LTE Ranging Measurement Using Uplink Opportunistic Signals and the SAGE algorithm

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***Abstract*—With the increase of services that need accurate location of the user, new techniques that cooperate with the Global Navigation Satellite System (GNSS) are necessary. GNSS suffers poor performance in indoor and dense urban environments, due to high signal attenuation and severe multipath propagation. The current release of the 3rd Generation Partnership Project (3GPP) LTE specification, supports the Uplink Time Difference Of Arrival (UTDOA) localization technique, which uses as reference the Sounding Reference Signal (SRS). Local Measurement Units (LMUs) devices use knowledge of the SRS to perform time difference measures. This paper studies the possibility of performing radio localization using a new UTDOA technique that exploits the uplink Demodulation Reference Signal (DM-RS) in 4G Long Term Evolution (LTE) cellular networks. We point out the advantages of our proposal and evaluate its feasibility by measuring the distance between two antennas using real DM-RS signals generated by an LTE module.**

***Index Terms*—LTE, Time Difference of Arrival, Ranging Measure, Opportunistic Positioning, Uplink DM-RS, SAGE algorithm**

I. Introduction

GLOBAL Navigation Satellite System (GNSS) terminals provide outdoor positions within meters accuracy, but location-based services and emergency call localization ask for the development and enhancement of positioning techniques based on wireless radio signals. In fact, in some environments such as indoor or in urban canyons, the GNSS signal cannot be available or can be affected by significant multipath components. In these cases, it is not possible to achieve the same accuracy as in the outdoor environment. The 3GPP LTE protocol (3rd Generation Partnership Project Long Term Evolution) introduced the support for positioning and, indeed, 4G LTE can provide good coverage in those scenarios where the GNSS fails. The network usually measures the Time of Arrival (TOA), the Time Difference of Arrival (TDOA), the Received Signal-Strength (RSS), or the Angle of Arrival (AOA) [1]. These measurements are used to perform positioning, using techniques like Enhanced Cell Identity (E-CID), Assisted GNSS (A-GNSS), Observed Time Difference of Arrival (OTDOA) and Uplink Time Difference of Arrival (UTDOA) [1].

3GPP introduces two stand-alone network-based TDOA measures, i.e., OTDOA and UTDOA, which exploit downlink and uplink transmissions, respectively. In OTDOA, the UE measures the TDOA of neighbour eNodeBs with respect to a reference eNodeB, exploiting the Positioning Reference Signal (PRS). The UTDOA positioning method in LTE is similar to OTDOA, except that it uses the uplink Sounding Reference Signal (SRS). In this case, UE signals are measured at some eNodeBs or at some Location Measurement Units (LMUs), which are standalone units integrated in the eNodeB or, alternatively, located in some known strategic location. In many cases, however, the eNodeB does not trigger the UE to transmit the SRS, thus making it impossible to provide positioning information. The main contribution of this paper is to define a new opportunistic method to estimate the TOA using the uplink demodulation reference signal (DM-RS) instead of the SRS. As a matter of fact, the DM-RS signal is transmitted in every slot during UE data transmission, e.g., in the transmission of a simple “ping” signal. A possible drawback of the use of DM-RS with this strategy, however, is that data transmission could occupy a smaller bandwidth than SRS, thus making TOA estimation less precise. We show that the combined use of DM-RS and of the SAGE algorithm [2] can perform range measurements with a good precision, confirming that DM-RS can be effectively used for positioning using TDOA techniques. In case of a strong line of sight component, good accuracy can be obtained with a simpler DM-RS signal correlation peak detector.

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II. Reference Signals

In the LTE uplink, there are two types of reference signals (RSs), i.e., DM-RS and SRS. The DM-RS is designed for coherent demodulation of the data and for channel estimation. The SRS is designed to determine the channel quality in order to use a frequency selective scheduling of the uplink transmission [3].

The DM-RS of a given UE is the reference signal associated to data or control transmission and occupies the same transmission bandwidth of the data, and it has the advantage that it is transmitted on every slot used. On the contrary, the SRS in some configurations is never sent. When the SRS is transmitted, however, it fills the last SC-FDMA symbol of the subframe and occupies different subcarriers with respect to the ones assigned to data transmission, usually with a larger bandwidth.

