Adaptive Beamforming Based on Time Modulated Array with Harmonic Characteristic Analysis

1st Chong He
Dept. of Electronic Engineering
Shanghai Jiao Tong University
Shanghai, China
hechong@sjtu.edu.cn

2nd Guanli Yi
No.722 Research Institute
China Shipbuilding Industry Corp.
Wuhan, China
yiguanli@sina.com

3rd Jingfeng Chen
Dept. of Electronic Engineering
Shanghai Jiao Tong University
Shanghai, China
laowu3917@sjtu.edu.cn

***Abstract*—Adaptive beamforming based on the time modulated array (TMA) is proposed and verified, which exploits the characteristic of fundamental and harmonic components after the time modulation. By establishing the relationship between time modulation sequences added to RF switches of the TMA and generated harmonic components of received signals, the complex weights needed in adaptive beamforming can be calculated. Then the weights are mapped to time modulation sequences added to RF switches to generate the beam pointing to the received signal at the frequency of the positive first harmonic component. Compared to the adaptive beamforming based on the digital array, the proposed method exploits only a single RF channel, while its main calculation locates the Discrete Fourier Transform (DFT) of several harmonic components. Therefore, the proposed method has a simple structure and a concise signal processing. Numeric simulations are provided to show its effectiveness.**

***Index Terms*—adaptive beamforming, time modulation, discrete Fourier transform, harmonic analysis, pattern synthesis**

I. Introduction

ADAPTIVE beamforming plays an important role in cellular mobile communications and military communications. In 3G mobile communication, adaptive beamforming is one of key techniques in TD-SCMDA systems. For 5G mobile communication, it has an potential application to improve the frequency efficiency by suppressing signals from the other directions. In military communication systems, adaptive beamforming helps to track the target, and to restrain the jamming in the meanwhile. In general, adaptive beamforming methods can be divided into three kinds. The first kind is based on the direction-of-arrival (DOA) estimation. A lots of super-resolution DOA algorithms, such as the multiple signal classification (MUSIC) algorithm [1] and the estimating signal parameters via rotational invariance techniques (ESPRIT) algorithm [2] have been proposed to estimate the incident direction of the signal since 1980s. Then the DOA estimation result is used to generate weights used in the adaptive beamforming. The second kind is based on the expected direction, whose weights are calculated according to some criteria, such as the minimum variance distortionless response (MVDR) method [3], the minimum mean square error (MMSE) method, and the linearly constrained minimum variance (LCMV) method [4]. The third kind is the blind beamforming, which exploits the characteristic of the signal or the channel. The typical algorithms includes the sample matrix inversion (SMI) method [5].

The basic hardware platform implementing adaptive beamforming includes multiple RF channels, multiple ADCs and DACs, which has a high complexity and cost. On the other hand, the mismatches among all the channels influence the effect of adaptive beamforming. Therefore, the complicated algorithm is required to calibrate all the channels. In this conference paper, we have proposed a simple adaptive beamforming method based on the TMA, which has advantage in hardware and algorithm complexity. TMA adds high speed switches to the RF fronts of the antenna array, which are modulated with 'ON-OFF' states periodically. After the combiner, only a single RF channel is exploited to transmit and receive the signal. The concept of the TMA was first proposed by H. E. Shanks and R. W. Bickmore in 1950s [6], in which the TMA was exploited to synthesize ultra-low sidelobe patterns. In 1961, H. E. Shanks noticed that the harmonic components generated by the TMA could be exploited in the electronic scanning [7]. The development of TMA was limited by the switching speed of the RF switch in the next 40 years. With the development of the microelectronic technology, TMA has attracted attention again since 2000 [8], [9]. Nowadays, the switching speed of RF switches can be less than 1 nS, which promotes the TMA from theory research to engineering application.

