Weighted Sum Rate Maximization for Hybrid Beamforming Design in Multi-Cell Massive MIMO OFDM Systems

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***Abstract*—In this paper, we look at hybrid beamforming (HBF) for the MIMO Interfering Broadcast Channel (IBC), i.e. the Multi-Input Multi-Output (MIMO) Multi-User (MU) Multi-Cell downlink channel, in an orthogonal frequency-division multiplexing (OFDM) system. While most of the existing works on wideband hybrid systems focus on single-user systems and a few on multi-user single-cell systems, we consider HBF design for OFDM systems in the case of multi-cell. We look at the maximization of weighted sum rate (WSR) using minorization and alternating optimization, the main advantage of which compared to the Weighted Sum Mean Squared Error (WSMSE) based methods is it's faster convergence to a local optimum and user streams selection. Through Simulation results, we show that the proposed deterministic annealing based approach for phase shifter constrained analog BF performs significantly better than state of the art Weighted Sum Mean Squared Error (WSMSE) or WSR based solutions in a wideband OFDM setting. We show that the optimal analog BF can be frequency flat and also provide an analysis of the minimum number of RF chains required to obtain fully digital performance.**

I. Introduction

In this paper, the term Tx may denote transmit/transmitter/transmission and Rx may denote receive/receiver/reception. Hybrid beamforming (HBF) [1] which is a potential candidate of massive MIMO systems involves a two stage beamforming architecture. The outer beamforming (BF) stage contains a network of phase shifters whose values are chosen such that it forms a set of narrow beams equal to the number of radio-frequency (RF) chains thus providing a beamforming gain. The inner digital BF provides spatial multiplexing gain. An extensive literature on the single user multi-input multi-output (MIMO) HBF systems exist and few interesting HBF designs can be found in [2], [3]. The authors in [2] look at near-optimal solutions and propose compressed sensing based solutions for channel estimation and HBF design in a single user mmWave system.

Recent studies on HBF designs for multi-user systems can be found in [4]–[10]. In a very recent paper [8], analog beamformer is designed using average channel statistics and switches (designed using instantaneous channel knowledge) are used to select the analog beams.

Prior work for the design of HBF design for OFDM systems can be found in [11]–[14]. In [13], for a Multi-User system, an iterative algorithm is proposed for the hybrid beamformer in which the digital beamformer is derived using the WSMSE approach which is optimal. But for the analog beamformer a suboptimal method is introduced based on certain approximations for the weighted sum rate (WSR). The analog precoder design is based only on a quadratic sum of the channel matrices across all the subcarriers. In [11], again for a MU-MISO system, the hybrid beamforming design is based on the QR decomposition of the all-digital beamformer. The authors also derive the number of RF chains and phase shifting components required to realize a hybrid beamformer which is equivalent to an all-digital beamformer. In [15], a multi-beam transmission diversity scheme is proposed for an OFDM system. The analog beamformer is chosen on the basis of the beam steering angle which maximizes the sum rate.

The unit modulus constraint on the analog BF coefficients makes the cost function for an HBF design highly non-convex. This results in the convergence of BF solutions to a local optimum [10] depending on the initialization and convergence to the global optimum cannot be guaranteed.

A. Contributions of this paper

- Using alternating minorization approach to optimize the WSR, we propose an HBF design with deterministic annealing (DA) to optimize the analog phasors. Compared to the WSMSE based solution [10], the proposed iterative algorithm converges faster (no ping-pong between Tx and Rx optimization) and involves direct power optimization. Compared to our previous work [16] which was for a narrowband system, we consider a wideband OFDM system in this paper and furthermore provide a convergence proof for the minorization algorithm.
- To achieve optimal fully digital performance, we show that the number of RF chains can be as small as the total number of multi-paths across all the users (thanks to the sparsity of the mmWave channels) and it doesn't depend on the number of antennas or the sub-carriers. Using this result, we explain why a frequency flat precoding for analog BF may be sufficient to guarantee optimal performance.
- Monte-Carlo simulations (provided for multi-cell systems also) suggest that the proposed DA based wideband HBF design performs very close to the optimal fully digital solutions [17], [18] and far better than the existing WSMSE based alternating optimization of phasors.