A. DM-RS Specification

The time domain DM-RS is generated starting from a base reference sequence (RS). The length of the RS, N_{rs}, is equal to the number of assigned subcarriers M_{sc}^{RB}. The M_{sc}^{RB} value is always a multiple of 12, which is the number of sub-carriers

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assigned to a resource block (RB). Therefore, we have $N_{rs} = M_{sc}^{RB} = m \cdot 12,\ 1 \leq m \leq N_{RB}$, where N_{RB} is the total number of RBs in the uplink bandwidth.

When only one or two resource blocks are allocated for data transmission, a special QPSK sequence is used as the base sequence (more details can be found in [4]). When the RBs allocation is greater than two, the base RS sequence is a Zadoff-Chu (ZC) sequence. This sequence is defined as

$$x_q(k) = e^{-i\pi q \frac{k(k+1)}{N_{ZC}^{RS}}}, \quad k = 0, 1, \cdots, N_{ZC}^{RS}, \tag{1}$$

where $x_q(k)$ is the ZC sequence, N_{ZC}^{RS} is the largest prime number smaller than N_{rs}, and q is the root index of the ZC sequence. The q index depends on the group index u and the sequence index v, which are assigned depending on the base station configuration [4]. The reference sequence of length N_{rs} is obtained as a cyclical extension of the ZC sequence $x_q(k)$, namely

$$\bar{r}_{u,v}(n) = x_q\left(n \mod N_{ZC}^{RS}\right) \quad n = 0, 1, \cdots, N_{rs}. \tag{2}$$

After the cyclical extension, the sequence is multiplied by the exponential

$$\bar{r}_{u,v}^{\alpha}(n) = \bar{r}_{u,v}(n) \cdot e^{-i\alpha n}, \tag{3}$$

corresponding to a shift in the time domain. The reference sequence is then mapped to the transmission subcarriers and transmitted in the time domain through an OFDM modulator.

In order to measure the TOA at the antennas receiving the DM-RS, knowledge of some parameters is necessary. A detailed description of the parameters can be found in [4]. In the experiment setup described below, these parameters can be obtained from the LTE equipment. In this way, we are able to reconstruct the original signal at the receiver, and estimate its TOA. In a real implementation, this calculation can be done by the eNodeBs, which know the parameters, and also by the LMUs, coordinated by the reference eNodeB.

III. SAGE Algorithm

TOA-based positioning systems require the estimation of the time of arrival of the first path. To this purpose, we adapt the Space-Alternating Generalized Expectation-Maximization (SAGE) algorithm [2], [5] to the particular characteristics of the DM-RS uplink signals.

The SAGE algorithm is an iterative method to separate and estimate the multipath components of the received signal. In our experiment we do not use information about the AOA, so the channel impulse response (CIR) is modeled as [6]

$$h(t;\tau;\boldsymbol{\vartheta}) = \sum_{l=1}^{L} \alpha_l \delta(\tau - \tau_l) e^{i2\pi f_{D,l} t}, \tag{4}$$

where we consider L paths propagating from the transmitter to the receiver and define the channel parameter vector $\boldsymbol{\vartheta} = [\boldsymbol{\vartheta_1}, \cdots, \boldsymbol{\vartheta_L}] \in \mathbb{R}^{4L}$, with $\boldsymbol{\vartheta_l} = [\Re\{\alpha_l\}, \Im\{\alpha_l\}, \tau_l, f_{D,l}]$. Each path has a complex amplitude α_l, a delay τ_l, and a Doppler shift $f_{D,l}$. The channel frequency response (CFR) corresponding to (4) is

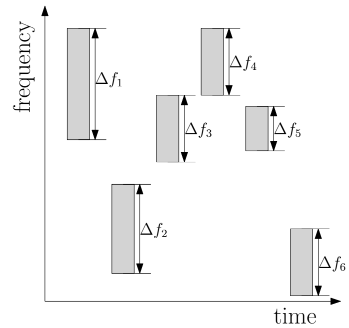

Fig. 1. Time-frequency pattern.

$$H(t;f;\boldsymbol{\vartheta}) = \sum_{l=1}^{L} H_l(t;f;\boldsymbol{\vartheta_l}) = \sum_{l=1}^{L} \alpha_l e^{i2\pi(f_{D,l}t - f\tau_l)}. \tag{5}$$

Note that the DM-RS is transmitted inside an uplink data transmission, and the band occupied by the data is scheduled by the eNodeB. In correspondence of a bandlimited DM-RS, the CFRs can be estimated, in any given subframe, only in those portions of the band where the data transmission is scheduled. Fig. 1 shows a possible time-frequency pattern for the bandwidth occupied by the DM-RS in different subframes. Note that the available information is fragmented in the time-frequency domain.

