Semidefinite Programming for MIMO Radar Target Localization using Bistatic Range Measurements

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***Abstract*—In this paper, we investigates the target localization problem based on bistatic range measurements in multiple input multiple output (MIMO) radar system with widely separated antennas. Under the assumption of uncorrelated Gaussian distributed measurement noises, The maximum likelihood estimator (MLE) is derived for this problem, which is highly nonconvex and difficult to solve. Weighted least squares (WLS) and Semidefinite programming (SDP) are two research directions for solving this problem. However, existing studies can not provide a high-quality solution over a large range of measurements noise. In this work, we propose to add a penalty term to improve the performance of the original SDP method. We further address the issue of robust localization in the case of non-accurate transimitter/receiver position. The corresponding Cramér Rao lower bound (CRLB) is also derived. Simulation results show the superiority of our proposed methods by comparing with other exiting algorithms and CRLB.**

***Index Terms*—Multiple-inputmultiple-output (MIMO) radar, Target localization, Range measurements, Semidefinite programming (SDP), Cramér-Rao Lower Bound (CRLB)**

I. Introduction

Nowadays, multiple-input multiple output (MIMO) radar systems have been widely used in military monitoring field due to its advantages over traditional phased-array radar systems [1]–[4]. Specifically, MIMO radars have the ability to jointly fuse the received signals at multiple receive antennas using multiple transmit waveforms [5]. Generally speaking, MIMO radar system can be classified into two categories, that is, the colocated antennas (CLA) and the widely separated antennas (WSA). The MIMO radar with CLA antennas ultilizes waveform diversity, whereas in the WSA structure, its performance gain mainly comes from the spatial diversity. Both of these two structures have significant advantages in target detection and localization over conventional phased-array radar systems. In this work, we focus on target localization using the WSA structure.

Basically, there are two kinds of approaches for MIMO radar localization, namely, direct and indirect localization. In the direct form, the target position is directly estimated by fusing the original received signals. Maximum likelihood estimation (MLE) is one of the representative methods in direct localization [6], [7]. The MLE is asymptotically optimal, but the formulated ML optimization problem is highly nonlinear and nonconvex. Although it can be solved iteratively by using numerical methods, a significant computational effort is involved. What's more, the nonconvexity also implies multiple local minima and hence an appropriate initialization is very crucial. On the other hand, the indirect form divides the localization procedure into two steps, i.e. range estimation and position estimation. The former, estimating the sum of transmitter-to-target and target-to-receiver distances from the received signals, which is referred to as bistatic range measurements. The latter procedure is to estimate the target location from the bistatic range measurements. To achieve this goal, MIMO target localization techniques using bistatic range measurements can be divided into three categorizes: ML [8], least squares (LS) based [9]–[12] and semidefinite programming (SDP) based [13], [14]. As aforementioned, the ML problem is very difficult to solve due to the existence of multiple local minima. The LS based method linearizes the observation equations by introducing auxiliary parameters such that LS (WLS) based solutions can be easily obtained. This type of methods are usually computationally efficient and accurate when the measurement noise is small, but are very susceptible to a large noise. The SDP-based method deals with the nonconvexity of the ML problem by performing semidefinite relaxation. As a result, a convex SDP problem is obtained, which can be efficiently solved using the interior-point methods. However, this class of methods require a tight relaxation to guarantee an accurate estimate.

In this paper, the addressed localization problem belongs to the indirect approach which assumes the bistatic range measurements are obtained using other estimation methods. Our contributions can be summarized as (i) we have proposed an improved SDP method for target localization in MIMO radar systems by introducing a penalty term in the objective function; (ii) we have extended the proposed SDP method to the localization scenario in which the antenna positions are subject to errors. Also, the corresponding Cramér Rao lower bound (CRLB) for this scenario is derived.

The rest of the paper is organized as follows. In Section II, the target localization problem considered in this work is formulated. In Section III, we first derive the SDP solution for the localization scenario with accurate antenna positions, then provide an extension for the localization case that the MIMO system with the presence of antenna position errors. In Section IV, simulation results are given to demonstrate the effectiveness of the proposed estimators. Finally, section IV

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summarizes this paper.