Notations: Throughout the paper, boldface lower-case and upper-case characters denote vectors and matrices, respec-

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tively. The operators $\mathrm{E}(\cdot)$, $\mathrm{tr}\{\cdot\}$, $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^*$ represent expectation, trace, conjugate transpose, transpose and complex conjugate, respectively. $\mathbf{V}_{max}(\mathbf{A},\mathbf{B})$ or $\mathbf{V}_{1:d_k}(\mathbf{A},\mathbf{B})$ represents (normalized) dominant generalized eigenvector or the matrix formed by the (normalized) d_k dominant generalized eigenvectors of $\mathbf{A}$ and $\mathbf{B}$. $\mathbf{\Sigma}_{1:d_k}(\mathbf{A},\mathbf{B})$ represent the d_k generalized eigen values matrix which is diagonal. $\mathbf{x} = vec(\mathbf{X})$ represents the vectorization of a matrix obtained by stacking each of the columns of $\mathbf{X}$ and $unvec(\mathbf{x})$ represents the inverse operation of $vec(.)$. $\mathbf{I}_N$ or $\mathbf{I}$ represents the identity matrix of size N or with appropriate dimensions. $\mathrm{diag}\,(\mathbf{x})$ represents the diagonal matrix obtained by the vector $\mathbf{x}$ as its entries.

II. Multi-User Mulit-Cell MIMO System Model

We consider a multi-cell MU downlink (i.e. Interfering BroadCast Channel (IBC)) OFDM system of C cells with a total of K users. We constrain the total transmit power to be P_c at BS c and N_t^c transmit antennas in cell c. N_s represents the total number of subcarriers which is shared across all the users. User k is equipped with N_k antennas. The number of streams intended for user k is d_k. Let $\mathbf{H}_{k,c}[n]$ represents the $N_k \times N_t^c$ MIMO downlink channel between user k and BS c and we define the channel covariance to be $\mathrm{E}(\mathbf{H}_{k,c}^H[n]\mathbf{H}_{k,c}[n]) = \mathbf{\Theta}_k^c[n]$. n represents the subcarrier index throughout the paper. It is important to emphasize here that the analog precoder is assumed to be frequency flat (same for all subcarriers) and digital precoder to be frequency selective. Note that we consider the Rx to be a fully digital system since N_k is not very high at the UE. User k receives

$$\mathbf{y}_k[n]=\mathbf{H}_{k,b_k}[n]\mathbf{V}^{b_k}\,\mathbf{G}_k[n]\,\mathbf{s}_k[n]+ \sum_{i\neq k}\mathbf{H}_{k,b_i}[n]\mathbf{V}^{b_i}\,\mathbf{G}_i[n]\,\mathbf{s}_i[n]+\mathbf{v}_k[n], \tag{1}$$

where $\mathbf{s}_k[n]$, of size $d_k\times 1$, is the transmit symbol vector with $\mathbf{s}_k[n]\sim\mathcal{CN}(\mathbf{0},\mathbf{I})$. b_i refers to the serving base station of user i. BS c serves U_c users and $K=\sum_{c=1}^{C}U_c$. Assuming that $\mathbf{y}_k[n]$ represent a noise whitened signal model, we get for the noise $\mathbf{v}_k\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_{N_k})$ (circularly complex Gaussian random vector). The analog BF which is same across all the subcarriers, $\mathbf{V}^c$ for base station c is of dimension $N_t^c\times M^c$ where M^c is the number of RF chains at BS c. The $M^c\times d_k$ digital beamformer is $\mathbf{G}_k[n]$, where $\mathbf{G}_k[n] = \left[\mathbf{g}_k^{(1)}[n]\ ...\ \mathbf{g}_k^{(d_k)}[n]\right]$ and $\mathbf{g}_k^{(s)}[n]$ represents the beamformer for stream s of user k.

A. MIMO OFDM Channel Model

In this sub-section, we omit the user and cell indices for simplicity. We consider a geometric channel model for a mmWave propagation environment [19] with L_s scattering clusters and L_r scatterers or rays in each cluster. In a more compact form, we can represent the channel matrix at a subcarrier n as,

$$\mathbf{H}[n]=\mathbf{H}_r\sum_{d=1}^{D}\mathbf{A}_d[n]\mathbf{H}_t^H,\ \text{ where} \tag{2}$$