Because of the periodical modulation, the fundamental and harmonic components are generated simultaneously in each element. For generated harmonic components, their amplitudes and phases are decided by the time modulation sequence added to the corresponding element. Therefore, the beams of harmonic components are controlled by the opening and closing instants added to all the RF switches. In recent years, many works were reported to achieve this goal. In 2008, Gang Li *et al.* proposed an adaptive beamforming method based on a hybrid analog-digital structure [10], in which the weights needed in the beamforming were calculated by a DSP chip. In 2010, Y. Tong analyzed the harmonic beam steering method by the TMA with sidelobe level control [11]. In 2011, L. Poli synthesized multi-beams of TMA simultaneously by Particle Swarm algorithm (PSO) [12]. In 2012, Y. Tong proposed the adaptive beamforming method by a two-channel TMA [13].

978-9-0827-9703-9/19/$31.00 ©2019 IEEE

The above research exploits the ability of harmonic beamforming of the TMA, which makes the patterns of harmonic components point to the desired directions by adjusting time modulation sequences added to RF switches. However, most existing works require that the direction for beamforming is known, which synthesize the time modulation sequences to point to the certain direction. In this conference paper, we discuss the adaptive beamforming based on the TMA when the incident direction of the received signal is unknown. The proposed method exploits the ability of direction finding of the TMA [14], [15], whose principle is as follows. By establishing the relationship between time modulation sequences added to RF switches and the harmonic characteristic of the received signal, the array manifold can be estimated. Then the complex weights for adaptive beamforming can be calculated. Finally they are mapped to new time modulation sequences added to RF switches, which make the beam of selected harmonic component point to the direction of received signal.

The remainder of this conference paper is organized as follows. First, the principle of harmonic beamforming by the TMA is discussed. Second, the complex weights for adaptive beamforming is deduced mathematically in theory. Third, Numeric simulation is provided to verify the effectiveness of the proposed method. Finally, some conclusions are drawn.

II. Theory

The block diagram of the adaptive beamforming based on the TMA is schematically shown in Fig. 1. Considering that the TMA works on the receiving state, the source signal enters the TMA from the far-field. For each element, the received signal is periodically modulated by a high speed single-pole single-throw (SPST) switch. After the power combiner, the modulated signal is input to the band-pass filter, the mixer, and the low-pass filter in sequence. Then it is sampled into the digital domain by the analog-to-digital converter (ADC). In the digital domain, the received signal is demodulated by the base-band module after a digital filter. Meanwhile, it is exploited to calculate the complex weights for the adaptive beamforming. Then, the calculated results are mapped to time modulation sequences added to each RF switch, which control the selected harmonic component to point to the direction of source signal.

We analyze the theory and procedure of the proposed adaptive beamforming method by following two steps. First, the harmonic beamforming method by the TMA is analyzed, which demonstrates the principle and method to synthesize the patterns of harmonic components by controlling the periodical modulation sequences added to RF switches. Second, the proposed method is discussed in theory detailedly, and the formula to calculate the complex weights for the adaptive beamforming is deduced.

A. Harmonic Beamforming by the TMA

Consider an N-element linear TMA with the element spacing D. Assume that a far-filed sinusoidal source with the carrier frequency F_c enters in the array from the incident direction θ. In the initial state, the periodical time modulation function added to the n^{th} element is

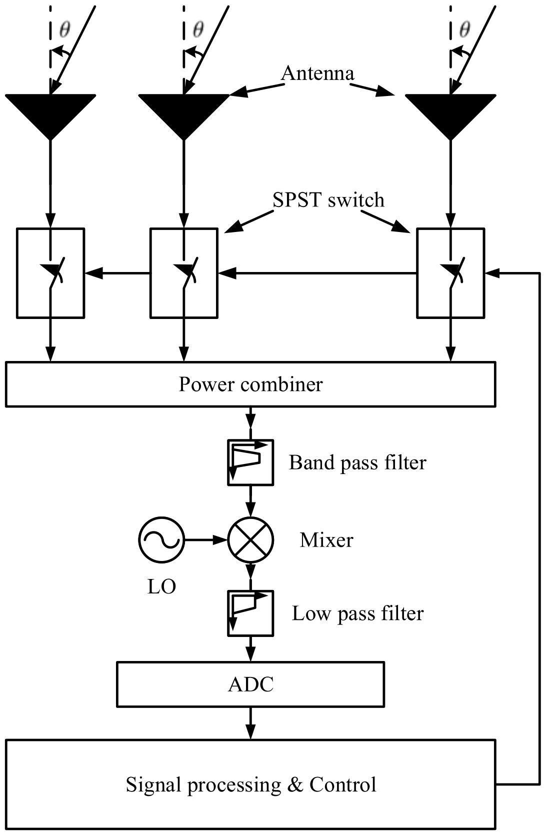

Fig. 1. Block diagram of the adaptive beamforming with TMA.