Suppose therefore that the channel is observed at different subcarriers $f_k = k\Delta f$, and at different time snapshots $t_n = n\Delta t$, (corresponding to different measurements of the CFR) in a non-uniform grid of coordinate pairs $I = \{(n,k)\}$. Let us denote with $N = |I|$ the cardinality of set I, i.e., the number of available channel observations in the time-frequency domain. In practice, an estimate of the CFR can be obtained in the receiver at the Discrete Fourier Transform (DFT) output used to demodulate the OFDM signal (assumed to deploy a Cyclic Prefix with length longer than the channel impulse response duration). In the following, we also implicitly assume that the channel parameters are constant within the time-frequency observation window. This is a reasonable assumption for short transmissions and/or small Doppler shifts. Then, N complex valued observations will be available at the receiver, with a fixed $\boldsymbol{\vartheta}$. These channel measurements can be described as $\boldsymbol{\hat{H}(t;f)} = \{\hat{H}(t_n;f_k) \in \mathbb{C}, (n,k) \in I\}$. Under the assumption of independent complex Gaussian noise samples $n(t_n;f_k) \in \mathbb{C}$, with variance $\sigma^2 = N_0/2$, we can write $\hat{H}(t_n;f_k) = H(t_n;f_k;\boldsymbol{\vartheta}) + n(t_n;f_k)$. The problem is reduced to finding an estimate of the unknown parameters $\boldsymbol{\gamma} = [\sigma^2, \boldsymbol{\vartheta}] \in \mathbb{R}^{4L+1}$ from the channel observations $\boldsymbol{\hat{H}(t;f)}$. The ML estimator of $\boldsymbol{\gamma}$ maximizes (the logarithm of) the likelihood function of $\boldsymbol{\hat{H}(t;f)}$, which under the Gaussian assumption is equal to

$$\gamma_{ML} = \min_{\gamma} \Big\{ 2N \log\left(2\pi\sigma^2\right) + \frac{1}{\sigma^2} \sum_{(n,k)\in I} \left| \hat{H}(t_n; f_k) - H(t_n; f_k; \boldsymbol{\vartheta}) \right|^2 \Big\} \quad (6)$$

As one can see, the noise variance σ^2 and the parameters $\boldsymbol{\vartheta}$ estimates can be separated, leading to

$$\hat{\boldsymbol{\vartheta}}_{ML} = \min_{\boldsymbol{\vartheta}} \{ \sum_{(n,k)\in I} |\hat{H}\left(t_n; f_k\right) - H\left(t_n; f_k; \boldsymbol{\vartheta}\right)|^2 \} \quad (7)$$

$$\hat{\sigma}^2_{ML} = \frac{1}{2N} \sum_{(n,k)\in I} |\hat{H}(t_n; f_k) - H(t_n; f_k; \hat{\boldsymbol{\vartheta}}_{ML})|^2. \quad (8)$$

The estimation of $\hat{\boldsymbol{\vartheta}}_{ML}$ is a $4L$ dimensional minimization problem and it is highly complex. The SAGE algorithm turns the $4L$ dimensional optimization problem iteratively into a sequence of one-dimensional problems [2], separately considering each path and its parameters. Assuming L multipath components, starting from the channel observations $\hat{\boldsymbol{H}}(\boldsymbol{t}; \boldsymbol{f})$, one can isolate the l-th term relating to the l-th path. This term can be calculated by canceling the interference caused by the other paths. Thus, having one previous estimate of the channel parameter $\hat{\boldsymbol{\vartheta}}'$, the term relating to the l-th path can be isolated as

$$\hat{H}_l\left(t_n; f_k\right) = \hat{H}\left(t_n; f_k\right) - \sum_{l'=1, l'\neq l}^{L} \hat{H}(t_n; f_k; \hat{\boldsymbol{\vartheta}}'_l). \quad (9)$$