where $\mathbf{H}_r = [\mathbf{h}_r(\theta_{1,1}),...,\mathbf{h}_r(\theta_{L_s,L_r})]$, $\mathbf{H}_t = [\mathbf{h}_t(\phi_{1,1}),...,\mathbf{h}_t(\phi_{L_s,L_r})]$, $\mathbf{A}_d[n] = \mathrm{diag}\,(\alpha_{1,1}p(dTs-\tau_1-\tau_{r1}),...,\alpha_{L_s,L_r}p(dTs-\tau_{L_s}-\tau_{rL_r}))e^{-j2\pi\frac{nd}{N_s}}$. Here $\phi_{s,l},\theta_{s,l}$ represent the angle of departure (AoD) and angle of arrival (AoA), respectively for the l^{th} path in the s^{th} cluster. $\mathbf{h}_r(\cdot),\mathbf{h}_t(\cdot)$ represent the antenna array responses at Rx and Tx respectively. The complex path gain which is an indicator of the channel power in each path is modeled as, $\alpha_{s,l}\sim\mathcal{CN}(0,\frac{N_tN_r}{L_sL_r})$ and $p(\tau)$ is the band-limited pulse shaping filter response evaluated at τ seconds. Each cluster has a time delay $\tau_s\in\mathcal{R}$ and each ray has a relative time delay τ_{rl}. Note that our HBF design which follows, is applicable for general MIMO channel models and the channel model outlined here is utilized for the simulations in Section VI. Another remark here is that, even though for an HBF system, at the baseband we have access to only the low-dimensional effective channel resulting from the combination of propagation channel and the analog precoder, it is still possible to estimate the individual components in a pathwise channel model as we consider here, for e.g. [3], [20].

III. WSR Maximization via Minorization and Alternating Optimization

For the convenience of analysis, we define the Tx covariance matrix as $\mathbf{Q}_i[n] = \mathbf{V}^{b_i}\mathbf{G}_i[n]\mathbf{G}_i[n]^H\mathbf{V}^{b_i\,H}$. HBF design using WSR maximization of the Multi-cell MU-MIMO OFDM system can be formulated as follows,

$$\begin{aligned}[\mathbf{V}\ \ \mathbf{G}] &= \arg\max_{\mathbf{V},\mathbf{G}} WSR(\mathbf{G},\mathbf{V})\\ &= \arg\max_{\mathbf{V},\mathbf{G}}\sum_{k=1}^{K}u_k\sum_{n=1}^{N_s}\ln\det(\mathbf{R}_{\overline{k}}[n]^{-1}\mathbf{R}_k[n]),\\ \text{s.t.}\ &\sum_{k:b_k=c}\sum_{n=1}^{N_s}\mathrm{tr}\,\{\mathbf{Q}_k[n]\}\ \le\ P_c\,.\end{aligned} \tag{3}$$

where the u_k being the weight for user k (can represent the priority), $\mathbf{G}$ represents the collection of digital BFs $\mathbf{G}_k[n]$ and $\mathbf{V}$ the collection of analog BFs $\mathbf{V}^{b_k}$. From [17], [18], we can write,

$$\begin{aligned}\mathbf{R}_{\overline{k}}[n] &= \sum_{i=1,i\neq k}^{K}\mathbf{H}_{k,b_i}[n]\mathbf{Q}_i[n]\mathbf{H}_{k,b_i}^H[n]\ +\ \mathbf{I}_{N_k},\\ \mathbf{R}_k[n] &= \sum_{i=1}^{K}\mathbf{H}_{k,b_i}[n]\mathbf{Q}_i[n]\mathbf{H}_{k,b_i}^H[n]\ +\ \mathbf{I}_{N_k},\end{aligned} \tag{4}$$

where $\mathbf{R}_{\overline{k}}[n]$ is the interference plus noise covariance matrix.The WSR problem considered above (3) is non-concave in the $\mathbf{Q}_k[n]$ due to the interference terms. Therefore finding the global optimum is challenging. In order to compute a feasible solution for this non-convex problem, we consider the difference of convex functions (DC programming) approach as in [21] in which the WSR is written as the summation of a convex and a concave term. Consider the dependence of the WSR on $\mathbf{Q}_k[n]$ alone:

$$\begin{aligned}&WSR(\mathbf{G},\mathbf{V})\ =\ u_k\ln\det(\mathbf{R}_{\overline{k}}[n]^{-1}\mathbf{R}_k[n])\ +\ WSR_{\overline{k}}[n]+\\ &\sum_{s=1,s\neq n}^{N_s}\sum_{k=1}^{K}u_k\ln\det(\mathbf{R}_{\overline{k}}[s]^{-1}\mathbf{R}_k[s]),\\ &WSR_{\overline{k}}[n]\ =\sum_{i=1,\neq k}^{K}u_i\ln\det(\mathbf{R}_{\overline{i}}[n]^{-1}\mathbf{R}_i[n]),\end{aligned} \tag{5}$$

where $\ln\det(\mathbf{R}_{\overline{k}}[n]^{-1}\mathbf{R}_k[n])$ is concave in $\mathbf{Q}_k[n]$, $WSR_{\overline{k}}[n]$ is convex in $\mathbf{Q}_k[n]$ and the third summation across subcarriers other than n is independent of $\mathbf{Q}_k[n]$. To get a feasible solution, we consider the first order Taylor series expansion of $WSR_{\overline{k}}[n]$ in $\mathbf{Q}_k[n]$ around $\widehat{\mathbf{Q}}[n]$ (i.e. all $\widehat{\mathbf{Q}}_i[n]$, $\widehat{\mathbf{R}}_{\overline{i}}[n], \widehat{\mathbf{R}}_i[n]$ corresponds to $\widehat{\mathbf{Q}}_i[n]$). This approximation makes the convex part linear in $\mathbf{Q}_k[n]$ and since a linear function is simultaneously convex and concave, we can make use of conventional convex optimization techniques further.