$$U_n(t) = \sum_{m=-\infty}^{\infty} g_n(t - mT_p), m \in Z, \tag{1}$$

where T_p is the modulation period. $g_n(t)$ is a gate function, which is expressed as

$$g_n(t) = \begin{cases} 1, \tau_{n,on} < t \le \tau_{n,off} \\ 0, \text{otherwise} \end{cases}, \tag{2}$$

where $\tau_{n,on}$ and $\tau_{n,off}$ represent the opening and closing instants of the RF switch in the n^{th} element, respectively. The periodical function $U_n(t)$ is unfolded by the Fourier series as

$$U_n(t) = \sum_{k=-\infty}^{\infty} \alpha_{n,k} e^{j2\pi k F_p t}, k \in Z, \tag{3}$$

where F_p is the modulation frequency and equal to $1/T_p$. Because of the periodical modulation, the received signal includes the fundamental component at F_c and the harmonic components at $F_c + kF_p$. $\alpha_{n,k}$ is the Fourier coefficient of the k^{th} harmonic in the n^{th} element, which is calculated by

$$\alpha_{n,k} = \frac{1}{T_p}\int_0^{T_p} U_n(t)e^{-jk2\pi F_p t}dt = \begin{cases} \dfrac{\tau_{n,off}-\tau_{n,on}}{T_p}, k=0 \\ \dfrac{\sin k\pi F_p(\tau_{n,off}-\tau_{n,on})}{k\pi} e^{-jk\pi F_p(\tau_{n,off}+\tau_{n,on})}, k \neq 0 \end{cases} . \tag{4}$$

Referring to (4), the amplitude and phase of the k^{th} harmonic component are decided by the opening and closing instants of RF switches in the corresponding elements. For the fundamental component ($k = 0$), $\alpha_{n,0}$ is a real number. Therefore, its pattern points to the normal direction of the TMA, which is independent of the time modulation sequences. For the harmonic components ($k \neq 0$), $\alpha_{n,k}$ is a complex number, which can be exploited in the beamforming to make the corresponding harmonic components to point to different directions.

Setting the first element as the reference, its received signal is $e^{j2\pi F_c t}$. Assume that the complex gains of all the elements are the same and equal to one. After the time modulation, the output of the power combiner is

$$S_r(t) = \sum_{n=1}^{N} U_n(t)e^{j2\pi F_c t}e^{-j(n-1)KD\sin\theta} = \sum_{n=1}^{N}\sum_{k=-\infty}^{\infty} \alpha_{n,k}e^{j2\pi(F_c+kF_p)t}e^{-j(n-1)KD\sin\theta}, \tag{5}$$

where K is the wavenumber and equal to $2\pi/\lambda$, λ the wavelength corresponding to F_c. Referring to (5), the array pattern of the fundamental component is

$$AF_0 = \sum_{n=1}^{N} \alpha_{n,0}e^{-j(n-1)KD\sin\theta} = \sum_{n=1}^{N} \frac{\tau_{n,off}-\tau_{n,on}}{T_p}e^{-j(n-1)KD\sin\theta}, \tag{6}$$

and the array pattern of the k^{th} harmonic component is

$$AF_k = \sum_{n=1}^{N} \alpha_{n,k}e^{-j(n-1)KD\sin\theta} = \sum_{n=1}^{N} \frac{\sin k\pi F_p(\tau_{n,off}-\tau_{n,on})}{k\pi} \times e^{-j[(n-1)KD\sin\theta + k\pi F_p(\tau_{n,off}+\tau_{n,on})]}. \tag{7}$$