After isolating the CFR relative to the single path l, its parameters $\boldsymbol{\vartheta}_l$ can be estimated by solving problem (7), with $\hat{H}_l\left(t_n; f_k\right)$ instead of $\hat{H}\left(t_n; f_k\right)$, namely

$$\boldsymbol{\vartheta}_l = \min_{\boldsymbol{\vartheta}_l} \{ \sum_{(n,k)\in I} |\hat{H}_l\left(t_n; f_k\right) - \alpha_l e^{i2\pi(f_{D,l} t_n - f_k \tau_l)}|^2 \}. \quad (10)$$

The SAGE algorithm uses alternate optimization over the three parameters τ_l, $f_{D,l}$ and α_l, starting from an initial guess of their values. It optimizes only one variable at a time, keeping the others fixed. The process is then repeated until the variables have converged. The value of α_l can be determined assuming that the delay value τ_l, and the Doppler shift value $f_{D,l}$, have been specified. The problem becomes

$$\alpha_{l,ML}(\tau_l, f_{D,l}) = \min_{\alpha_l} \{ \sum_{(n,k)\in I} |\hat{H}_l\left(t_n; f_k\right) - \alpha_l e^{i2\pi(f_{D,l} t_n - f_k \tau_l)}|^2 \}. \quad (11)$$

We can arrange values $\hat{H}_l\left(t_n; f_k\right)$ and $e^{i2\pi(f_{D,l} t_n - f_k \tau_l)}$ in the complex vectors $\boldsymbol{H}_l$ and $\boldsymbol{W}_l$, respectively, so that the minimization problem can be written as

$$\alpha_{l,ML}(\tau_l, f_{D,l}) = \min_{\alpha_l} \{ \|\boldsymbol{H}_l - \alpha_l \boldsymbol{W}_l\|^2 \} \quad (12)$$

It is well known that the vector $\alpha_l \boldsymbol{W}_l$ minimizing (12), is the orthogonal projection $\boldsymbol{U}_l$ of $\boldsymbol{H}_l$ onto $\boldsymbol{W}_l / \|\boldsymbol{W}_l\|$, namely

$$\boldsymbol{U}_l = \frac{\langle \boldsymbol{H}_l, \boldsymbol{W}_l \rangle}{\|\boldsymbol{W}_l\|^2} \boldsymbol{W}_l \qquad \alpha_{l,ML}(\tau_l, f_{D,l}) = \frac{\langle \boldsymbol{H}_l, \boldsymbol{W}_l \rangle}{\|\boldsymbol{W}_l\|^2}. \quad (13)$$

The inner product can be written as

$$\langle \boldsymbol{H}_l, \boldsymbol{W}_l \rangle = \sum_{(n,k)\in I} \hat{H}_l\left(t_n; f_k\right) e^{-i2\pi(f_{D,l} t_n - f_k \tau_l)}, \quad (14)$$

and, furthermore, $\|\boldsymbol{W}_l\|^2 = N$. Interestingly enough, note that, even in the fragmented case we are considering, (14) can be easily computed via FFT algorithms on a *regular* grid, including I, in the time-frequency domain, by setting to zero all the missing values.

We can estimate τ_l, assuming knowledge of $f_{D,l}$, as $\tau_{l,ML} = \min_{\tau_l} \|\boldsymbol{H}_l - \alpha_{l,ML}(\tau_l, f_{D,l}) \boldsymbol{W}_l\|^2$. Substituting the $\alpha_{l,ML}(\tau_l, f_{D,l})$ value of (13), we can simplify the problem formulation and obtain

$$\tau_{l,ML} = \max_{\tau_l} \{ | \sum_{(n,k)\in I} \hat{H}_l\left(t_n; f_k\right) e^{-i2\pi(f_{D,l} t_n - f_k \tau_l)}|^2 \} \quad (15)$$

The same procedure, assuming knowledge of $\tau_{l,ML}$, can be used to determine $f_{D,l,ML}$, so that

$$f_{D,l,ML} = \max_{f_{D,l}} \{ | \sum_{(n,k)\in I} \hat{H}_l\left(t_n; f_k\right) e^{-i2\pi(f_{D,l} t_n - f_k \tau_l)}|^2 \} \quad (16)$$

Again, we can compute the estimates via FFT algorithms on a regular grid by setting to zero the missing values. Finally, we calculate $\alpha_{l,ML} = \alpha_{l,ML}(\tau_{l,ML}, f_{D,l,ML})$.

