Sampling Rate and Bits Per Sample Tradeoff for Cloud MIMO Radar Target Detection

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***Abstract*—In this paper, target detection is studied for a cloud multiple-input multiple-output (MIMO) radar system, where each receiver communicates with a fusion center (FC) through a backhaul network. To reduce communication burden, local measurements at each receiver are quantized before they are sent to the FC. Under a bitrate constraint for each local sensor, we derive the detection probability of the cloud radar and analyze effects of the sampling rate and bits per sample on the detection performance. The quantizer output is initially modeled using direct analysis (DA), and then the Gaussian quantization error approximation (GQEA) method is employed to facilitate theoretical analysis. We verify that these two methods lead to close enough detection performance for large enough number of bits per sample. The tradeoff between the sampling rate and bits per sample is presented analytically and numerically.**

***Index Terms*—MIMO radar, quantization, detection, sampling rate**

I. Introduction

Cloud radar, where multiple receivers send local data to a fusion center (FC) via a backhaul communication network, has been studied in [1] and [2] for code vector optimization with a single transmitter. The work in [3] extends the study on cloud radar to the MIMO case with multiple transmitters and multiple receivers, and parameter estimation performance has been presented. To reduce communication burden, the local data are usually quantized [1], [3] before being sent to the FC. It has been shown that when the other system parameters are fixed, increasing the number of bits per sample improves the estimation performance [3]. In this work, we discuss the target detection problem for the cloud MIMO radar.

Considering that in the cloud radar, a large number of local sensors may communicate with the FC wirelessly, the backhaul capacity associated with a local sensor-to-FC path is limited, so that the bitrate allowed for each path must be limited [4]. The bitrate R equals the product of bits per sample b and sampling rate f_s, i.e. $R = bf_s$. While higher sampling rate and larger number of bits per sample are more favorable, the limitation on bitrate implies a tradeoff between the sampling rate and bits per sample. In this paper, we analyze the effects of the sampling rate and bits per sample on the detection performance under a bitrate constraint.

The detection performance depends heavily on the quantization output. Two popular ways to model the quantization output are the direct analysis (DA) [5], [6], [7], [8], [9], [10], [11], [12], [13] and Gaussian quantization error approximation (GQEA) [4], [14], [15], [16]. The DA method considers the exact quantization process to model the quantizer output as a discrete random variable. The GQEA method attributes the quantization effect to the introduction of an additive Gaussian error to the input, so that the quantizer output is modeled as the input plus a Gaussian noise, leading to a continuous random variable which is usually more tractable for further analysis. In this work, we employ the DA method to compute the exact detection performance, and then use the GQEA method for comparison. We show that the detection performance obtained from the GQEA method approaches that from the DA method, as long as the bits per sample is large enough. Unlike most of the existing work where the quantizer is applied to real data [5], [6], [7], [8], [9], [10], [11], [14], [15], [16], we consider the quantization of complex data, which is more appropriate for many common baseband communication signals.

II. Signal Model

Consider a cloud MIMO radar system that has M transmitters and N receivers, which are widely spaced. The $m-th$, $m = 1, ..., M$ transmitter and the $n-th$, $n = 1, ..., N$ receiver are located at (x^t_m, y^t_m) and (x^r_n, y^r_n) respectively, in a two-dimensional Cartesian coordinate system. The lowpass equivalent of the signal emitted from the $m-th$ transmitter is $\sqrt{E/M}s_m(kT_s)$, where E denotes the total transmitted energy, T_s is the sampling period, k ($k = 1, ..., K, K = \lceil Tf_s \rceil$, where $\lceil \cdot \rceil$ means round up) is an index running over the different time samples, f_s is the sampling rate, and T is the observation time. The signals transmitted from different transmitters are assumed to be orthogonal and maintain orthogonality for the delays shifts and Doppler shifts of interest [17]. The target, if present, is located at (x, y) moving with velocity (v_x, v_y). The signal received at receiver n contributed by the $m-th$ transmitter at time kT_s is [17]

$$r_{nm}[k] = \sqrt{\frac{E}{M}} \varsigma_{nm} s_m(kT_s - \tau_{nm}) e^{j2\pi f_{nm} kT_s} + w_{nm}[k], \quad (1)$$

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where the clutter-plus-noise $w_{nm}[k]$ is assumed to be zero-mean white complex circular Gaussian distributed with $\mathbb{E}\{w_{nm}[k]w_{nm}^*[k']\} = \sigma_w^2\delta[k-k']$ for all k and k' before and after sampling[1], $\mathbb{E}[\cdot]$ denotes mathematical expectation. To simplify the analysis, suppose the reflection coefficient ς_{nm} is a deterministic known parameter. The τ_{nm} represents the time delay associated with the $nm-th$ path and f_{nm} represents the Doppler shift of the received signal corresponding to the $nm-th$ path.