The following notations are used through the paper. Upper (lower) bold-face letters stand for matrices (vectors), respectively. The $\mathbb{E}(\cdot)$, $\|\cdot\|$, $(\cdot)^T$, $(\cdot)^{-1}$, trace$(\cdot)$, rank$(\cdot)$ and $\otimes$ stand for the expectation, l_2 norm, transpose, inverse, trace, rank and Kronecker product operators, respectively. The $\mathbf{1}$ and $\mathbf{I}$ denote the all-one vector and identity matrix (size indicated in the subscript if necessary), respectively. The ith element of $\mathbf{a}$ and (i,j) entry of $\mathbf{A}$ are represented as $[\mathbf{a}]_i$ and $[\mathbf{A}]_{i,j}$, respectively. Additionally, $[\mathbf{A}]_{i:j,k:l}$ contains entries in the intersection of the ith to the jth rows and the kth to the lth columns. The diag$(a_1, a_2, ..., a_k)$ is a diagonal matrix with diagonal elements $a_1, a_2, ..., a_k$. blkdiag$(\cdot)$ denotes the block diagonal matrix.

II. Problem formulation

We consider the target localization problem in a noncoherent MIMO radar system, which consists of M transmit and N receiver antennas. For simplicity, we consider the two-dimensional scenario, extension for the three-dimensional case is straightforward. Let $\mathbf{x}_m^t = [x_m^t, y_m^t]^T, m = 1, ..., M$, $\mathbf{x}_n^r = [x_n^r, y_n^r]^T, n = 1, ..., N$ and $\mathbf{u} = [x, y]^T$ be the known coordinates of the ith transmitter antenna, the jth receiver antenna and the unknown target position, respectively. In MIMO radar localization process, the transmit antennas send a set of mutually orthogonal waveforms which are reflected by the target and then collected at the receiver antennas. Denoting $\tau_{m,n}$ as the time delay measurement from the mth transmitter and the nth receiver, which is the sum of the signal propagation time from the mth transmitter to the target and from the target to the nth receiver. Let $s_m(t)$ be the low-pass equivalent of the emitted signal from the mth transmitter. For non-coherent processing where the receivers are not phase-synchronized, the signal measured at the nth antenna, denoted by $z_n(t)$, can be modeled as [6]

$$z_n(t) = \sum_{m=1}^{M} \alpha_{m,n} s_m(t - \tau_{m,n}) + w_n(t), \tag{1}$$

where $\alpha_{m,n}$ represents the signal amplitude and $w_n(t)$ is the zero-mean Gaussian noise. Assuming the time delays $\{\tau_{m,n}\}$ have been estimated from the received signals using the expectation-maximization method [15].

Subsequently, after multiplying by the wave propagation speed, the total NM bistatic range measurments are collected in a fusion center for estimating the position of the target. The bistatic range measurments, denoted by $\{r_{m,n}\}$, are

$$r_{m,n} = R_m^t + R_n^r + \varepsilon_{m,n}, m = 1, ..., M, n = 1, ..., N, \tag{2}$$

where $R_m^t = \|\mathbf{x}_m^t - \mathbf{u}\|$ and $R_n^r = \|\mathbf{x}_n^r - \mathbf{u}\|$ are the true distances between the target and the mth transmitter and the nth receiver, respectively. The term $\varepsilon_{m,n}$ is the measurement noise which is assumed to be independent and identically distributed (i.i.d) Gaussian random variables with zero mean and variance σ^2. Given the MN bistatic range measurements as well as $\{\mathbf{x}_m^t\}$ and $\{\mathbf{x}_n^r\}$, the localization task is to find the target position $\mathbf{u}$.

III. Proposed semidefinite programming approach

This section will first present our improved SDP localization approach under the assumption that the network knowledge of the radar antenna positions is accurate. Then, extension for the case of non-accurate antenna position as well as its corresponding CRLB are derived.

A. Localization with accurate antenna position

Under the i.i.d Gaussian noise assumption in (2), the joint conditional probability density function of the measurement data $\{r_{m,n}\}$ is given as

$$p(r_{m,n}|\mathbf{u}) = \prod_{m=1}^{M}\prod_{n=1}^{N}\left(2\pi\sigma^2\right)^{-1/2} \times \exp\left(-\tfrac{1}{2\sigma^2}\left(r_{m,n} - R_m^t - R_n^r\right)^2\right)$$

and the maximum likelihood estimation is

$$\mathbf{u}_{ml} = \arg\max_{\mathbf{u}} p(r_{m,n}|\mathbf{u}).$$

Then, $\mathbf{u}_{ml}$ can be written explicitly as

$$\mathbf{u}_{ml} = \arg\min_{\mathbf{u}} \left(\sum_{m=1}^{M}\sum_{n=1}^{N}\left(r_{m,n} - R_m^t - R_n^r\right)^2\right). \tag{3}$$

The above optimization problem is nonlinear and nonconvex, consequently the optimal solution is hard to achieve. Next, we will show that this cost minimization problem admits a SDP relaxation problem and can be solved by solving and rounding its SDP relaxation.

