# Robust ToA-Based Localization in a Mixed LOS/NLOS Environment Using Hybrid Mapping Technique

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Abstract—A two-stage hybrid method based on the machine learning approach is proposed for source localization using time of arrival (ToA) measurements in a mixed line of sight (LOS) and non-line of sight (NLOS) environment. The first stage applies an artificial neural network (NN) to detect the NLOS measurements that are outliers and the second stage passes the identified LOS measurements to an inverse weighted self-organizing network (IWSON) for determining the source location. The NN NLOS detector is able to take care of a variable number of NLOS measurements while the IWSON handles naturally a variable number of inputs and yields a solution without explicitly solving the nonlinear estimation problem. Simulations validate the good performance of the system with a different number of NLOS measurements. It provides a solution in reaching the Cramèr-Rao lower bound (CRLB) accuracy under a harsh multipath noisy environment, except over the small error region where it can act as an initialization for the iterative MLE to refine accuracy if necessary.

Index Terms—ToA, localization, neural network, outlier, correct detection, false alarm.

## I. INTRODUCTION

Source localization in a multipath environment has attracted the attention of many researchers over the years. Non-Line of Sight (NLOS) observation, referring to outlier in this work, may occur due to severe environmental conditions [1], sensor failure [2], channel impairments [3], or a malicious attack [4], etc. These measurements are statistically inconsistent with the normal line of sight (LOS) data, and as such, often lead to significant deterioration in the localization performance [5]-[9]. Analyzing the data to identify the NLOS measurements becomes essential in maintaining an acceptable level of localization performance. Traditional localization with LOS measurements uses explicit algebraic solution that works well only under low noise condition, or iterative solution that yields good result only when the initialization is near the actual. This paper took a pure and hybrid machine learning approach, in particular using an artificial neural network (NN) for NLOS detection and a self-organizing network for source localization in an environment in which NLOS measurements could appear.

Artificial NNs have been successfully adopted in solving highly nonlinear problems in pattern recognition, prediction,

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system identification, nonlinear processing, fault tolerance, feature generalization, and control [10]-[13]. Related to outlier identification, [14] proposed an online technique based on hyper-ellipsoid one-class support vector machine (SVM). It used the spatio-temporal correlation in the sensor data to identify outliers and showed that the technique gives better performance than the spherical SVM. [15] developed a nonparametric unsupervised outlier detection algorithm for wireless sensor network (WSN) based on single-hop communication. In contrast to detecting outlier, [16] employed a NN to estimate the states of an outlier-free system and achieved fault detection by comparing the observed states with those expected from the outlier-free system. Recently, [17] suggested a back-propagation (BP) NN model for improving a time difference of arrival (TDOA) algorithm in NLOS environment by utilizing a variable gradient training algorithm. Related to estimation. [18] proposed a k-means clustering NN to find the extent of overlap among the users in a community for social network application. [19] proposed a cloud computing-based self-organized localization (SOL) system to address the misestimation problem that may occur in a mixture space and to decrease the amount of inter-node communication exchanges. These prior research for machine learning approaches consider the outlier detection and the localization problems separately.

In this work, we propose a unified hybrid approach as shown in Fig. 1 for the source localization problem using time of arrival (ToA) measurements, where outliers resulting from NLOS propagation may occur. The suggested approach is based on the mapping technique that maximizes the objective score when learning environments are reinforced. First, we propose the features to be used in conjunction with a supervised NN for outlier detection. Second, an inverse weight selforganizing network (IWSON) is developed to accommodate seamlessly a variable number of input elements and produce an accurate source location estimate. The proposed method integrates the two mapping techniques together without the limitations caused by degenerated sensor geometry, high computational complexity, and the need for additional information about the statistics or the number of outliers.

The presentation of this work concentrates on the 2-D scenario, the proposed method can be applied directly for the 3-D case as well. Following the introduction, we next describe

the problem in Section II. Section III presents the proposed hybrid two stage localization method. Section IV gives the simulation results for supporting the proposed system and we draw the conclusion in Section V. In the paper, bold lower case letter denotes column vector and bold upper case letter represents matrix.  $\mathbf{a}(i)$  is the *i*-th element of a.  $\mathbf{1}_N$  is a vector of unity having length N.  $\| \bullet \|$  is the Euclidean norm and  $\odot$ represents the operation of element by element multiplication.