It is easy to see that $r_{nm}(k)$ is complex Gaussian with mean

$$\mu_{nm}[k] = \sqrt{\frac{E}{M}}\varsigma_{nm}s_m\left(kT_s-\tau_{nm}\right)e^{j2\pi f_{nm}kT_s}, \tag{2}$$

and variance σ_w^2. The real and imaginary parts of $r_{nm}[k]$ are independent Gaussian distributed, $Re\{r_{nm}[k]\} \sim \mathcal{N}(\mu_{nm,k}^{real},\sigma_{nm}^2)$, and $Im\{r_{nm}[k]\} \sim \mathcal{N}(\mu_{nm}^{imag},\sigma_{nm}^2)$, where

$$\sigma_{nm}^2 = \frac{1}{2}\sigma_w^2,\ \mu_{nm,k}^{real} = Re\{\mu_{nm}[k]\},\ \mu_{nm,k}^{imag} = Im\{\mu_{nm}[k]\}, \tag{3}$$

$Re\{\cdot\}$ represents an operator taking the real part of a complex number and $Im\{\cdot\}$ represents the imaginary part accordingly.

III. Quantization under Constrained Bitrate

Under a bitrate constraint for each local sensor,

$$R = bf_s \tag{4}$$

is fixed, where b denotes the number of bits for each sample and f_s is the sampling rate. Next we discuss the detection performance under certain b and f_s.

A common approach to quantize complex data is to feed its real and imaginary parts to two quantizers of the same design separately [18], [19]. After $r_{nm}[k]$ is quantized, the quantization output $q_{nm}[k]$ can be obtained

$$q_{nm}[k] = \mathbb{Q}\{Re\{r_{nm}[k]\}\} + j\mathbb{Q}\{Im\{r_{nm}[k]\}\}, \tag{5}$$

where $\mathbb{Q}\{\cdot\}$ represents quantization of a real number. Denote the quantized vector by $\mathbf{q}[k] = \left[q_{11}[k]\ q_{12}[k]\ \cdots\ q_{NM}[k]\right]^\dagger$, where the superscript "†" denotes transpose. The overall quantized vector is

$$\mathbf{q}=\left(\mathbf{q}^\dagger[1]\ \ \mathbf{q}^\dagger[2]\ \cdots\ \ \mathbf{q}^\dagger[K]\right)^\dagger. \tag{6}$$

The local sensors can communicate with the FC through a backhaul network, and the FC uses the quantized data to complete the target detection. To simplify analysis, we assume that this backhaul is ideal [2], [20]. Thus, the observation vector received at the FC is

$$\mathbf{y}=\mathbf{q}. \tag{7}$$

[1]The correlated clutter-plus-noise case will be analyzed in future work.

Therefore, under the H_1 hypothesis (target present in the cell-under-test) and the H_0 hypothesis (target absent), the detection problem at the FC is

$$\begin{aligned}
H_0 &: \mathbf{y}=\mathbf{q} = (q_{11}[1]\ q_{12}[1]\ \cdots\ q_{NM}[K])^\dagger,\\
&\quad q_{nm}[k] = \mathbb{Q}\{Re\{w_{nm}[k]\}\} + j\mathbb{Q}\{Im\{w_{nm}[k]\}\},\\
H_1 &: \mathbf{y}=\mathbf{q} = (q_{11}[1]\ q_{12}[1]\ \cdots\ q_{NM}[K])^\dagger,\\
&\quad q_{nm}[k] = \mathbb{Q}\{Re\{\mu_{nm}[k]+w_{nm}[k]\}\}\\
&\qquad\qquad + j\mathbb{Q}\{Im\{\mu_{nm}[k]+w_{nm}[k]\}\}.
\end{aligned} \tag{8}$$

Next, we discuss the target detection for quantizing the received signals in two ways, one using the DA method, and the other GQEA method.