$$\begin{aligned}&\underline{WSR_{\overline{k}}}[n](\mathbf{Q}_k[n],\widehat{\mathbf{Q}}[n]) \approx WSR_{\overline{k}}[n](\widehat{\mathbf{Q}}_k[n],\widehat{\mathbf{Q}}[n])-\\ &\text{tr}\left\{(\mathbf{Q}_k[n]-\widehat{\mathbf{Q}}_k[n])\widehat{\mathbf{A}}_k[n]\right\},\\ &\widehat{\mathbf{A}}_k[n] = -\left.\frac{\partial WSR_{\overline{k}}[n]\big(\mathbf{Q}_k[n],\widehat{\mathbf{Q}}[n]\big)}{\partial \mathbf{Q}_k[n]}\right|_{\widehat{\mathbf{Q}}_k[n],\widehat{\mathbf{Q}}[n]}\\ &= \sum_{i=1,\neq k}^{K} u_i\, \mathbf{H}_{i,b_k}^H[n]\left(\widehat{\mathbf{R}}_{\overline{i}}[n]^{-1}-\widehat{\mathbf{R}}_i[n]^{-1}\right)\mathbf{H}_{i,b_k}[n]\,. \end{aligned} \tag{6}$$

Note that the linearized tangent expression $\underline{WSR_{\overline{k}}}[n]$ constitutes a (touching) lower bound for $WSR_{\overline{k}}[n]$ via $-\text{tr}\{\mathbf{R}^{-1}[n]\boldsymbol{\Delta}\} \leq -\ln\det(\mathbf{R}^{-1}[n](\mathbf{R}[n]+\boldsymbol{\Delta}))$ and $\mathbf{R}_k[n] \geq \mathbf{R}_{\overline{k}}[n]$. Hence the DC approach is also a minorization approach [22], regardless of the (re)parameterization of $\mathbf{Q}$. Now, dropping constant terms, reparameterizing the $\mathbf{Q}_k[n] = \mathbf{V}^{b_k}\mathbf{G}_k[n]\mathbf{G}_k[n]^H\mathbf{V}^{b_k\,H}$, performing this linearization for all users and across all subcarriers, we get the Lagrangian after including the Tx power constraints,

$$\begin{aligned}&\mathcal{L}(\mathbf{G},\mathbf{V},\boldsymbol{\Lambda}) =\\ &\sum_{k=1}^{K}\sum_{n=1}^{N_s}[u_k \ln\det(\mathbf{I}+\mathbf{G}_k^H[n]\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k[n])\\ &-\text{tr}\{\mathbf{G}_k^H[n]\mathbf{V}^{b_k\,H}(\widehat{\mathbf{A}}_k[n]+\lambda_{b_k}\mathbf{I})\mathbf{V}^{b_k}\mathbf{G}_k[n]\}]+\sum_{j=1}^{C}\lambda_j P_j,\end{aligned} \tag{7}$$

where $\widehat{\mathbf{B}}_k[n] = \mathbf{H}_{k,b_k}^H[n]\mathbf{R}_{\overline{k}}^{-1}[n]\mathbf{H}_{k,b_k}[n]$. $\boldsymbol{\Lambda}$ represents the set of Lagrange multipliers λ_c. It can be verified that the summation across all the subcarriers after the DC approximation is still a minorizer of the original WSR using the same argument as before for a single subcarrier. Further, we shall optimize this minorizer function under perfect CSIT by alternating optimization between digital and analog BFs.

