noise exceeds 0 dBW. The MLE on the other hand exhibits an RMSE performance very close to the CRLB for the entire noise range, but it does not provide a closed-form solution, is computationally expensive, and can be vulnerable to divergence problems. In contrast, the proposed IV-based estimator produces smaller RMSE than the 2-Step WLS and A-BLUE especially for large σ, almost achieving the CRLB.

Fig. 4 shows the bias versus noise. The U-BLUE, 2-Step WLS and A-BLUE all suffer from severe bias problems for noise larger than -6 dBW. The proposed IV based estimator exhibits the best bias performance even outperforming the MLE for large noise.

Figs. 5–6 show the performance metrics versus the number of sensors (N) when $\gamma = 4$ and $\sigma = 4$ dBW. For the simulated

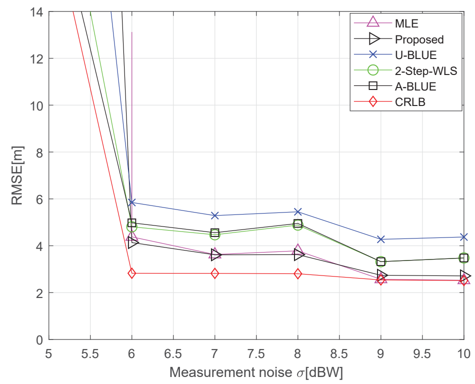

Fig. 5: RMSE versus N when $\gamma = 4$ and $\sigma = 4$ dBW.

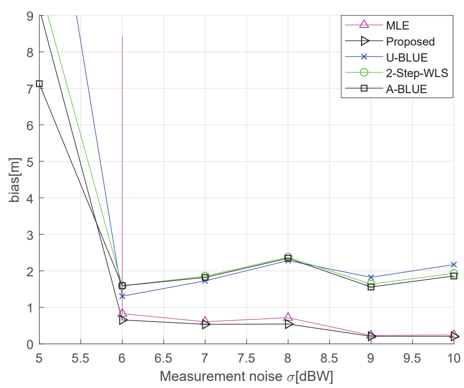

Fig. 6: Bias versus N when $\gamma = 4$ and $\sigma = 4$ dBW.

geometry the IV based estimator achieves a good performance if 6 or more sensors are used.

VI. Conclusion

Using DRSS measurements to perform localization in wireless sensor networks is promising because it obviates the need to determine many transmitter and environmental parameters, and it can be formulated as a linear estimation problem. However, existing solutions are either not closed-form, may diverge, or suffer from significant bias problems due to the injection of noise into the data matrix by the linearization process. This paper proposes a linear estimator for DRSS-based localization by enhancing the 2-Step WLS estimator with an IV matrix, whose function is to eliminate the correlation between the data matrix and the measurement noise, which is the root cause of bias. Simulation results show that the proposed IV-based estimator outperforms the original 2-Step WLS, as well as the U-BLUE and A-BLUE estimators over a wide range of noise levels, producing very small bias and an RMSE performance close to the CRLB. Furthermore, the proposed estimator has smaller computational complexity than the MLE, and does not suffer from divergence problems.

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