Stacking the MN bistatic range measurements and the distances $\{R_m^t, R_n^r\}$ into $\mathbf{r} = [r_{1,1}, ..., r_{1,N}, ..., r_{M,1}, ..., r_{M,N}]^T$ and $\mathbf{g} = [R_1^t, ..., R_M^t, ..., R_1^r, ..., R_N^r]^T$, respectively. Defining three matrixes $\mathbf{G} = \mathbf{g}\mathbf{g}^T$, $\mathbf{D} = [\mathbf{I}_M \otimes \mathbf{1}_N, \mathbf{1}_M \otimes \mathbf{I}_N]$ and

$$\mathbf{C} = \begin{bmatrix} \mathbf{D}^T\mathbf{D} & -\mathbf{D}^T\mathbf{r} \\ -\mathbf{r}^T\mathbf{D} & \mathbf{r}^T\mathbf{r} \end{bmatrix}.$$

With these notations, the objective function of (3) for minimization can be rewritten as

$$\text{trace}\left\{\mathbf{C}\begin{bmatrix} \mathbf{G} & \mathbf{g} \\ \mathbf{g}^T & 1 \end{bmatrix}\right\}. \tag{4}$$

Notice that the objective function of (4) is a linear function of both $\mathbf{G}$ and $\mathbf{g}$ and is convex. However, the constraints $R_m^t = \|\mathbf{x}_m^t - \mathbf{u}\|$, $R_n^r = \|\mathbf{x}_n^r - \mathbf{u}\|$ and $\mathbf{G} = \mathbf{g}\mathbf{g}$, are nonconvex, the solution remains difficult. Next, We will relax these constraints into convex constraints as tighten as possible.

To begin, we introduce a dummy variable $z = \mathbf{u}^T\mathbf{u}$. Then the square of distance constrains which relates z and $\mathbf{u}$ are

$$\begin{aligned} (R_m^t)^2 &= z - 2\mathbf{u}^T\mathbf{x}_m^t + (\mathbf{x}_m^t)^T\mathbf{x}_m^t, m = 1, ..., M, \\ (R_n^r)^2 &= z - 2\mathbf{u}^T\mathbf{x}_n^r + (\mathbf{x}_n^r)^T\mathbf{x}_n^r, n = 1, ..., N. \end{aligned} \tag{5}$$

We further notice that

$$\begin{aligned} [\mathbf{G}]_{m,m} &= (R_m^t)^2, m = 1, ..., M, \\ [\mathbf{G}]_{M+n,M+n} &= (R_n^r)^2, n = 1, ..., N. \end{aligned} \tag{6}$$

With the use of all developed constraints, the original optimization problem is equivalent to the following formulation

$$\begin{aligned}&\min_{\mathbf{G},\mathbf{g},z,\mathbf{u}}\left\{\text{trace}\left\{\mathbf{C}\begin{bmatrix}\mathbf{G} & \mathbf{g}\\ \mathbf{g}^T & 1\end{bmatrix}\right\}\right\}\\ &s.t. [\mathbf{G}]_{m,m}=z-2\mathbf{u}^T\mathbf{x}_m^t+(\mathbf{x}_m^t)^T\mathbf{x}_m^t, m=1,...,M,\\ &\quad [\mathbf{G}]_{M+n,M+n}=z-2\mathbf{u}^T\mathbf{x}_n^r+(\mathbf{x}_n^r)^T\mathbf{x}_n^r, n=1,...,N,\\ &\quad z=\mathbf{u}^T\mathbf{u},\\ &\quad \mathbf{G}=\mathbf{g}\mathbf{g}^T.\end{aligned} \tag{7}$$