A. Digital BF Design

By Hadamard's inequality [23, p. 233], it can be seen that for the maximization problem above, $\mathbf{G}_k^H[n]\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k[n]$ should be diagonal and thus maximizing w.r.t $\mathbf{G}_k[n]$ leads to the following dominant generalized eigen vector solution. Also note that the gradient w.r.t. $\mathbf{G}_k[n]$ of (7) is still the same as that of (3).

$$\mathbf{G}_k^{'}[n]=\mathbf{V}_{1:d_k}(\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k},\mathbf{V}^{b_k\,H}(\widehat{\mathbf{A}}_k[n]+\lambda_{b_k}\mathbf{I})\mathbf{V}^{b_k}), \tag{8}$$

with associated generalized eigenvalues $\boldsymbol{\Sigma}_k[n] = \boldsymbol{\Sigma}_{1:d_k}(\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k},\mathbf{V}^{b_k\,H}(\widehat{\mathbf{A}}_k[n] + \lambda_{b_k}\mathbf{I})\mathbf{V}^{b_k})$. Let $\boldsymbol{\Sigma}_k^{(1)}[n] = \mathbf{G}_k^{'\,H}[n]\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k^{'}[n]$ and $\boldsymbol{\Sigma}_k^{(2)}[n] = \mathbf{G}_k^{'\,H}[n]\mathbf{V}^{b_k\,H}\widehat{\mathbf{A}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k^{'}[n]$. Intuitively, (8) represents a compromise between increasing the signal part and reducing the interference. Now we introduce stream powers in the diagonal matrices $\mathbf{P}_k[n] \geq 0$. The Lagrangian formulation (7) allows us to optimize the stream powers. Further substituting $\mathbf{G}_k[n] = \mathbf{G}_k^{'}[n]\,\mathbf{P}_k^{\frac{1}{2}}[n]$ in (7) yields the following interference leakage aware water filling (WF) (jointly for the $\mathbf{P}_k[n]$ and λ_c)

$$\mathbf{P}_k[n] = (u_k(\boldsymbol{\Sigma}_k^{(2)}[n]+\lambda_{b_k}\mathbf{V}^{b_k\,H}\mathbf{V}^{b_k})^{-1}-\boldsymbol{\Sigma}_k^{-(1)}[n])^{+}, \tag{9}$$

where $(\mathbf{X})^{+}$ denotes the positive semi-definite part of Hermitian $\mathbf{X}$ (so by removing the terms with negative eigen values to zero) and the Lagrange multipliers (per BS) are computed using bisection to satisfy the power constraints.

B. Design of Unconstrained Analog BF

At first we investigate the case in which the analog BF is unconstrained. One remark here is that the resulting HBF design is also applicable to general two-stage BF design [24], where the higher dimensional outer BF stage ($\mathbf{V}^c$) is common to all users in a cell. To optimize $\mathbf{V}^c$, we equate the gradient of (7) w.r.t. $\mathbf{V}^c$ to zero. Using the result $\partial \ln\det\mathbf{X} = \text{tr}(\mathbf{X}^{-1}\partial\mathbf{X})$ and $\det(\mathbf{I}_M+\mathbf{AB}) = \det(\mathbf{I}_N+\mathbf{BA})$ from [25], we get

$$\begin{aligned}&\sum_{k:b_k=c}\sum_{n=1}^{N_s}(\widehat{\mathbf{B}}_k[n]\mathbf{V}^c\mathbf{G}_k[n]\boldsymbol{\zeta}_k[n]\mathbf{G}_k^H[n]-\\ &(\widehat{\mathbf{A}}_k[n]+\lambda_c\mathbf{I})\mathbf{V}^c\mathbf{G}_k[n]\mathbf{G}_k^H[n]) = 0,\\ &\text{with } \boldsymbol{\zeta}_k[n] = u_k(\mathbf{I}+\mathbf{G}_k^H[n]\mathbf{V}^{b_k H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k[n])^{-1}\end{aligned} \tag{10}$$

Now with $vec(\mathbf{AXB}) = (\mathbf{B}^T\otimes\mathbf{A})vec(\mathbf{X})$ [25], where $\otimes$ represents the Kronecker product between the two matrices, we get

$$\begin{aligned}&\mathbf{V}^c = unvec(\mathbf{V}_{max}(\mathbf{B}_c[n],\mathbf{A}_c[n])),\ \text{with}\\ &\mathbf{B}_c[n] = \sum_{k:b_k=c}\sum_{n=1}^{N_s}(\mathbf{G}_k[n]\boldsymbol{\zeta}_k[n]\mathbf{G}_k^H[n])^T\otimes\widehat{\mathbf{B}}_k[n],\\ &\mathbf{A}_c[n] = \sum_{k:b_k=c}\sum_{n=1}^{N_s}(\mathbf{G}_k[n]\mathbf{G}_k^H[n])^T\otimes(\widehat{\mathbf{A}}_k[n]+\lambda_c\mathbf{I})\,.\end{aligned} \tag{11}$$

We emphasize here that the extension to the partially connected HBF architecture is quite straightforward and we include the comparison of both in section VI.

