Beam-Steering in Switched 4D Arrays Based on the Discrete Walsh Transform

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***Abstract*—4D arrays provide cost-effective beam steering capabilities considering radio-frequency switches (controlled by periodic sequences) instead of variable phase shifters. Their synthesis is based on the Fourier coefficients (which depend on configurable time parameters) of the corresponding periodic modulating sequences. Whereas Fourier series use trigonometric functions to synthesize any periodic continuous waveform, Walsh series relies on bipolar orthogonal sequences and, if such a series is truncated, the expanded function is approximated by a stair-step signal. In this paper we present a novel method to synthesize 4D arrays by means of a finite set of Walsh functions. The technique allows for implementing the analog time-modulated feeding network of an array employing single-pole double-throw switches and achieves excellent rejection levels of the undesired harmonics.**

***Index Terms*—4D arrays, time-modulated arrays, beam steering, Walsh functions.**

I. Introduction

Among 4D antenna arrays, time-modulated arrays (TMAs) employ in their feeding network radio frequency (RF) switches governed by periodical sequences instead of conventional variable phase shifters (VPSs) [1]. TMAs constitute an attractive alternative for the design of smart antenna solutions, mainly due to simplicity and cost-effectiveness reasons. The TMA design is subject to a set of handicaps related to efficiency issues that are not present in conventional arrays. More specifically, the array designer must take into consideration critical aspects such as the level of the unexploited harmonics [2], [3], the presence of mirrored frequency diagrams (negative harmonics) [4], [5] or the transmitted (and received) signal energy wasted during the zero state of the switches [6], [7].

Some strategies to overcome the previous drawbacks focus on the generation of a single pattern diagram over the first positive harmonic frequency while minimizing the level of the remaining harmonics and avoiding mirrored diagrams. The idea behind such strategies is to approximate the TMA modulating waveforms by time-delayed sine functions. This can be done either by using single-pole multiple-throw (SPMT) switches and one-bit VPSs, by generating tristate (-1, 0, 1) sequences [5], [8] (thus not solving the problem of the loss of energy during the zero state), or by employing single-pole dual-throw (SPDT) switches to generate bipolar (-1, 1) sequences [9].

This latter solution exhibits an attractive efficiency because the switches are never in zero state. Inspired by this idea, we focus the contents of this paper on the synthesis of the

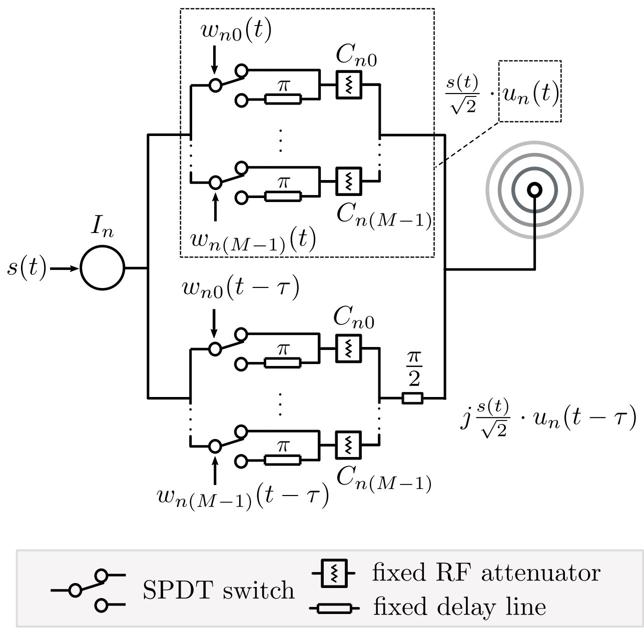

Fig. 1. Generalized architecture of the n-th element of the feeding network of a TMA designed with Walsh functions.

approximated time-delayed sine waveforms which modulate the individual elements of a TMA by means of bipolar sequences, more specifically, by means of Walsh functions [10], [11], which constitute a complete set of orthogonal functions. Therefore, any given function that fulfills certain requirements (see Section II-B) can be expressed as a linear combination of them. The coefficients of this linear combination are obtained solving integrals similar to those of the Fourier coefficients but using the corresponding Walsh function instead of sines or cosines. Furthermore, analogously to the discrete Fourier Transform (DFT), we can use the discrete Walsh Transform (DWT) to obtain, in a more agile way, the Walsh coefficients. Indeed, since Walsh functions may be written in matrix notation using a Hadamard matrix, given a vector with samples of the sine waveform to be approximated, the calculation of the Walsh coefficients is reduced to products of matrices. Accordingly, the degree of fidelity of a given approximated sine waveform depends on the number of functions considered in the Walsh expansion and raises a trade-off between the efficiency of the beam steering and the complexity of the antenna feeding network.

An additional advantage of using Walsh functions in the

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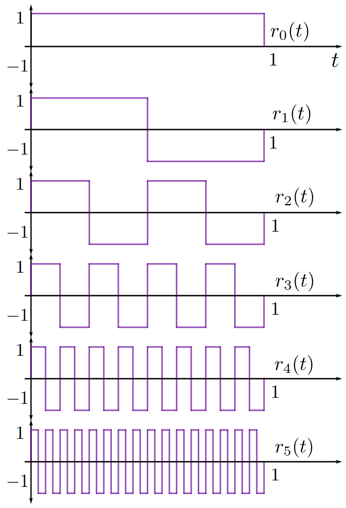

Fig. 2. The first six Rademacher functions.