As is shown in (4), the amplitude of the k^{th} harmonic component accords with the $Sinc(\cdot)$ function when the order k increases. The first harmonic component is relatively large among all the generated harmonic components. Therefore, it is appropriate to select the first harmonic component to do beamforming, whose array pattern is as follows

$$AF_1 = \sum_{n=1}^{N} \alpha_{n,1}e^{-j(n-1)KD\sin\theta} = \sum_{n=1}^{N} \frac{\sin \pi F_p(\tau_{n,off}-\tau_{n,on})}{\pi} \times e^{-j[(n-1)KD\sin\theta + \pi F_p(\tau_{n,off}+\tau_{n,on})]}. \tag{8}$$

It is worth noting that the $+k^{\text{th}}$ and $-k^{\text{th}}$ harmonic components have the same amplitudes and the inverse phases, according to (4). Therefore, their array patterns AF_k and AF_{-k} are symmetric about the normal direction of the TMA. Assume that

$$\begin{cases} \tau_{n,off} = u_{n,2}T_p \\ \tau_{n,on} = u_{n,1}T_p \end{cases}, \tag{9}$$

and the normalized amplitude and phase needed in the n^{th} element is A_n and Φ_n, respectively. Referring to (8), we get the following equations

$$\begin{cases} A_n = \sin\pi[u_{n,2} - u_{n,1}] \\ \Phi_n = 2\pi - \pi(u_{n,2} + u_{n,1}) \end{cases}. \tag{10}$$

Considering that $u_{n,1}$ and $u_{n,2}$ are in the range of [0,1], they can be solved from (10) as

$$\begin{cases} u_{n,2} = \text{mod}(1 - \dfrac{\Phi_n}{2\pi} + \dfrac{\arcsin A_n}{2\pi}, 1) \\ u_{n,1} = \text{mod}(1 - \dfrac{\Phi_n}{2\pi} - \dfrac{\arcsin A_n}{2\pi}, 1) \end{cases}. \tag{11}$$

B. Complex Weights Calculation for Adaptive Beamforming by the TMA

This subsection deduces the complex weights needed for the adaptive beamforming from the received signal after the time modulation. After the power combiner, the signal contains the fundamental component at F_c and harmonic components at $F_c + kF_p$. Both the fundamental component and harmonic components are the summation of corresponding components generated in all the N elements. A far-field sinusoidal signal enters the TMA from the direction θ. Set the first element as the reference, in which the complex amplitude of the received signal is A_0. Assume that the fundamental component and the k^{th} harmonic component after the power combiner are γ_0 and γ_k, respectively, and we can get the following system of linear equations

$$\begin{bmatrix} \alpha_{1,0} & \alpha_{2,0} & \cdots & \alpha_{N,0} \\ \alpha_{1,1} & \alpha_{2,1} & \cdots & \alpha_{N,1} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{1,K} & \alpha_{2,K} & \cdots & \alpha_{N,K} \end{bmatrix} \begin{bmatrix} 1 \\ e^{-jKD\sin\theta} \\ \vdots \\ e^{-j(N-1)KD\sin\theta} \end{bmatrix} = \frac{1}{A_0}\begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \vdots \\ \gamma_K \end{bmatrix}. \tag{12}$$

Referring to (12), the matrix on the left side is named as the harmonic characteristic matrix (HCM), which is composed of the Fourier coefficients calculated by (4). Therefore, the HCM is only related to time modulation sequences added to

RF switches. The vector on the left side of (12) is the array manifold vector, and the vector on the right side of (12) is made up of the fundamental and harmonic components of the received signal, which can be calculated by the DFT. If the HCM has an inverse matrix, the array manifold can be calculated by

$$\begin{bmatrix} 1 \\ e^{-jKD\sin\theta} \\ \vdots \\ e^{-j(N-1)KD\sin\theta} \end{bmatrix} = \frac{1}{A_0}\begin{bmatrix} \alpha_{1,0} & \alpha_{2,0} & \cdots & \alpha_{N,0} \\ \alpha_{1,1} & \alpha_{2,1} & \cdots & \alpha_{N,1} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{1,K} & \alpha_{2,K} & \cdots & \alpha_{N,K} \end{bmatrix}^{-1} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \vdots \\ \gamma_K \end{bmatrix}. \tag{13}$$