The above optimization problem is convex except the two constraints $z=\mathbf{u}^T\mathbf{u}$ and $\mathbf{G}=\mathbf{g}\mathbf{g}^T$. These two nonconvex constrains will be replaced by the inequality $z\succeq\mathbf{u}^T\mathbf{u}$ and $\mathbf{G}\succeq\mathbf{g}\mathbf{g}^T$, respectively, to meet the convex specification. Further, they can be written as linear matrix inequalities

$$\begin{bmatrix}\mathbf{I}_2 & \mathbf{u}\\ \mathbf{u}^T & z\end{bmatrix}\succeq 0, \begin{bmatrix}\mathbf{G} & \mathbf{g}\\ \mathbf{g}^T & 1\end{bmatrix}\succeq 0. \tag{8}$$

Now, the nonconvex optimization problem of (7) is approximated as a convex optimization problem

$$\begin{aligned}&\min_{\mathbf{G},\mathbf{g},z,\mathbf{u}}\left\{\text{trace}\left\{\mathbf{C}\begin{bmatrix}\mathbf{G} & \mathbf{g}\\ \mathbf{g}^T & 1\end{bmatrix}\right\}\right\}\\ &s.t.\ [\mathbf{G}]_{m,m}=z-2\mathbf{u}^T\mathbf{x}_m^t+(\mathbf{x}_m^t)^T\mathbf{x}_m^t, m=1,...,M,\\ &\quad [\mathbf{G}]_{M+n,M+n}=z-2\mathbf{u}^T\mathbf{x}_n^r+(\mathbf{x}_n^r)^T\mathbf{x}_n^r, n=1,...,N,\\ &\quad \begin{bmatrix}\mathbf{I}_2 & \mathbf{u}\\ \mathbf{u}^T & z\end{bmatrix}\succeq 0,\\ &\quad \begin{bmatrix}\mathbf{G} & \mathbf{g}\\ \mathbf{g}^T & 1\end{bmatrix}\succeq 0.\end{aligned} \tag{9}$$

However, we note that the convex optimization formulation of (9) is still prone to ambiguities. Specifically, $[\mathbf{G}]_{i,i}$ and $[\mathbf{g}]_i$ represent $(R_i^t)^2$ (or $(R_i^r)^2$) and R_i^t (or R_i^r) respectively. From the minimization problem formulated in (3), we observe that a large $(R_m^t+R_n^r)^2$ and a large $(R_m^t+R_n^r)$ could yield the same cost value as a small $(R_m^t+R_n^r)^2$ and a small $(R_m^t+R_n^r)$. In fact, it has been observed that the estimate of $[\mathbf{G}]_{i,i}$ always becomes very large while that of $[\mathbf{g}]_i$ close to zero. Therefore, we add a penalty term to avoid the ambiguity, i.e. $\eta\text{trace}(\mathbf{G})$, where η is the penalty coefficient. Note that, as far as we known, the optimal selection strategy of η has not been addressed so far. Thus, we choose the value of η based on the previous work [16]. Finally, we formulate our proposed SDP optimization problem as

$$\begin{aligned}&\min_{\mathbf{G},\mathbf{g},z,\mathbf{u}}\left\{\text{trace}\left\{\mathbf{C}\begin{bmatrix}\mathbf{G} & \mathbf{g}\\ \mathbf{g}^T & 1\end{bmatrix}\right\}+\eta\text{trace}(\mathbf{G})\right\}\\ &s.t.\ [\mathbf{G}]_{m,m}=z-2\mathbf{u}^T\mathbf{x}_m^t+(\mathbf{x}_m^t)^T\mathbf{x}_m^t, m=1,...,M,\\ &\quad [\mathbf{G}]_{M+n,M+n}=z-2\mathbf{u}^T\mathbf{x}_n^r+(\mathbf{x}_n^r)^T\mathbf{x}_n^r, n=1,...,N,\\ &\quad \begin{bmatrix}\mathbf{I}_2 & \mathbf{u}\\ \mathbf{u}^T & z\end{bmatrix}\succeq 0,\\ &\quad \begin{bmatrix}\mathbf{G} & \mathbf{g}\\ \mathbf{g}^T & 1\end{bmatrix}\succeq 0.\end{aligned} \tag{10}$$

In the optimization literature, there are readily available solvers for finding the globally optimum SDP solution of (9), such as SEDUMI, SDPT3 and CVX [17].

B. Localization with antenna position errors

In preceding development of the SDP target localization algorithm, we have made the assumption that the antenna positions are accurate. In fact, it is difficult to obtain precise locations of the antennas due to imperfections of deployment, measurement, and position updating. Therefore, we would like to develop robust target localization algorithm in MIMO radar systems.