Fig. 1. Block diagram of the proposed hybrid mapping method for source localization.

## **II. PROBLEM FORMULATION**

We are given a 2-D sensor network that has M sensors placed at  $\mathbf{s}_i \in \mathbb{R}^2$ , where  $i = 1, 2, \dots, M$ . Each of them is able to obtain ToA measurement through signal time stamping and/or message exchanges with an object at unknown location  $\mathbf{u}^o = [x, y]^T$ . Some of the measurements could be experiencing NLOS effect and become outliers. We shall use k to represent the number of outliers, where k is not known.

The range (equivalent to ToA) measurement between the object and the i-th sensor is modeled as

$$m_i = \|\mathbf{u}^o - \mathbf{s}_i\| + \gamma_i, \quad i = 1, 2, \dots, M$$
 (1)

where  $\gamma_i$  is the measurement noise that is IID and Gaussian distributed. We model the outlier error through the distribution of  $\gamma_i$ . For measurement from direct LOS,  $\gamma_i \sim \mathcal{N}(0, \sigma^2)$ . For outlier resulted from NLOS,  $\gamma_i \sim \mathcal{N}(\mu_{NL}, \sigma_{NL}^2)$ , where  $\mu_{NL} > 0$  and  $\sigma_{NL}^2 > \sigma^2$ . We do not expect  $\sigma^2$ ,  $\mu_{NL}$ , and  $\sigma_{NL}^2$  are known. The collection of all measurements is  $\mathbf{m} = [m_1, m_2, \dots, m_M]^T$ .

We would like to obtain the object location  $\mathbf{u}^o$ , using the M range measurements where an unknown number of them are outliers. We next present the proposed method to solve this localization problem.

#### III. PROPOSED HYBRID METHOD

The proposed method has two stages as shown in Fig. 1. A. First Stage

The first stage extracts a set of features and uses a supervised NN for the detection of outliers in the M range measurements. During training, the ability of an NN converging toward the desired output (correct state) depends on the availability of sufficient information presented to it that can correlate well to the desired output state [20]. Deriving the meaningful features is crucial for the effectiveness of the NN outlier detector.

The features are devised based on the following concept. The Maximum Likelihood (ML) location estimate from a number of LOS measurements, say  $\check{\mathbf{u}}$ , is expected to be near the true value. Indeed,  $f = (\check{\mathbf{u}} - \mathbf{u}^o)^T \mathbf{Q}^{-1} (\check{\mathbf{u}} - \mathbf{u}^o)$  follows the central  $\chi^2$  distribution with 2 degrees of freedom according to the LOS noise model [21], where  $\mathbf{Q}$  denotes the CRLB of the location estimate. On the other hand, if the estimate  $\check{\mathbf{u}}$  is from a set of range measurements that has one or more outliers, it will deviate significantly from the true value; the resulting f will not follow the central  $\chi^2$  distribution and will be large.

Among the M ToAs, the number of possible subsets of measurements that can give a location estimate is

$$L = \sum_{i=3}^{M} \binom{M}{i},\tag{2}$$

considering that at least 3 range measurements are needed to yield a unique estimate in 2-D. For each subset of measurements, we can obtain the location estimate and the residual square error statistic f. A small value of f indicates the subset does not have outliers and has otherwise.

The exact f value cannot be evaluated since  $\mathbf{u}^o$  and  $\mathbf{Q}$  are not known. We shall use a reference solution to approximate  $\mathbf{u}^o$ . Inspired by [21], [22], we define the reference (conservative) solution  $\bar{\mathbf{u}}$  as the best estimate among those from using any 3 among the M measurements. Using the minimum number of measurements has the highest probability of having the reference solution not from outlier measurement(s). We mean here the best is the one that yields the smallest trace of the CRLB with the noise power  $\sigma^2$  ignored, with the true value  $\mathbf{u}^o$  in the CRLB replaced by the estimate. The CRLB matrix for ToA positioning with 3 LOS measurements from sensors i, j, k is simply

$$\operatorname{CRLB}(\mathbf{u}^{o}) = \sigma^{2} \sum_{p=\{i,j,k\}} \frac{(\mathbf{u}^{o} - \mathbf{s}_{p})(\mathbf{u}^{o} - \mathbf{s}_{p})^{T}}{\|\mathbf{u}^{o} - \mathbf{s}_{p}\|^{2}} \,.$$
(3)