A. Direct analysis

We first analyze the quantization output directly, and assume the output of the quantizer to input γ is

$$\mathbb{Q}\{\gamma\} = \begin{cases} 0 & ,\gamma_0<\gamma<\gamma_1\\ 1 & ,\gamma_1<\gamma<\gamma_2\\ \vdots & \\ D-1 & ,\gamma_{D-1}<\gamma<\gamma_D \end{cases}, \tag{9}$$

where $D = 2^b$ is the number of quantized values. The quantized observations are therefore

$$q_{nm}[k] \triangleq q_{nm}^{real}[k] + jq_{nm}^{imag}[k] = \mathbb{Q}\{Re\{r_{nm}[k]\}\} + j\mathbb{Q}\{Im\{r_{nm}[k]\}\}$$

Under two different hypotheses, the likelihood function of $\mathbf{y}$ is

$$p(\mathbf{y}|H_0) = \prod_{n=1}^{N}\prod_{m=1}^{M}\prod_{k=1}^{K} p(q_{nm}[k]|H_0), \tag{10}$$

$$p(\mathbf{y}|H_1) = \prod_{n=1}^{N}\prod_{m=1}^{M}\prod_{k=1}^{K} p(q_{nm}[k]|H_1). \tag{11}$$

Based on (3) and (9) , it can be obtained that for $d_{real,k} = 0,1,\cdots,D-1, d_{imag,k} = 0,1,\cdots,D-1$

$$\begin{aligned}
&p(q_{nm}[k]|H_0) = p(q_{nm}^{real}[k], q_{nm}^{imag}[k]|H_0)\\
&= p(q_{nm}^{real}[k]=d_{real,k}|H_0)p(q_{nm}^{imag}[k]=d_{imag,k}|H_0)\\
&= \left[Q(\frac{\gamma_{d_{real,k}}}{\sigma_{nm}}) - Q(\frac{\gamma_{d_{real,k}+1}}{\sigma_{nm}})\right]\left[Q(\frac{\gamma_{d_{imag,k}}}{\sigma_{nm}}) - Q(\frac{\gamma_{d_{imag,k}+1}}{\sigma_{nm}})\right],
\end{aligned} \tag{12}$$

where $Q(\cdot)$ is the complementary distribution function of the standard Gaussian distribution defined as

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt. \tag{13}$$

Similarly, we can obtain

$$\begin{aligned}
p(q_{nm}[k]|H_1) = &\left[Q(\frac{\gamma_{d_{real,k}}-\mu_{nm,k}^{real}}{\sigma_{nm}}) - Q(\frac{\gamma_{d_{real,k}+1}-\mu_{nm,k}^{real}}{\sigma_{nm}})\right]\\
&\times\left[Q(\frac{\gamma_{d_{imag,k}}-\mu_{nm,k}^{imag}}{\sigma_{nm}}) - Q(\frac{\gamma_{d_{imag,k}+1}-\mu_{nm,k}^{imag}}{\sigma_{nm}})\right].
\end{aligned} \tag{14}$$

Then, the log-likelihood ratio of $\mathbf{y}$ is

$$L(\mathbf{y}) = \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} \ln \frac{\left[Q(\frac{\gamma_{d_{real,k}} - \mu_{nm,k}^{real}}{\sigma_{nm}}) - Q(\frac{\gamma_{d_{real,k}+1} - \mu_{nm,k}^{real}}{\sigma_{nm}})\right]}{\left[Q(\frac{\gamma_{d_{real,k}}}{\sigma_{nm}}) - Q(\frac{\gamma_{d_{real,k}+1}}{\sigma_{nm}})\right]} \times \frac{\left[Q(\frac{\gamma_{d_{imag,k}} - \mu_{nm,k}^{imag}}{\sigma_{nm}}) - Q(\frac{\gamma_{d_{imag,k}+1} - \mu_{nm,k}^{imag}}{\sigma_{nm}})\right]}{\left[Q(\frac{\gamma_{d_{imag,k}}}{\sigma_{nm}}) - Q(\frac{\gamma_{d_{imag,k}+1}}{\sigma_{nm}})\right]}. \quad (15)$$