C. Algorithm Convergence

The convergence proof follows in the same direction as in [26]. For the WSR cost function for a wideband system, we construct the minorizer as in (6), (7) leading to

$$\begin{aligned}&WSR(\mathbf{Q}) \geq \underline{WSR}(\mathbf{Q},\widehat{\mathbf{Q}}) = \sum_{k=1}^{K}\sum_{n=1}^{N_s}[u_k\\ &\ln\det(\mathbf{I}+\widehat{\mathbf{B}}_k[n]\mathbf{Q}_k[n]) - \text{tr}\{\widehat{\mathbf{A}}_k[n](\mathbf{Q}_k[n]-\widehat{\mathbf{Q}}_k[n])\}],\end{aligned} \tag{12}$$

where $\underline{WSR}(\widehat{\mathbf{Q}},\widehat{\mathbf{Q}}) = WSR(\widehat{\mathbf{Q}})$. The resulting minorizer above is a concave function in $\widehat{\mathbf{Q}}$ and has the same gradient as $WSR(\widehat{\mathbf{Q}})$. Hence the KKT conditions are unaffected. Now reparameterizing $\mathbf{Q}$ in terms of $\mathbf{P}, \mathbf{G}', \mathbf{V}$ as in (4) and adding the power constraints to the minorizer, we get the Lagrangian (7). Every alternating update of $\mathcal{L}$ w.r.t. $\mathbf{V}$, $\mathbf{G}'$, or $(\mathbf{P},\boldsymbol{\Lambda})$ increases the WSR since the approximate problem is a concave function, which ensures convergence within each of these 3 parameter goups and we further alternate between each user or BS. Also, at the convergence point, the gradients of $\mathcal{L}$ w.r.t. $\mathbf{V}$ or $\mathbf{G}'$ correspond to the gradients of the Lagrangian of the original WSR and hence the KKT conditions remain unaffected. For fixed $\mathbf{V}$ and $\mathbf{G}'$, $\mathcal{L}$ is concave in $\mathbf{P}$, hence strong duality is satisfied for the saddle point $\max_{\mathbf{P}}\min_{\boldsymbol{\Lambda}}\mathcal{L}$. Also, at the convergence point the solution to $\min_{\mathbf{\Lambda}} \mathcal{L}(\mathbf{V}^o, \mathbf{G}'^o, \mathbf{P}^o, \mathbf{\Lambda})$ satisfies the gradient KKT condition for $\mathbf{P}$ and the complementary slackness conditions for $c = 1, \ldots, C$

$$\lambda_c^o \, (P^c - \sum_{k:b_k=c} \sum_{n=1}^{N_s} \text{tr}\{\mathbf{V}^{co}\mathbf{G}_k'^o[n]\mathbf{P}_k^o[n]\mathbf{G}_k'^{o\,H}[n]\mathbf{V}^{co\,H}\} = 0, \tag{13}$$

where all individual factors in the products are nonnegative. In the proposed approach, $g(\mathbf{\Lambda}|\mathbf{V}, \mathbf{G}') = \max_{\mathbf{P}} \mathcal{L}(\mathbf{V}, \mathbf{G}', \mathbf{P}, \mathbf{\Lambda})$.

IV. Analysis on the number of RF Chains and HBF Performance

In this section, we derive an analytical solution for the analog phasors to achieve a fully digital BF performance. In short, we prove that it is possible to achieve using sufficient number of RF chains under certain conditions on the massive MIMO channel being considered. For notational simplicity we shall consider a uniform $L = L_s L_r$ and $N_k = N_r, \forall k, N_t^c = N_t, M^c = M, \forall c$. Let us represent the concatenated antenna array response matrix of all user channel from BS c as, $\overline{\mathbf{H}}_t^c = \left[\mathbf{H}_{t,1}^c \; \mathbf{H}_{t,2}^c \; ... \; \mathbf{H}_{t,K}^c\right]$, of dimension $N_t \times N_p$, where we denote the total number of paths $N_p = LK$. We define $\mathbf{A}_{d,k}^c[n]$ as the diagonal path amplitude matrix for the channel from BS c to user k for subcarrier n. Similarly we define $\overline{\mathbf{H}}_r^c$ and $\overline{\mathbf{A}}^c[n] = \text{diag}\,(\sum_{d=1}^{D} \mathbf{A}_{d,1}^c[n], ..., \sum_{d=1}^{D} \mathbf{A}_{d,K}^c[n])$ of size $N_p \times N_p$ for the concatenated Rx antenna array responses and complex path amplitudes. $\overline{\mathbf{H}}_r^c$ is a $KN_r \times N_p$ block diagonal matrix with blocks of size $N_r \times L$. Finally, we can write the $KN_r \times N_t$ MIMO channel from BS c to all a users as $\mathbf{H}^{c\,H}[n] = \overline{\mathbf{H}}_t^c \, \overline{\mathbf{A}}^{c\,H}[n] \, \overline{\mathbf{H}}_r^{c\,H}$.