The estimation of the number of paths L may not be a trivial task and is important for a successful use of the algorithm. In our experiments, we estimate L using a simple procedure based on the amplitude α_l of the different paths. In particular, we run the algorithm for $L = 1, 2, ...$ and stop the procedure when, in correspondence to the current value $L = L_s$, one of the paths has an amplitude $|\alpha_l|$ which is 20 dB below the value $|\alpha_{MAX}|$ of the path with maximum amplitude. Indeed, when this happens, it means that one of the L_s paths is negligible, so we keep the channel parameters calculated for $L = L_s - 1$. In order to safely stop the procedure in any case, we limit the search to $L \leq L_{MAX}$. This threshold choice is justified by the fact that the delay profiles described in [7] have a minimum relative power of -20.8 dB.

IV. Experimental Setup

In this section, we describe the setup of an experiment to determine the distance between two antennas using the DM-RS signals and the adapted SAGE algorithm. The experiment was carried out inside a building of the University of Udine. As one can see in Fig. 3 the two antennas are connected to two different inputs of the same USRP. The connection cables have the same length, in order to guarantee, to a first approximation, that we do not have to compensate the delay introduced by the cables. Furthermore, in order to allow synchronised acquisition of the signals coming from the LTE module, playing the role of the transmitting UE, the two acquisition channels of the USRP share the same reference clock. The two antennas are positioned at a distance d and are aligned with the LTE module, so that we can estimate d by measuring the TDOA

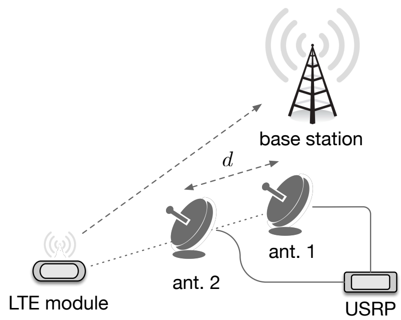

Fig. 2. Arrangement of the experimental setup.

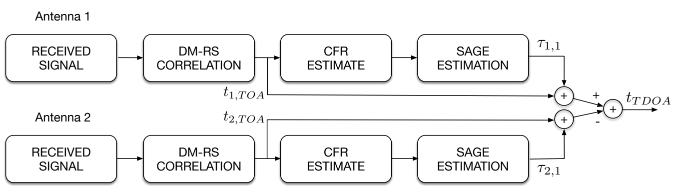

Fig. 3. Block diagram of the proposed system.

between the two antennas and multiplying the result by the speed of light. In this way, we do not need to know the transmission start time, thus making the setup simpler. A good estimate of d implies that the proposed system can provide good TDOA measurements, which are the basis of hyperbolic positioning systems [1]. A sketch of the setup arrangement is shown in Fig. 2.

The DM-RS is always associated with a data transmission. In our experiment, we force the LTE module Toby L2 [8] to transmit a "ping". After receiving the signal from the two antennas at the USRP, we find the arrival time of the first DM-RS signal inside the received signals by correlating them with the known transmitted DM-RS. This estimate is rather coarse, since it does not take into account the effect of the multipath channel. We denote this coarse TOA as $t_{n,TOA}$ where n indicates the n-th antenna. Considering that the LTE standard ensures a fixed time $\Delta t = 0.5$ ms between a DM-RS and the one transmitted in the next slot, knowledge of $t_{n,TOA}$ allows the estimation of the time position of successive DM-RSs in a given number of consecutive slots. Knowing the positions in time and the bandwidth occupied by the DM-RSs, we can estimate the CFRs for each DM-RS using the cyclic prefix and the deconvolution method in frequency, obtaining estimates $\hat{H}(t_n; f_k)$ as defined before. The values $\hat{H}(t_n; f_k)$ for frequencies outside the DM-RS bandwidth at each t_n are set to zero.