Therefore, the detection probability can be obtained

$$P_D = P\left(L(\mathbf{y}) \geq \alpha | H_1\right), \quad (16)$$

where α is the detection threshold determined by the false alarm level P_{FA},

$$P_{FA} = P\left(L(\mathbf{y}) > \alpha | H_0\right). \quad (17)$$

To simplify the analysis and obtain the closed-form detection probability for further theoretical analysis, we adopt a uniform quantizer next. In this case, $\gamma_0 = -\infty, \gamma_D = \infty$, $\gamma_d = [d - (D-1)/2 - 1]\Delta, d = 1, \cdots, D-1$, and Δ is the quantization step.

B. Gaussian quantization error approximation

Assuming the real and imaginary parts of the complex data use the same uniform quantizer and the amplitude of values is bounded within the interval $[-A_{max}, A_{max}]$, the quantization error for each part may be conveniently modeled as a zero mean process uniformly distributed. When $\sigma_{nm} > 0.25\Delta$ ($\Delta = 2A_{max}/2^b$ is the quantization step , σ_{nm} is the standard deviation of the real or imaginary parts of $r_{nm}[k]$), Gaussian quantization error approximation can be safely adopted for the quantized complex data in most practical cases [21] and the variance of the quantization error is $\Delta^2/12$. Assume the Gaussian approximation applies here, then after $r_{nm}[k]$ is quantized, the quantization output $q_{nm}[k]$ can be modeled as

$$q_{nm}[k] = \mathbb{Q}\left\{Re\left\{r_{nm}[k]\right\}\right\} + j\mathbb{Q}\left\{Im\left\{r_{nm}[k]\right\}\right\} = r_{nm}[k] + \varepsilon_{nm}[k],$$

where $\varepsilon_{nm}[k]$ is the quantization error which is a zero-mean white complex Gaussian random process, $\mathbb{E}\{\varepsilon_{nm}[k]\varepsilon_{nm}^*[k']\} = \frac{\Delta^2}{6}\delta[k-k']$. Therefore, under two different hypotheses, the likelihood function of $\mathbf{y}$ is

$$\begin{aligned} p(\mathbf{y}|H_0) &= \prod_{n=1}^{N}\prod_{m=1}^{M}\prod_{k=1}^{K} p(q_{nm}[k]|H_0) \\ &= \prod_{n=1}^{N}\prod_{m=1}^{M}\prod_{k=1}^{K} \frac{1}{\pi\sigma^2} e^{-\frac{1}{\sigma^2}(q_{nm}[k])^H(q_{nm}[k])}, \\ p(\mathbf{y}|H_1) &= \prod_{n=1}^{N}\prod_{m=1}^{M}\prod_{k=1}^{K} p(q_{nm}[k]|H_1) \\ &= \prod_{n=1}^{N}\prod_{m=1}^{M}\prod_{k=1}^{K} \frac{1}{\pi\sigma^2} e^{-\frac{1}{\sigma^2}(q_{nm}[k]-\mu_{nm}[k])^H(q_{nm}[k]-\mu_{nm}[k])}, \end{aligned}$$

where $\sigma^2 = \sigma_w^2 + \frac{\Delta^2}{6}$. The log-likelihood ratio of $\mathbf{y}$ is

$$L_G(\mathbf{y}) \propto \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} 2Re\left\{q_{nm}^H[k]\,\mu_{nm}[k]\right\} - |\mu_{nm}[k]|^2. \quad (18)$$

Based on (18), the test statistic is given by

$$T_G = \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} Re\left\{q_{nm}^H[k]\,\mu_{nm}[k]\right\}, \quad (19)$$

and

$$T_G|H_0 \sim \mathcal{N}\left(0, \sigma_t^2\right),\ T_G|H_1 \sim \mathcal{N}\left(\mu_t, \sigma_t^2\right), \quad (20)$$

where

$$\mu_t = \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} |\mu_{nm}[k]|^2, \sigma_t^2 = \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{K} \frac{\sigma^2}{2}|\mu_{nm}[k]|^2. \quad (21)$$