Theorem 1. *Consider a multi-cell MU MIMO OFDM system with the number of RF chains being less than the total number of paths across all user channels from any BS and assume phasor antenna responses. In order to achieve optimal all-digital precoding performance, an analytical solution for the analog beamformer can be obtained as the Tx side concatenated antenna array response and thus frequency flat assuming no beam squint effect.*

Proof: From [18], the optimal all-digital beamformer for any subcarrier n is of the form

$$\begin{aligned}&(\mathbf{H}^{c\,H}[n]\mathbf{D}_1^c[n]\mathbf{H}^c[n] + \lambda_c \mathbf{I})^{-1}\mathbf{H}^{c\,H}[n]\mathbf{D}_2^c[n] \\ &= \mathbf{H}^{c\,H}\mathbf{B}^c = \overline{\mathbf{H}}_t^c \; \overline{\mathbf{A}}^{c\,H}[n] \, \overline{\mathbf{H}}_r^{c\,H} \, \mathbf{B}^c[n],\end{aligned} \tag{14}$$

where $\mathbf{B}^c[n] = \left(\lambda_c \mathbf{I} + \mathbf{D}_1^c[n]\mathbf{H}^c[n]\mathbf{H}^{c\,H}[n]\right)^{-1} \mathbf{D}_2^c[n]$, $\mathbf{D}_1^c[n]$, $\mathbf{D}_2^c[n]$ are block diagonal matrices and we used the identity $(\mathbf{I} + \mathbf{XY})^{-1}\mathbf{X} = \mathbf{X}(\mathbf{I} + \mathbf{YX})^{-1}$. Under the Theorem assumptions we can then separate the BFs as

$$\mathbf{V}^c = \overline{\mathbf{H}}_t^c, \;\; \mathbf{G}^c[n] = \overline{\mathbf{A}}^{c\,H}[n]\overline{\mathbf{H}}_r^{c\,H}\mathbf{B}^c[n]\,. \tag{15}$$

Hence $\mathbf{V}$ is a function of only the frequency flat antenna array responses which are slow fading components. So it is independent of the subcarrier number and this explains why it is optimal to consider a frequency flat design for analog BF. However, note that the digital BF $\mathbf{G}$ in (15) is a function of the instantaneous CSIT and needs to be updated every channel use in the time and frequency domain. Also, while the spatial angles in antenna array responses may include a frequency dependency called as beam-squint in the literature [27], we don't consider this factor at the moment.

For the case when $M < N_p$, we utilize the DA based approach proposed earlier in our own work [16]. We refer the reader for a more detailed discussion on this to our paper. In the below table Algorithm 1, we describe in detail the HBF algorithm which combines minorization and DA.

Algorithm 1 Minorization and DA based HBF design

Given: $P_c, \mathbf{H}_{k,c}[n], u_k \, \forall k, c, n$, b is a constant < 1, say 0.9.
Initialization: $\mathbf{V}^c = e^{j\angle \mathbf{V}_{1:M^c}(\sum_{k:b_k=c} \mathbf{\Theta}_k^c[n], \sum_{i:b_i \neq c} \mathbf{\Theta}_i^c[n])}$,
The $\mathbf{G}_k^{(0)}[n]$ are initialized to be ZF precoders for the effective channels $\mathbf{H}_{k,b_k}[n]\mathbf{V}^{b_k}$, with uniform power distribution across the streams. **Iteration** (j) :

1) Compute $\widehat{\mathbf{B}}_k[n], \widehat{\mathbf{A}}_k[n], \, \forall k, n$ from (6), (7).
2) Update $\mathbf{G}_k^{'(j)}[n]$ from (8), and $\mathbf{P}_k[n]$ from (9), $\forall k, n$.
3) Update $(\mathbf{V}_{p,q}^c)^{(j)}$, $\forall c, \forall (p, q)$, using DA (phasor constrained) or from (11) (unconstrained).
4) If the algorithm is converged, exit the loop, otherwise go to step 1).
5) Scale $\forall \, (i, j) : |\mathbf{V}_{i,j}^c| \leftarrow e^{b \ln |\mathbf{V}_{i,j}^c|}$ $(\mathbf{V}_{i,j}^c = |\mathbf{V}_{i,j}^c| e^{j\theta_{i,j}^c})$.
6) Reoptimize all $\theta_{i,j}^c$ and all digital BFs using 1)-4).
7) Update stream powers and Lagrange multipliers.
8) Go to 5) for a number of iterations.
9) Finally redo 6)-7) a last time with all $|\mathbf{V}_{i,j}^c| = 1$ in 5).