In the presence of antenna position errors, our antenna position observations are

$$\begin{aligned}\tilde{\mathbf{x}}_m^t&=\mathbf{x}_m^t+\varepsilon_m^t, m=1,...,M,\\ \tilde{\mathbf{x}}_n^r&=\mathbf{x}_n^r+\varepsilon_n^r, n=1,...,N,\end{aligned} \tag{11}$$

where $\{\varepsilon_m^t,\varepsilon_n^r\}$ are assumed to be i.i.d Gaussian noise with zero mean variance δ^2. Under the condition that the noises $\{\varepsilon_{m,n},\varepsilon_m^t,\varepsilon_n^r\}$ are mutually independent, the joint conditional probability density function of $\{r_{m,n},\tilde{\mathbf{x}}_m^t,\tilde{\mathbf{x}}_n^r\}$ is give as

$$\begin{aligned}p(r_{m,n},\tilde{\mathbf{x}}_m^t,\tilde{\mathbf{x}}_n^r|\,\mathbf{X})=&\prod_{m=1}^{M}\prod_{n=1}^{N}\left(2\pi\sigma^2\right)^{-1/2}\\ &\times\exp\left(-\tfrac{1}{2\sigma^2}(r_{m,n}-R_m^t-R_n^r)^2\right)\\ &\times\prod_{m=1}^{M}\left(2\pi\delta^2\right)^{-1/2}\exp\left(-\tfrac{1}{2\delta^2}(\tilde{\mathbf{x}}_m^t-\mathbf{x}_m^t)^2\right)\\ &\times\prod_{n=1}^{N}\left(2\pi\delta^2\right)^{-1/2}\exp\left(-\tfrac{1}{2\delta^2}(\tilde{\mathbf{x}}_n^r-\mathbf{x}_n^r)^2\right),\end{aligned} \tag{12}$$

where $\mathbf{X}=[\mathbf{u},\mathbf{x}_1^t,...,\mathbf{x}_M^t,\mathbf{x}_1^r,...,\mathbf{x}_N^r]$. Maximizing (12) leads to the ML solution of $\mathbf{X}$, which can be equivalently written as

$$\begin{aligned}\mathbf{X}_{ml}=\arg\max_{\mathbf{X}}&\left\{\sum_{m=1}^{M}\sum_{n=1}^{N}\tfrac{1}{\sigma^2}(r_{m,n}-R_m^t-R_n^r)^2\right.\\ &\left.+\sum_{m=1}^{M}\tfrac{1}{\delta^2}(\tilde{\mathbf{x}}_m^t-\mathbf{x}_m^t)^2+\sum_{n=1}^{N}\tfrac{1}{\delta^2}(\tilde{\mathbf{x}}_n^r-\mathbf{x}_n^r)^2\right\}.\end{aligned} \tag{13}$$

Comparing with (3), the weight factor of (13) can not be ignored since σ^2 is not necessarily equal to δ^2. In order to form a tight constraint in the later relaxation procedure, we define five variables $\mathbf{W}=\sigma^{-1}\mathbf{I}_{MN}$, $\tilde{\mathbf{r}}=\mathbf{W}\mathbf{r}$, $\tilde{\mathbf{D}}=\mathbf{W}\mathbf{D}$,

$$\tilde{\mathbf{C}}=\begin{bmatrix}\tilde{\mathbf{D}}^T\tilde{\mathbf{D}} & -\tilde{\mathbf{D}}^T\tilde{\mathbf{r}}\\ -\tilde{\mathbf{r}}^T\tilde{\mathbf{D}} & \tilde{\mathbf{r}}^T\tilde{\mathbf{r}}\end{bmatrix}$$

and $\mathbf{Y}=\mathbf{X}^T\mathbf{X}$. Thus, the optimization problem of (13) can be expressed as