The trace of the CRLB provides a good measure to indicate the accuracy of a solution estimate.  $\sigma^2$  is not known but it is irrelevant to select the good solution. As for **Q**, it is logical to replace it by the CRLB matrix, with the true source location replaced by the estimate.

For each subset of measurements, we can now form

$$f(h) = \frac{[\check{u}_h(1) - \bar{u}(1)]^2}{c_{xx,h}} + \frac{[\check{u}_h(2) - \bar{u}(2)]^2}{c_{yy,h}}$$
(4)

for h = 1, 2, ..., L and  $h \neq \bar{h}$ , where  $\bar{h}$  is the measurement subset that gives the reference solution.  $\check{\mathbf{u}}_h$  is the solution obtained by measurement subset number h using a typical ToA localization algorithm [22] that has the ML accuracy.  $c_{xx,h}$  and  $c_{yy,h}$  are the (1,1) and (2,2) diagonal elements of CRLB $(\check{u}_h)/\sigma^2$ . Note that we ignore the off-diagonal components of the CRLB when forming f(h) in (4).  $\mathbf{f} = [f(1), f(2), \ldots, f(L)]^T$  gives L features to the NN.

The feature vector for the NN outlier detection is  $[\mathbf{f}^T, \mathbf{m}^T]^T$ , as the original measurements may contain useful information. The NN output is represented by the length M indication vector  $\mathbf{v}$ , whose elements are either 1 or 0. Having a value 1 in element j indicates the j-th element is an outlier.

A number of experiments have been performed with various training algorithms to determine the structure of the NN by maximizing the percentage of correct classification of outliers while minimizing the amount of false detection. The structure is shown in Fig. 2. It has four layers, one input, two hidden,



Fig. 2. The neural network architecture for outlier detection.

and one output layers. The input layer has L + M nodes and output layer M nodes. Each of the two hidden layers has 75 neurons. Among the algorithms examined, we select the Bayesian Regularization (BR) with an adaptive learning rate as the training algorithm. BR describes a good generalization model, avoids costly cross-validation to solve the overfitting problem and explores complex model by effectively penalizing the intricate architecture [23], [24].

During training, a set of samples are generated according to the known sensor positions and a random sampling over the area of coverage for creating the location of the object. The measurements for the training samples are synthesized using (1), with  $\sigma^2$  and  $\sigma_{NL}^2$  set to  $10^{-5}$  and  $4\sigma^2$ , respectively. The training data contains equal proportion of the measurements of 0, 1, ..., up to the maximum number of outliers expected. The outliers are simulated according to Section IV.

### B. Second Stage

The second stage removes the outlier measurements and uses only the detected LOS measurements to obtain the object location. This is accomplished by using the IWSON that has the benefit for accepting a variable number of inputs for localization.

The traditional self-organized map (SOM) [25] computes the weight matrices using a training algorithm started with initial random weights. We developed a systematic approach based on the structure of SOM to construct the weight matrices from the clean synthetic measurements having the object at each of a number of hypothesized positions. The hypothesized positions are random locations generated by dividing uniformly the area of interests. The IWSON algorithm for localization has two phases summarized as follows:

Phase1: Setting Up

This phase is being done offline before localization.

- 1) Generate  $N_y \times N_x$  hypothesized positions  $\mathbf{p}_{\alpha,\beta}$  that are chosen randomly over the region  $[x_{min}, x_{max}] \times [y_{min}, y_{max}]$  with uniform distribution for the x- and y-coordinates, where  $\alpha = 1, \dots, N_x$  and  $\beta = 1, \dots, N_y$ .
- Compute the clean synthetic measurements (ranges) for all the hypothesized positions with respect to each sensor position

$$d_{\alpha,\beta,i} = ||\mathbf{p}_{\alpha,\beta} - \mathbf{s}_i|| \tag{5}$$

for  $\alpha = 1, \cdots, N_x$ ,  $\beta = 1, \cdots, N_y$ , and  $i = 1, 2, \dots, M$ .