After obtaining the array manifold, the adaptive beamforming based on the TMA can be implemented as follows. It takes the calculated array manifold as phase weights. Combined with the amplitude weighting methods, such as the Dolph-Chebyshev method and the Taylor method, the TMA can make the beam of the corresponding harmonic component point to the incident direction, while its side-lobe level can be controlled simultaneously. Assuming that the array manifold vector is $\vec{V}_{AM}$, we take its conjugate vector $\vec{V}_{AMC}$ as the phase weights, which is

$$\vec{V}_{AMC} = \left[1, e^{jKD\sin\theta}, \cdots, e^{j(N-1)KD\sin\theta}\right]^T, \tag{14}$$

and the amplitude weight vector by the Dolph-Chebyshev method is

$$\vec{A}_{DC} = [a_{DC1}, a_{DC2}, \cdots, a_{DCN}]^T. \tag{15}$$

Therefore, the complex weight vector for the adaptive beamforming is the Hardmard product of the phase weight vector and the amplitude weight vector, which is expressed as

$$\begin{aligned} \vec{W} =& \vec{V}_{AMC} \circ \vec{A}_{DC} \\ =& [a_{DC1}, a_{DC2}e^{jKD\sin\theta}, \cdots a_{DCN}e^{j(N-1)KD\sin\theta}]^T \\ =& [A_1 e^{\Phi_1}, A_2 e^{\Phi_2}, \cdots A_N e^{j\Phi_N}]^T. \end{aligned} \tag{16}$$

Substituting the A_n and Φ_N in (16) into (11), the opening and closing instants of all the elements can be calculated to make the beam of first harmonic component point to the incident direction.

The proposed TMA can work on the direction finding state alone, with the same hardware but different software. To interchange two states, RF switches should be controlled flexibly to reducing the transient time.

III. Numeric simulation

In this section, a numeric simulation is provided to verify the propose adaptive beamforming method. Assume that the sinusoidal signal enters into the TMA from the far-field. The purpose is to make the beam of the first harmonic component point to the incident direction, meanwhile to make its sidelobe less than -20dB. A far-field sinusoidal signal with the carrier frequency 1GHz enters into the 8-element TMA from the direction $-15°$. The element spacing of the TMA is 15cm. In the initial state, the SPST switches in 8 elements are opened and closed sequentially in one modulation period, and the modulation function added to the n^{th} element is

$$U_n(t) = \begin{cases} 1, & 0 < t \le \dfrac{n-1}{N}T_p \text{ or } \dfrac{n}{N}T_p < t \le T_p \\ 0, & \text{others} \end{cases}, \tag{17}$$

where the modulation period T_p is 1μs, and the modulation frequency F_p is 1MHz. The time modulation sequences of 8 elements in one period is plotted in Fig. 2. The solid bar for each element represents the time when the SPST RF switch is open. Assume that the signal-to-noise ratio (SNR) is 10dB and the sampling frequency F_s is 10GHz. 20 periods' data are sampled for analysis. After the combiner, the spectrum of the received signal is plotted in Fig. 3. As is shown, the combined signal after the time modulation contains the fundamental component at the carrier frequency 1GHz and the harmonic components with the frequency stepping F_p.

The fundamental component γ_0 and the first seven harmonic components γ_k $(k = 1, 2, \ldots, 7)$ of the combined signal are substituted into (13) to calculate the phase weights needed in the adaptive beamforming by the TMA, where the HCM is calculated by (4). The Dolph-Chebyshev amplitude weights are used to reduce the sidelobe of the TMA. According to (16), the calculated amplitudes and phases of 8 weights for the adaptive beamforming is shown in Table I. Then the amplitudes A_n and the phase Φ_n are mapped to the opening and closing instants of 8 elements by (11), in order to make the beam of the first harmonic component point to the desired direction.The mapping result for time modulation sequences of 8 elements in one period is plotted in Fig. 4.