$$\begin{aligned}&\min_{\mathbf{G},\mathbf{g},\mathbf{Y},\mathbf{X}}\left\{\text{trace}\left\{\tilde{\mathbf{C}}\begin{bmatrix}\mathbf{G} & \mathbf{g}\\ \mathbf{g}^T & 1\end{bmatrix}\right\}+t_1+t_2\right\}\\ &s.t.\ t_1=\tfrac{1}{\delta^2}\sum_{m=1}^{M}\left([\mathbf{Y}]_{1+m,1+m}-2(\tilde{\mathbf{x}}_m^t)^T[\mathbf{X}]_{1:2,m+1}\right)\\ &\quad t_2=\tfrac{1}{\delta^2}\sum_{n=M+1}^{M+N}\left([\mathbf{Y}]_{1+n,1+n}-2\left(\tilde{\mathbf{x}}_{n-M}^r\right)^T[\mathbf{X}]_{1:2,n+1}\right)\\ &\quad [\mathbf{G}]_{m,m}=[\mathbf{Y}]_{1,1}-2[\mathbf{Y}]_{1,1+m}+[\mathbf{Y}]_{1+M,1+m},\\ &\quad m=1,...,M,\\ &\quad [\mathbf{G}]_{n,n}=[\mathbf{Y}]_{1,1}-2[\mathbf{Y}]_{1,1+n}+[\mathbf{Y}]_{1+n,1+n},\\ &\quad n=M+1,...,M+N,\\ &\quad \mathbf{G}=\mathbf{g}\mathbf{g}^T,\\ &\quad \mathbf{Y}=\mathbf{X}^T\mathbf{X}.\end{aligned} \tag{14}$$

Similar to (10), applying the relaxation procedure and adding the penalty term, Our proposed SDP optimization problem under radar antenna position uncertainties can be formulated as

$$\begin{aligned}
&\min_{\mathbf{G},\mathbf{g},\mathbf{Y},\mathbf{X}} \left\{ \operatorname{trace}\left\{ \tilde{\mathbf{C}} \begin{bmatrix} \mathbf{G} & \mathbf{g} \\ \mathbf{g}^T & 1 \end{bmatrix} \right\} + t_1 + t_2 + \eta \operatorname{trace}(\mathbf{G}) \right\} \\
&s.t.\ t_1 = \frac{1}{\delta^2} \sum_{m=1}^{M} \left([\mathbf{Y}]_{1+m,1+m} - 2(\tilde{\mathbf{x}}_m^t)^T [\mathbf{X}]_{1:2,m+1} \right) \\
&\quad t_2 = \frac{1}{\delta^2} \sum_{n=M+1}^{M+N} \left([\mathbf{Y}]_{1+n,1+n} - 2\left(\tilde{\mathbf{x}}_{n-M}^r\right)^T [\mathbf{X}]_{1:2,n+1} \right) \\
&\quad [\mathbf{G}]_{m,m} = [\mathbf{Y}]_{1,1} - 2[\mathbf{Y}]_{1,1+m} + [\mathbf{Y}]_{1+M,1+m}, \\
&\quad m = 1, ..., M, \\
&\quad [\mathbf{G}]_{n,n} = [\mathbf{Y}]_{1,1} - 2[\mathbf{Y}]_{1,1+n} + [\mathbf{Y}]_{1+n,1+n}, \\
&\quad n = M+1, ..., M+N, \\
&\quad \begin{bmatrix} \mathbf{G} & \mathbf{g} \\ \mathbf{g}^T & 1 \end{bmatrix} \succeq 0, \\
&\quad \begin{bmatrix} \mathbf{Y} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{I}_2 \end{bmatrix} \succeq 0.
\end{aligned} \tag{15}$$

The above optimization problem can be efficiently solved by the CVX toolbox. Once we obtain the solution of $\mathbf{X}$, the target position estimate is readily given as $\hat{\mathbf{u}} = [\mathbf{X}]_{1:2,1}$.

To end this section, we would like to derive the CRLB of the considered localization problem with antenna position errors. Let $\mathbf{z}$ be a vector containing all available observations $\{r_{m,n}, \tilde{\mathbf{x}}_m^t, \tilde{\mathbf{x}}_n^r\}$, i.e.

$$\mathbf{z} = \left[r_{1,1}, ..., r_{M,N}, (\tilde{\mathbf{x}}_1^t)^T, ..., (\tilde{\mathbf{x}}_M^t)^T, (\tilde{\mathbf{x}}_1^r)^T, ..., (\tilde{\mathbf{x}}_N^r)^T \right]^T.$$

From the measurement model (2) and (11), we see that $\mathbf{z}$ is Gaussian distributed with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{V}$:

$$\mathbf{z} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{V}), \tag{16}$$

where

$$\begin{aligned}
\boldsymbol{\mu} = &\Big[R_1^t + R_1^r, ..., R_M^t + R_N^r, (\mathbf{x}_1^t)^T, ..., (\mathbf{x}_M^t)^T, \\
&(\mathbf{x}_1^r)^T, ..., (\mathbf{x}_N^r)^T \Big]^T \\
\mathbf{V} = &\operatorname{blkdiag}\left\{ \sigma^2 \mathbf{I}_{M\times N}, \mathbf{V}^t, \mathbf{V}^r \right\}, \\
\mathbf{V}^t = &\mathbf{I}_M \otimes \delta^2 \mathbf{I}_2, \mathbf{V}^r = \mathbf{I}_N \otimes \delta^2 \mathbf{I}_2.
\end{aligned}$$