3) Form M weighting matrices, one for each sensor position having size  $N_y \times N_x$  given by  $\mathbf{W}_i = [d_{\alpha,\beta,i}]$ .



Fig. 3. Performance comparison of IWSON and SOM with the CRLB accuracy.

## Phase2: Determining Location

Let  $\widetilde{\mathbf{m}}$  be the identified LOS measurements found from the first stage with the corresponding sensors located at  $\{\mathbf{s}_{i_1}, \mathbf{s}_{i_2}, \ldots, \mathbf{s}_{i_{\widetilde{M}}}\}$ , where  $i_k \in \{1, 2, \ldots, M\}$ ,  $k = 1, 2, \ldots, \widetilde{M}$  and  $\widetilde{M}$  is the length of  $\widetilde{\mathbf{m}}$ . The location solution  $\hat{\mathbf{u}}$  is obtained as follows.

1) Evaluate the matrix of total error measure by computing the  $\ell_2$ -norm of each element in the identified LOS measurement vector  $\tilde{\mathbf{m}}$  with each of the elements in  $\mathbf{W}_{i_k}$  by

$$\mathbf{E} = \sum_{k=1}^{M} \left( m_{i_k} \mathbf{1}_{N_y} \mathbf{1}_{N_x}^T - \mathbf{W}_{i_k} \right) \odot \left( m_{i_k} \mathbf{1}_{N_y} \mathbf{1}_{N_x}^T - \mathbf{W}_{i_k} \right)$$
(6)

- 2) Find the first q smallest elements of E, denoted by  $\hat{\varepsilon}_n$ , and identify their corresponding hypothesized positions  $\hat{\mathbf{u}}_n$  based on the element indexes, where  $n = 1, \dots, q$ .
- 3) Obtain the object position estimate  $\hat{\mathbf{u}}$  by:

$$\hat{\mathbf{u}} = \frac{\sum_{n=1}^{q} \varepsilon_n^{-1} \times \hat{\mathbf{u}}_n}{\sum_{n=1}^{q} \varepsilon_n^{-1}}.$$
(7)

Compared with the traditional SOM algorithm, IWSON has lower computational complexity and there is no need to retrain to take into account the variable number of inputs (the identified LOS measurements from Stage 1). The grid size of the network  $(N_y \times N_x)$  and the number of q are selected according to the required accuracy of the solution and the acceptable level of the computational complexity.

To examine the performance difference between IWSON and SOM, we generate a grid with a resolution of 4 uniformly distributed points over an area of  $[-100, 100] \times [-100, 100]$ . The sensor positions were set the same as in [21] after being scaled down by a factor of 61. The weight matrices of IWSON are generated using the proposed Phase1 IWSON algorithm while the initial weight matrices for SOM are updated using competitive learning algorithm. The parameter q = 4 for IWSON. Fifty trials each with the object at a different location chosen from the grid were conducted and the results are plotted as shown in Fig. 3. The number of ensemble runs in each trial is 100. IWSON has better performance than SOM. Both have limited performance in the small noise region that is limited by the resolution of the grid size, but IWSON remains to be better. In this simulation, IWSON is about three times faster than SON.

TABLE IThe sensor (receiver) positions

$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	$s_7$
675	-675	-1000	-500	500	1000	0
1000	1000	-165	-835	-1000	0	835

#### IV. PERFORMANCE EVALUATION

We use a mix of two Gaussian distributions [7], [9] to simulate the measurement noise  $\gamma_i$  in (1),

$$p(\gamma_i) = (1 - \rho)N(0, \sigma^2) + \rho N(\mu_{NL}, \sigma_{NL}^2).$$
(8)

In (8),  $\rho$  is the outlier probability.  $\sigma^2$  is the noise power of LOS measurements, and  $\mu_{NL} > 0$  and  $\sigma_{NL}^2$  are the mean and variance of the noise in NLOS outlier measurements. In simulating outlier, we set  $\mu_{NL} = (1 + rand) \times$  true measurement value, where rand is sampled from U(0, 1), and  $\sigma_{NL} = 2\sigma$ . In an ensemble, each of the *M* measurements has a probability of  $\rho$  being an outlier. The number of ensemble runs is 100.