With the purpose of testing the effectiveness of the proposed adaptive beamforming method, the normalized power pattern of the first harmonic component under the time modulation sequences in Fig. 4 is plotted in Fig. 5. As is shown, the beam of the first harmonic component points to about $-15°$, while its sidelobe is about -20dB.

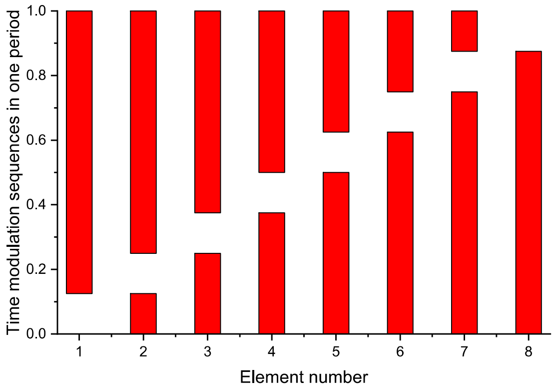

Fig. 2. Time modulation sequences in one period for simulation I.

IV. Conclusion

In this conference paper, a novel adaptive beamforming method based on the TMA has been proposed and verified. Its

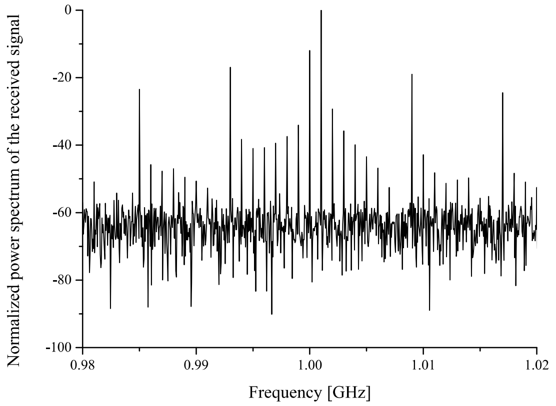

Fig. 3. Normalized power spectrum of the received signal for Simulation I.

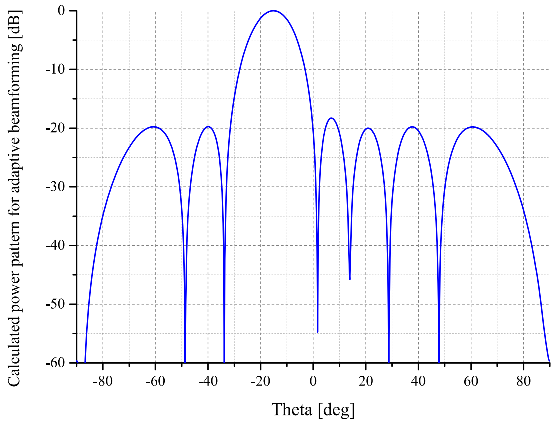

Fig. 5. Calculated power pattern for adaptive beamforming.

TABLE I
AMPLITUDES AND PHASES OF CALCULATED COMPLEX WEIGHTS

Element number	*Amplitude*	*Phase* [deg]
1	0.61	294.1
2	0.69	248.8
3	0.89	202.5
4	0.99	154.8
5	1.00	106.7
6	0.89	59.7
7	0.68	13.5
8	0.60	327.3

implementation is divided into two steps. The first step is to estimate the manifold of the array, by analyzing the harmonic characteristic of the received signal after time modulation. The second step is to synthesize the pattern of the $+1^{st}$ harmonic component to point to the direction of the received signal, by adjusting time modulation sequences added to all the elements. Numeric simulation is provided to examine the effectiveness of the proposed method, and the research to adaptive beamforming of the broadband signal based on the TMA and the experiment verification are still going on.