Using the SAGE algorithm, we can then process $\hat{\boldsymbol{H}}(\boldsymbol{t}; \boldsymbol{f})$ to estimate the delay. The most important delay for TOA estimation is that of the first arrival path $\tau_{n,1}$, where n denotes the n-th antenna. It is important to consider the fact that there is no certainty that the first path is the path with the larger amplitude $|\alpha|$. Notice that the "ping" signal occupies a relatively small number of subframes and that we can indeed assume that the channel parameters $\boldsymbol{\vartheta}$ remain constant during its transmission. After estimating the TOA and the delay for the two antennas, we can finally calculate the TDOA as $t_{TDOA} = (t_{1,TOA} + \tau_{1,1}) - (t_{2,TOA} + \tau_{2,1})$ and estimate the distance between the two antennas as $d = t_{TDOA}\, c$, where c is the speed of light, and we assume that the first antenna is further from the LTE module. Fig. 3 shows a block diagram of the main steps involved in the procedure.

V. Experimental Results

In this section, we describe the parameters of the experiment and present the results.

The LTE module was connected to a base station that uses the LTE band 3 (further details on LTE band definitions can be found in [7]), which used an uplink central frequency of 1.775 GHz. The available bandwidth is 18 MHz (which corresponds to 100 RBs). The time and frequency scheduling is managed by the BS. Typically, a ping transmission occupies from 30 to 40 subframes in time and from 2 to 50 RBs in frequency. The USRP was set to have a sample rate of 35 MHz which is then downconverted to 30.72 MHz in post processing. The measurements were made in three different scenarios, with a distance $d = 40$ m between the antennas, then with a distance $d = 35$ m and a distance $d = 30$ m. The SAGE algorithm was run with a maximum number of paths $L_{MAX} = 10$, in case the constraint described in Section III to estimate L is not achieved. Note that the transmission power may vary in different subframes [7], so we normalize the received signals by taking it into account, otherwise this would affect the estimated CFRs. The antennas were placed in a line of sight condition. In the experiment with $d = 40$ m, the furthest antenna was placed behind a open door, which could contribute to more multipath in this case. Distances were measured with a laser distance-meter, so that we can compare the value obtained with the distance-meter and the estimates obtained using the setup described above. For each value of d, we measure the distance obtained in 15 "ping" transmissions. We also show the mean distance obtained with the correlation and the mean distance obtained with SAGE.

The results of distance estimation with SAGE are compared with the average distance calculated as $(t_{1,TOA} - t_{2,TOA})\, c$, with $t_{1,TOA}$ and $t_{2,TOA}$ obtained with the correlation method.

Fig. 4 shows the distance estimates corresponding to the 15 "ping" transmissions when the two antennas were at a distance 40 m. As we can see, using SAGE we have only one estimate with an error greater than 15 m, while the error is always below 10 m in the other cases. The average distance estimate error is about 2 m. With the correlation estimate, instead, the error exceeds 10 m in three measures and the average error is about 6 m.

Fig. 5 shows the estimation corresponding to the "ping" transmissions when the two antennas were at distance of 35 m. The SAGE algorithm gives an error greater than 10 m in one case, but the other estimates are close to the correct distance 35 m. Also, the estimate average value is very close

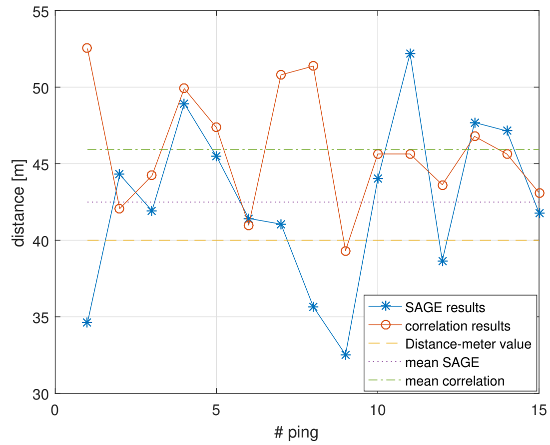

Fig. 4. Distance estimates using SAGE and the correlator, for a 40 m distance between the two antennas.

to 35 m and the error is below 1 m. The correlation estimate error never exceeds 5 m, but the mean value is slightly worse than the one obtained with SAGE. The improved precision of the correlation method in this case is probably due to the strong line of sight component in this case.