A dataset with 2000 possible object locations selected randomly in a square area of  $[-2000, 2000] \times [-2000, 2000]$ was created to train the NN. The sensor positions are fixed and summarized in Table I. The number of epochs that is used in the training phase is 42. The NN was trained with 0 and up to 3 outliers. A grid resolution of (40/3) and q = 4 have been applied for the IWSON in the second stage.

The performance of the proposed approach is evaluated in terms of the root of mean square error (RMSE) as the noise power varies. The accuracy of the IWSON solution, and its further refinement using the Gauss-Newton iterative MLE is compared with the Riba method [26] and the CRLB. In addition to the localization accuracy, we also provide the percentage of correct detection ( $P_{CD}$ ) and the percentage of false alarm ( $P_{FD}$ ) for outlier detection from Stage 1 that is defined as [9], [27]

$$P_{CD} = \frac{\sum_{l}^{L} \text{Outliers detected in ensemble } l}{\text{Total number of ensemble runs } L} \times 100\%,$$
(9)

$$P_{FD} = \frac{\sum_{l}^{L} At \text{ least one falsely detected in ensemble } l}{\text{Total number of ensemble runs } L} \times 100\%$$
(10)

We consider two possible object locations. One is inside the sensor area at  $\mathbf{u}^{o} = [200 \ 100]^{T}$  and the other outside at  $\mathbf{u}^{o} = [1500 \ 1400]^{T}$ . Among the seven ToA measurements, there can be 0, 1, 2, and 3 outliers, which corresponds to  $\rho$ equal to 0, 1/7, 2/7 and 3/7. Only the results of 1 and 3 outliers are presented for conciseness.

The performance of the proposed system is shown in Fig. 4 and Fig. 5 for the object inside and outside the sensor area. The performance is very poor without detecting and removing the outliers. The proposed IWSON solution behaves well and outperforms the Riba method except at low noise level ( $\sigma^2 = 1$ ) due to the limit in grid resolution. It also achieves the CRLB accuracy until the noise power reaches around  $10^5$ . Further refinement by the iterative MLE improves performance only at very low noise level where the accuracy is limited by the IWSON grid resolution. The performance of the proposed solution is consistent for the two object locations.

Figs. 6-7 are the results of the three outlier case. The false detection probability is lower than that for the one outlier case because we have more outliers and hence less chance of having false detection. In any case, the proposed NN outlier detector maintains nearly 100% correct detection and yields the CRLB performance even without MLE refinement until the thresholding effect occurs. The results for the 0 and 2 outlier cases are similar. The proposed method provides stable and consistent performance regardless the number of outliers present and is insensitive to the object location.

## V. CONCLUSION

A unified hybrid mapping approach for robust object localization in a mix of LOS and NLOS(outlier) environment is proposed, where the number of outliers and their statistics are not known. The first stage uses a NN for the detection of NLOS measurements and the second stage applies the IWSON for the estimation of the object location. In the first stage, we derived an effective feature vector for the NN detector, making it capable of handling the presence of an unknown number of outliers without requiring their statistics. The IW-SON developed in the second stage has the benefit of taking care of a variable number of identified LOS measurements from the first stage and providing an accurate object location estimate. The resulting positioning accuracy is able to reach the CRLB performance, unless the noise level is low where the accuracy is limited by the IWSON grid resolution. The proposed system is more effective if the NLOS measurements have more deviation from the actual values. We used ToA measurements and the 2-D scenario for illustration, and the proposed method can be extended directly for other types of measurements and the 3-D case. In the future work, we plan to investigate the generalization ability of this approach when the environment can be varying.

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Fig. 5. One outlier and object outside sensor configuration.

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Fig. 6. Three outliers and object inside sensor configuration.



Fig. 7. Three outliers and object outside sensor configuration.

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