Therefore, the detection probability can be obtained

$$P_D^G = P\left(T_G \geq \eta | H_1\right) = Q\left(\frac{\eta - \mu_t}{\sigma_t}\right) = Q\left(Q^{-1}\left(P_{FA}\right) - \frac{\mu_t}{\sigma_t}\right), \quad (22)$$

where η is the detection threshold determined by the false alarm level P_{FA},

$$P_{FA} = P\left(T_G > \eta | H_0\right) = Q\left(\frac{\eta}{\sigma_t}\right) \Rightarrow \eta = \sigma_t Q^{-1}\left(P_{FA}\right). \quad (23)$$

C. Trade-off Between Bits Per Sample and Sampling Rate

Substituting (21) into (22), we have

$$P_D^G = Q\left(Q^{-1}\left(P_{FA}\right) - \sqrt{\frac{2\sum\limits_{n=1}^{N}\sum\limits_{m=1}^{M}\sum\limits_{k=1}^{\lceil Tf_s \rceil} |\mu_{nm}[k]|^2}{\sigma_w^2 + \frac{2}{3}\frac{A_{max}^2}{4^b}}}\right). \quad (24)$$

When f_s or b is fixed, we can get the relationship between the detection probability and b or f_s.

It can be obtained from (24) that a higher f_s contributes to an improved detection performance, while a larger b lessens the performance deterioration induced by quantization. Thus, under a bitrate constraint, the sampling rate and bits per sample involve a compromise. When the bitrate is fixed to $R = bf_s$, the detection probability P_D^G can be transformed into a function of the sampling frequency f_s,

$$P_D^G = Q\left(Q^{-1}\left(P_{FA}\right) - \left(\frac{\mu_t}{\sigma_t}\right)_{f_s}\right), \quad (25)$$

where

$$\left(\frac{\mu_t}{\sigma_t}\right)_{f_s} = \sqrt{\frac{2\sum\limits_{n=1}^{N}\sum\limits_{m=1}^{M}\sum\limits_{k=1}^{\lceil Tf_s \rceil} |\mu_{nm}[k]|^2}{\sigma_u^2 + \frac{2}{3}\frac{A_{max}^2}{4^{R/f_s}}}}.$$

The optimal sampling rate f_s^* satisfies

$$\frac{\partial P_D^G}{\partial f_s}\Big|_{f_s=f_s^*} = \frac{\partial Q\left(Q^{-1}\left(P_{FA}\right) - \left(\frac{\mu_t}{\sigma_t}\right)_{f_s}\right)}{\partial f_s}\Big|_{f_s=f_s^*} = 0. \quad (26)$$

The optimal sampling rate is obtained according to (26) (that maximizes the detection probability).

Similarly, we can get the optimal b.

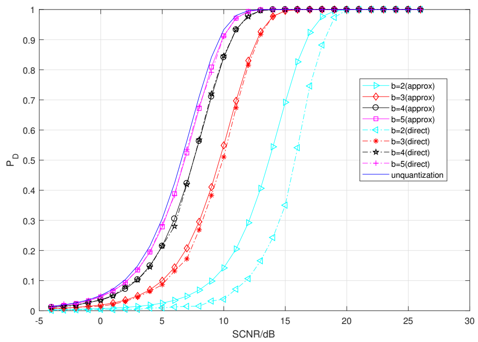

Fig. 1: P_D for direct analysis and GQEA method for different b for a cloud MIMO radar when sampling rate $f_s = 600Hz$.

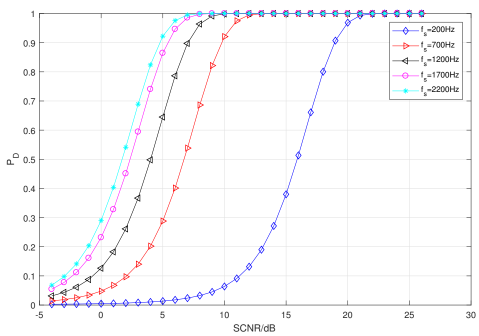

Fig. 2: P_D for GQEA method for different f_s for a cloud MIMO radar with $M = 2$ transmitters and $N = 3$ receivers.