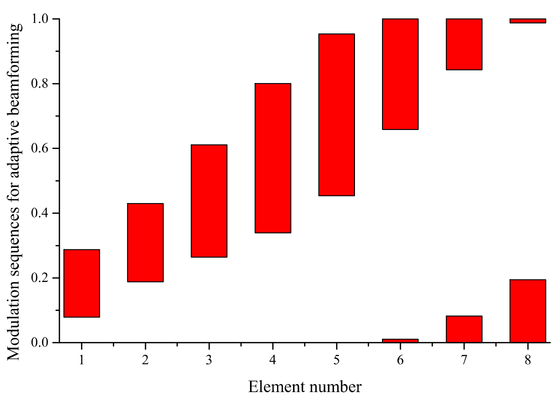

Fig. 4. Calculated time modulation sequences for adaptive beamforming.

REFERENCES

[1] H. Krim and M. Viberg, “Two decades of array signal processing research: the parametric approach,” *IEEE Signal Process Mag.*, vol. 13, no. 4, pp. 67–94, 1996.

[2] R. Roy and T. Kailath, “Esprit-estimation of signal parameters via rotational invariance techniques,” *IEEE Trans. Acoust. Speech Signal Process.*, vol. 37, no. 7, pp. 984–995, 2002.

[3] H. W. Chen and J. W. Zhao, “Wideband mvdr beamforming for acoustic vector sensor linear array,” *IET Radar Sonar Navig.*, vol. 151, no. 3, pp. 158–162, 2004.

[4] R. C. D. Lamare, L. Wang, and R. Fa, *Adaptive reduced-rank LCMV beamforming algorithms based on joint iterative optimization of filters: Design and analysis*. Elsevier North-Holland, Inc., 2010.

[5] T. Hunziker, T. Aono, and T. Ohira, “An iterative beamforming and decoding procedure for wireless networks with uncoordinated channel access,” *IEEE Commun. Lett.*, vol. 8, no. 4, pp. 256–258, 2004.

[6] H. E. Shanks and R. W. Bickmore, “Four-dimensional electromagnetic radiators,” *Can. J. Phys.*, vol. 37, no. 3, pp. 263–275, 1959.

[7] H. Shanks, “A new technique for electronic scanning,” *IRE Trans. Anntenas Propag.*, vol. 9, no. 2, pp. 162–166, 1961.

[8] S. Yang, Y.-B. Gan, and A. Qing, “Sideband suppression in time-modulated linear arrays by the differential evolution algorithm,” *IEEE Antennas Wirel. Propag. Lett.*, vol. 1, no. 1, pp. 173–175, 2002.

[9] S. Yang, Y. B. Gan, T. Peng Khiang, and T. Peng Khiang, “A new technique for power-pattern synthesis in time-modulated linear arrays,” *IEEE Antennas Wirel. Propag. Lett.*, vol. 2, no. 1, pp. 285 – 287, 2003.

[10] G. Li, S. Yang, Y. Chen, and Z. Nie, “An adaptive beamforming in time modulated antenna arrays,” in *2008 8th International Symposium on Antennas, Propagation and EM Theory*. IEEE, 2008, pp. 166–169.

[11] Y. Tong and A. Tennant, “Low sidelobe level harmonic beam steering using time-modulated linear arrays,” in *2010 Loughborough Antennas & Propagation Conference*. IEEE, 2010, pp. 249–252.

[12] L. Poli, P. Rocca, G. Oliveri, and A. Massa, “Harmonic beamforming in time-modulated linear arrays,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 7, pp. 2538 – 2545, 2011.

[13] Y. Tong and A. Tennant, “A two-channel time modulated linear array with adaptive beamforming,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 1, pp. 141 – 147, 2012.

[14] C. He, X. Liang, Z. Li, J. Geng, and R. Jin, “Direction finding by time-modulated array with harmonic characteristic analysis,” *IEEE Antennas Wirel. Propag. Lett.*, vol. 14, pp. 642–645, 2015.

[15] C. He, A. Cao, J. Chen, X. Liang, W. Zhu, J. Geng, and R. Jin, “Direction finding by time modulated linear array,” *IEEE Trans. Antennas Propag.*, 2018, in press.