The Fisher information matrix (FIM) for unknown vector $\mathbf{x} = \left[\mathbf{u}^T, (\mathbf{x}_1^t)^T, ..., (\mathbf{x}_M^t)^T, (\mathbf{x}_1^r)^T, ..., (\mathbf{x}_M^r)^T \right]^T$ can be computed as

$$[\mathbf{J}]_{i,j} = \left(\frac{\partial \boldsymbol{u}}{\partial [\mathbf{x}]_i} \right)^T \mathbf{V}^{-1} \left(\frac{\partial \boldsymbol{u}}{\partial [\mathbf{x}]_j} \right). \tag{17}$$

The calculation of $\frac{\partial \boldsymbol{u}}{\partial [\mathbf{x}]_i}, i = 1, ..., MN + M + N$ involves the equations $\frac{\partial (R_m^t + R_n^r)}{\partial x} = \frac{x - x_m^t}{R_m^t} + \frac{x - x_n^r}{R_n^t}$, $\frac{\partial (R_m^t + R_n^r)}{\partial y} = \frac{y - y_m^t}{R_m^t} + \frac{y - y_n^r}{R_n^t}$, $\frac{\partial (R_m^t + R_n^r)}{\partial x_m^t} = \frac{x_m^t - x}{R_m^t}$, $\frac{\partial (R_m^t + R_n^r)}{\partial x_n^r} = \frac{x_n^r - x}{R_n^r}$, $\frac{\partial (R_m^t + R_n^r)}{\partial y_m^t} = \frac{y_m^t - y}{R_m^t}$ and $\frac{\partial (R_m^t + R_n^r)}{\partial y_n^r} = \frac{y_n^r - y}{R_n^r}$, $m = 1, ...M, n = 1, ..., N$. Now, we can easily calculate the FIM for the unknown parameter $\mathbf{x}$ and hence the corresponding CRLB. We are interested in the estimating of the target position $\mathbf{u}$, and its CRLB is given by $\sqrt{\sum_{i=1}^{2} (\mathbf{J}^{-1})_{i,i}}$.

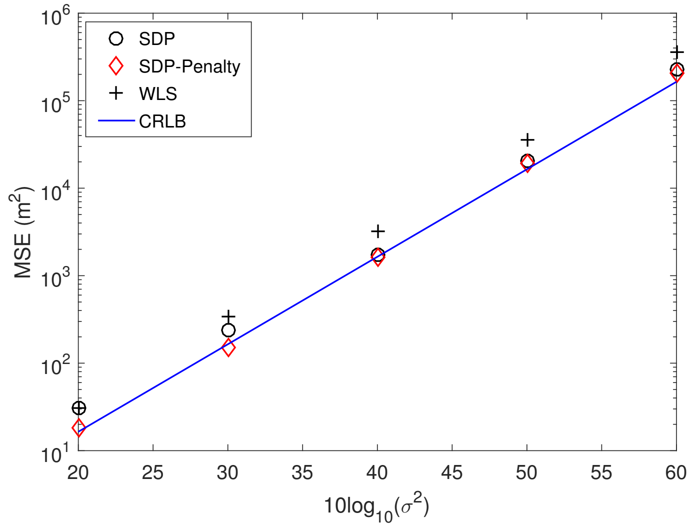

Fig. 1. MSE performance of the proposed SDP estimator with accurate antenna position.