IV. Simulation

In this section, the detection performance of the cloud MIMO radar system is investigated via numerical results. To define a general test set up that is easy to describe, assume each transmit and receive (single antenna) station is located 70 km away from the origin. Assume the number of transmitters and receivers are $M = 2$ and $N = 3$, and frequency spread single Gaussian pulse signals are adopted for transmission

$$s_m(kT_s) = (\frac{2}{T^2})^{1/4} e^{(-\pi(kT_s)^2/T^2)} e^{j2\pi m f_\Delta kT_s}, \tag{27}$$

where f_Δ is the frequency offset between adjacent radar transmit signals and T the pulsewidth. Set $f_\Delta = 150Hz$ and $T = 0.01s$. Suppose a target may be present at $(150, 130)m$ with velocity $(25, 20)km/h$. Define the signal to clutter-plus-noise ratio as SCNR $= 10\log_{10}(\sum_{n=1}^{N}\sum_{m=1}^{M} E|\varsigma_{nm}|^2/(N\sigma_w^2))$ and $\sigma_w^2 = 10^{-2}$. Set $P_{FA} = 10^{-3}$.

Fig. 1 plots the detection probability P_D versus SCNR for the direct analysis and GQEA method under different quantization bits when $f_s = 600Hz$. The results of direct analysis is obtained by 5000 Monte Carlo simulations. The unquantized results can be obtained from (22) by setting $\Delta = 0$. The figure shows that as the resolution increases, both models provide similar results and tend to the P_D for the unquantized case. When the number of bits used by the quantizer is large enough ($b > 2$), the result obtained by GQEA

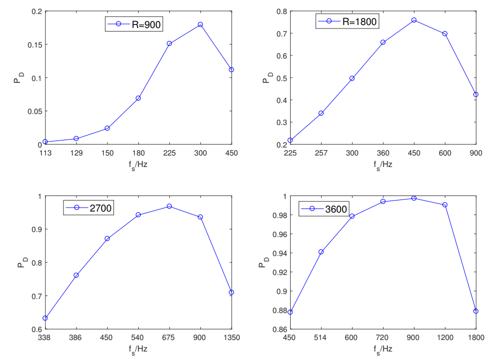

Fig. 3: P_D versus f_s for a fixed $R = [900, 1800, 2700, 3600]$ for a cloud MIMO radar.

method approaches to that obtained via direct analysis. When $b > 4$, there is almost no loss in performance for quantization. Fig. 2 plots the detection probability P_D versus SCNR under different f_s when $b = 4$. It can be seen from Fig. 2 that as the sampling rate f_s increases, the detection performance becomes better, while the gain of the detection performance due to the increase of the sampling rate becomes smaller. Taken together, Figs. 1 and Figs. 2 show that as b or f_s increases, the performance gain becomes smaller and smaller. Applying such analysis to clould MIMO radar system, the number of quantization bits or sampling rate needed[2] can be predicted.

Fig. 3 shows the P_D versus f_s for a given bitrate R when SCNR$=\ 11dB$. It can be seen from Fig. 3, the detection probability P_D increases as R increases, that is because the higher the f_s is, the better the detection performance is under the same b. Under fixed R, the detection probability increases first and then decreases with the increase of f_s. Therefore, there is a tradeoff between sampling rate and bits per sample and the sampling rate corresponding to the optimal detection performance may be larger than the Nyquist rate $f_N = 600Hz$, or smaller than the Nyquist rate. This method can be exploited to predict where the tradeoff occurs for cloud MIMO radar.

V. Conclusions

In this paper, target detection was studied for a cloud MIMO radar system using quantized measurements at the local sensor. Under a bitrate constraint for each local sensor, we derive the detection probability of the cloud radar and analyze effects of the sampling rate and bits per sample on the detection performance. The output of the quantizer is modeled in two different ways. The resulting detection performance is analyzed. The simulation results show that when the number of bits used by the quantizer is large enough ($b > 4$), there is almost no loss in performance for quantization and when $b > 2$, the result obtained by the GQEA method tends to that obtained via direct analysis. Under a fixed bitrate at the local sensor, the optimal sampling rate is obtained, and there is a tradeoff between the sampling rate and bits per sample.

[2]Theoretical analysis for the required number of quantization bits and sampling rate will be provided in the journal version.

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