V. Simulation Results

In this section, we validate the performance of the proposed HBF algorithms for a single cell and multi-cell system (Figure 2) with K single antenna users and for an OFDM system with $N_s = 32$ subcarriers using extensive Monte-Carlo simulations. We use the pathwise channel model in (2). We consider a Uniform Linear Array (ULA) of antennas with $\mathbf{h}_{t,k}\,(\phi_{c,l})$, the AoD $\phi_{c,l}$ are restricted to the interval $[0^o, 30^o]$ and uniformly distributed. For the multi-cell case in Figure 2, the parameters used are the same for both the cells, i.e. $M^1 = M^2 = M, N_t^1 = N_t^2 = N_t, U_1 = U_2 = K/2, L_s = 1, L_r = 4, L = L_s L_r$. Our system dimension are such that the number of RF chains satisfy the condition, $M < LK$ such that alternating optimization of phasors result in local optima issues. Notations used in the figure: "ABF" refers to the analog BF. "EV phasors" refers to the HBF design with analog phasors being chosen as the projection of eigen vectors of the sum of the user channel covariance matrices onto the unit modulus constraints. We compare the performance of the proposed algorithms with the optimal fully digital BF [18] (referred to as "Optimal Fully Digital"), approximate WSR based hybrid design [13] (referred to as "HBF with ABF based on Channel Average". For the multi-cell version of [13], channel average with only the direct user channels in a cell are considered. "HBF with Alternating Optimization of Phasors" refers to our own algorithm in the paper [10], but extended to an OFDM system.

It is evident from Figure 1 and 2 that our proposed unconstrained HBF has almost the same performance as that of the optimal fully digital BF. With phase shifter constrained analog precoder, the proposed DA based design narrows the

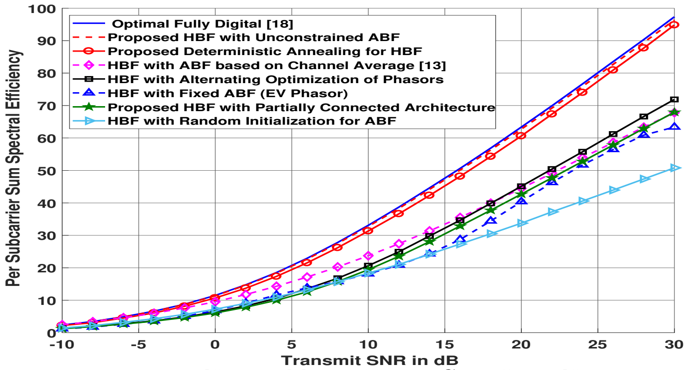

Fig. 1. Sum rate, $N_t = 32, M = 16, K = 16, C = 1, L = 4, N_s = 32$.

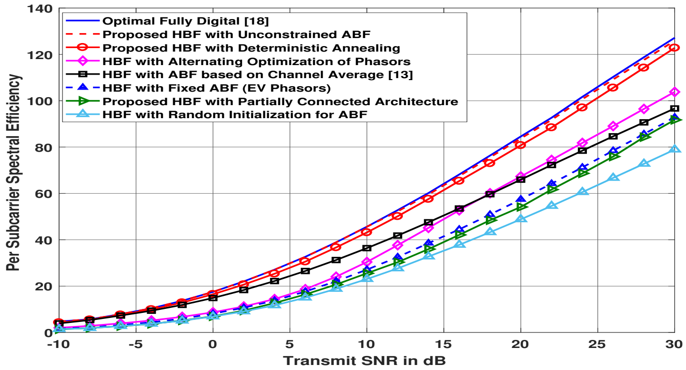

Fig. 2. Sum rate, $N_t = 64, M = 16, K = 16, C = 2, L = 4, N_s = 32$.

gap to the fully digital performance and performs much better than state of the art solutions such as WSMSE which suffer from the issue of local optima for analog phasors. Also, it is evident that the performance degrades for a partially connected architecture compared to the fully connected system. However, it is to be noted that the complexity of the proposed HBF design is slightly on the higher side and it is $O(N_t^3 N_{it})$, where N_{it} represent the number of iterations required for Algorithm 1 to converge.

VI. Conclusion

In this paper, we derived and presented an optimal BF algorithm for the HBF scenario in a Multi-cell MU-MIMO OFDM system. We optimized the WSR objective function using a difference of convex functions approach (which is also an instance of minorization) and the BF solutions are alternatively computed till convergence. Convergence to a local optimum is shown and through extensive simulations we show that our DA based approach for analog BF design performs far better than the existing state of the art solutions based on WSMSE or other suboptimal objective functions.

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