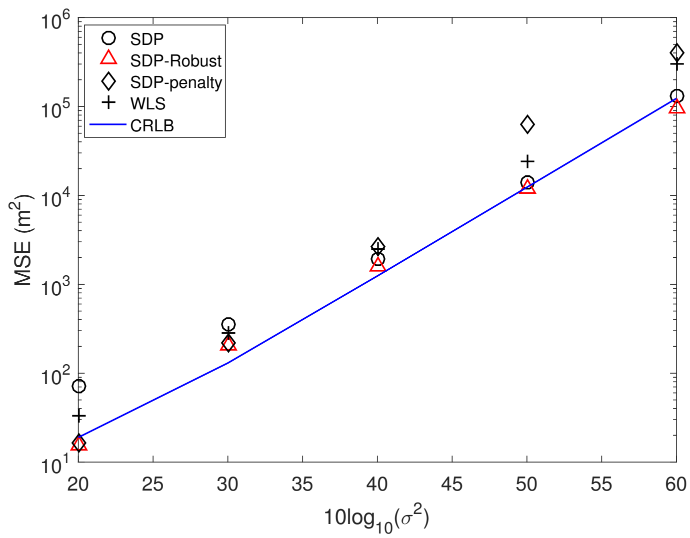

Fig. 2. MSE performance of the proposed SDP estimator with antenna position errors.

IV. Simulation Results

In this section, we conduct Monte Carlo simulations to evaluate the proposed SDP solutions. Comparison with the SDP method [14], WLS [12] and CRLB. The proposed SDP solutions based on (10) and (15), which are referred to as "SDP-penalty" and "SDP-Robust" respectively. We consider a MIMO radar system with four transmit and receive antennas, i.e. $M = N = 4$. The transmit antenna positions are $\mathbf{x}_1^t = [-1000, -1300]^T$m, $\mathbf{x}_2^t = [500, 2000]^T$m, $\mathbf{x}_3^t = [2500, 0]^T$m, $\mathbf{x}_4^t = [0, -1600]^T$m. The receive antenna positions are $\mathbf{x}_1^r = [1500, -1800]^T$m, $\mathbf{x}_2^r = [2100, 1500]^T$m, $\mathbf{x}_3^r = [-1200, 1000]^T$m, $\mathbf{x}_4^r = [1000, 1200]^T$m. The unknown-position target is located at $[0, 200]^T$m. The uncertainty of the antenna position is generated by adding the true position with an Gaussian random variable which has zero-mean and variance $\delta^2 = 12\mathrm{m}^2$. We scale $\{\varepsilon_{m,n}\}$ to produce different measurement noise conditions. All results provided are averages of 500 independent runs.

In the first simulation, the impact of measurement noise level on the mean square error (MSE) performance of the proposed SDP solution (10) is studied. We assume the antenna positions are accurate. We vary the variance of the measurement noise from $10^2\mathrm{m}^2$ to $10^6\mathrm{m}^2$. The corresponding MSEs are shown in Fig.1. It can be seen that the MSE performance of the proposed SDP solution attains the CRLB for the whole range of noise variance. The SDP solution without the penalty term can not guarantee its optimal MSE performance when the noise variance is small, that is, $\sigma^2 \leq 10^3\mathrm{m}^2$. Furthermore, the performance of the WLS solution is worse than SDP type solutions, which may be due to the ignoring of high order error terms in deducing the WLS method.

In the second simulation, we assume that the transmit/receive antenna positions are subject to uncertainties. The MSE performance results are ploted in Fig.2. It can be observed that the localization algorithms without considering the antenna position uncertainty show poor estimation accuracy under small or large noise condition. Specifically, the SDP method without penalty term shows worst MSE performance under small noise condition ($\sigma^2 \leq 10^4\mathrm{m}^2$), while the proposed SDP method of (10) shows worst MSE performance under large noise condition ($\sigma^2 \geq 10^4\mathrm{m}^2$). Thus, we conclude that our proposed robust SDP solution is significantly superior to other loclaization methods assuming perfect antenna position knowledge.

V. Conclusions

In this work, we have investigated the problem of target localization in MIMO radar system using bistatic range measurements. We first consider the localization scenario with accurate radar antenna positions. Under the assumption of Gaussian measurement noise, MLE objective function is formulated, and which turns out to be a highly nonconvex and nonlinear optimization problem. To solve this problem efficiently, we apply the SDP relaxation technique to convert the original MLE optimization problem into a convex problem. Furthermore, we propose to add a penalty term to improve the tightness of the original SDP algorithm. We then propose a robust localization method when the radar antenna positions are subject to errors. Finally, simulation results confirms the effectiveness and robustness of the proposed localization methods.

The study in this correspondence assumes Gaussian measurement and antenna position errors for ease of illustration and CRLB derivation. A possible extension of the current work is to consider different kinds of measurement or position errors, thus making the applicability of the localization algorithm more extensive.

Acknowledgment

This work was supported in part by National Natural Science Foundation of China (No. 61604176) and Major Research Development Program of Hunan Province of China (No. 2016JC2008).

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