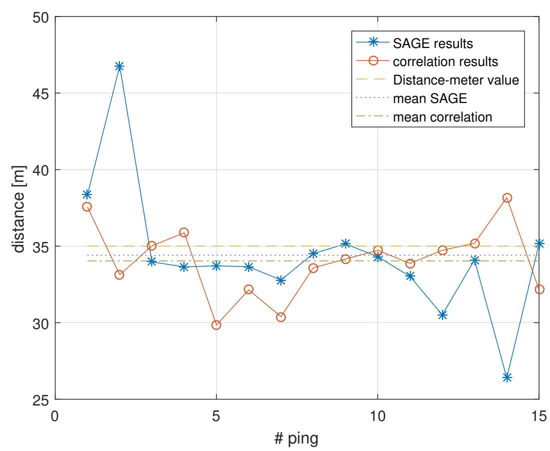

Fig. 5. Distance estimates using SAGE and the correlator, for a 35 m distance between the two antennas.

Fig. 6 shows the distance estimation corresponding to the "ping" transmissions when the two antennas were at a distance of 30 m. In this case, the error with the SAGE algorithm is always below 8 m, and the average error is around 1 m. The same happens for the correlation estimation, and in particular the average error is close to that one obtained with SAGE. Also in this case, the line of sight component is probably dominant.

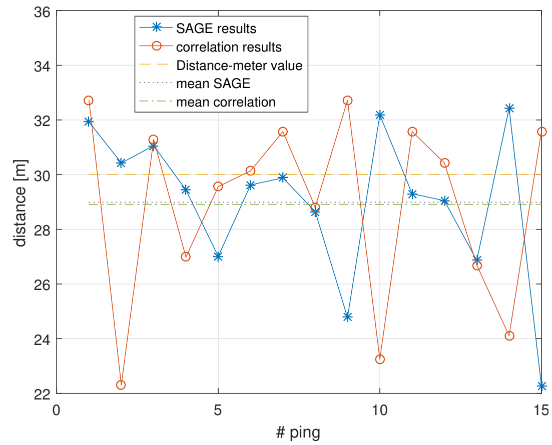

Fig. 6. Distance estimates using SAGE and the correlator, for a 30 m distance between the two antennas.

VI. Conclusions

A new method for the estimation of TOA using the uplink DM-RS instead of the SRS is proposed. We have demonstrated this by showing that we can use a simple "ping" uplink transmission to estimate the TDOA between two antennas. The advantage of the proposed method is that position can be estimated each time any data is transmitted, without the need to rely on the SRS signal. We demonstrated, by means of experiments, that using the SAGE algorithm and DM-RS, we obtain good positioning results, with a mean error varying between 4 m to below 1 m in experiments along an indoor corridor. We also showed that the accuracy obtained with a simpler correlation peak detector, still using the DM-RS, can be similar to the one obtained with SAGE, in cases in which there is of a strong line of sight component. The SAGE algorithm probably gives significantly better results when the line of sight component is not dominant. We are currently experimenting with further TDOA indoor positioning experiments and algorithms.

References

[1] R. Zekavat and R. M. Buehrer, *Handbook of Position Location: Theory, Practice and Advance.* Jhon Wiley & Sons, 2011.

[2] B. H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus, and K. I. Pedersen, "Channel parameter estimation in mobile radio environments using the SAGE algorithm," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 3, pp. 434–450, 1999.

[3] S. Sesia, I. Toufik, and M. Baker, *LTE - The UMTS Long Term Evolution From Theory to Practice*. John Wiley & Sons, 2011.

[4] *Physical Channels and Modulaton*, 3GPP Std. TS 36.211, Rev. 13.10.00, 2018. [Online]. Available: www.3gpp.org

[5] C. C. Chong, D. I. Laurenson, C. M. Tan, S. McLaughlin, M. A. Beach, and A. R. Nix, "Joint detection-estimation of directional channel parameters using the 2-D frequency domain SAGE algorithm with serial interference cancellation," in *Proc. International Conference on Communications (ICC)*. IEEE, Apr. 2002.

[6] R. Thoma, D. Hampicke, A. Richter, G. Sommerkorn, A. Schneider, and U. Trautwein, "Channel parameter estimation in mobile radio environments using the sage algorithm," *IEEE Transactions on Instrumentation and Measurement*, vol. 49, no. 2, pp. 357–364, 2000.

[7] *User Equipment (UE) Radio Transmission and Reception.*, 3GPP Std. TS 36.101, Rev. 13.12.00, 2018. [Online]. Available: www.3gpp.org

[8] u-blox. Toby-l2 series. [Online]. Available: www.u-blox.com/en/product/toby-l2-series