TMA design is that the on/off control switching signals capable of generating the Walsh waveforms are easily generated in the digital domain from a well known set of orthogonal squared bipolar sequences: the Rademacher functions [12].

II. Application of the Walsh Functions to TMA Design

In this section we analyze how Walsh functions can be applied to the design of the modulating waveforms in the TMA technique.

A. Generation of the Walsh functions from Square Waves

The Rademacher functions are a set of odd orthogonal bipolar square waves which can be derived from sinusoidal functions. More specifically, Rademacher functions have the form

$$r_i(t) = \begin{cases} 1 & i = 0 \\ \mathrm{sign}[\sin(2^i \pi t)] & i > 0, \end{cases} \tag{1}$$

being $i \in \mathbb{N}$ the order of the Rademacher function, $\mathrm{sign}[t] = 1$ if $t > 0$ and $\mathrm{sign}[t] = -1$ if $t < 0$ (see Fig. 2). These functions, with frequencies $0, 1, 2, 2^2, \ldots$ are easily generated from a binary counter and, since they are odd, do not constitute a complete set. However, a complete set of orthonormal functions, the so-called Walsh functions $w_i(t)$, can be generated from the Rademacher functions as follows [12], [13][1]:

- For $i \in \{0, 1, 2\}$, $w_i(t) = r_i(t)$.
- For $i \geq 3$:
  1) Write i in binary notation.
  2) Compute the product of the Rademacher functions corresponding to the positions with 1's in the binary representation of i, assigning $r_1(t)$ to the position of the least significant bit.

For instance, if $i = 5$, then $5_{10} = 101_2$, leading to $w_5(t) = r_3(t) \cdot r_1(t)$ (see Figs. 2 and 3).

[1]The Walsh functions generated by means of this procedure are said to be arranged in natural order. Another way of arranging such functions is the so-called sequence order, i.e., in ascending value of the number of zero crossings found within the time base.

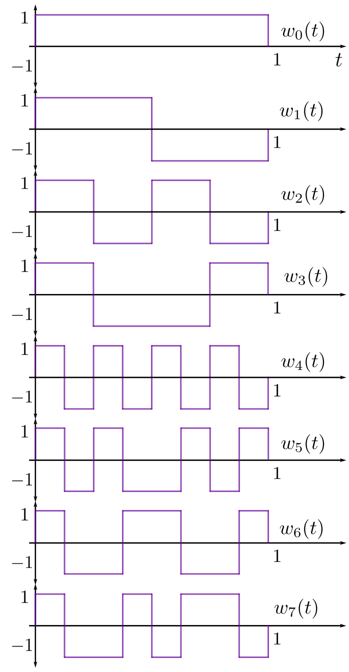

Fig. 3. The first eight Walsh functions arranged in natural order.

B. Determination of the Walsh Coefficients

A function $f(t)$ which is absolutely integrable in $[0, 1)$ can be represented by a Walsh series expansion [10]

$$f(t) = \sum_{i=0}^{\infty} C_i w_i(t) \tag{2}$$

being C_i the Walsh coefficients which satisfy the following condition with respect to the integral square error

$$\lim_{M \to \infty} \int_0^1 \left| f(t) - \sum_{i=0}^{M} C_i w_i(t) \right|^2 dt = 0. \tag{3}$$

Taking into account the orthonormal property of the Walsh functions, the coefficients of the Walsh series expansion are determined as follows

$$C_i = \int_0^1 f(t) w_i(t) dt, \quad i \in \mathbb{N}. \tag{4}$$

If the series in Eq. (2) is truncated up to a finite index M, we obtain an approximation of $f(t)$ and a non-zero integral square error. Furthermore, for numerical handling, it is interesting to consider a discrete series of $M = 2^p$ terms (with $p \in \mathbb{N}$) set up by sampling $f(t)$ at M equally spaced points t_k over $[0, 1)$ with $k \in \Psi = \{0, 1, \ldots, M-1\}$. Hence, the integration in Eq. (4) can be replaced by the finite sum[2]

$$C_i = \frac{1}{M} \sum_{k=0}^{M-1} f(t_k) w_i[t_k], \; i \in \Psi, \tag{5}$$

which constitutes the DWT. An additional key advantage in computational terms is that, for a given $i \in \Psi$, the values

[2]By applying the trapezium rule on M sampling points x_k and evaluating the function $f(t_k) w_i(t_k)$.

$w_i[t_k]$ with $k \in \Psi$ (see Fig. 3) form a sequence of 1's and -1's constituting a specific row of a Hadamard matrix with order $M = 2^p$. The Hadamard matrices are constructed according to the following recursive method

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix}, H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix} \cdots \tag{6}$$

By indexing the rows of a Hadamard matrix from 0 to $M-1$, the discrete version of the Walsh functions are arranged in the matrix according to a bit-reversal permutation of the row indices. For example, if we consider $p = 3$, and hence a Hadamard matrix with order $M = 8$, the row indices 0 to 7 have the corresponding binary representation 000 to 111; if we reverse the order of the binary digits, we have 000, 100, 010, 110, 001, 101, 011, and 111, which correspond to 0, 4, 2, 6, 1, 5, 3, and 7. Hence, we have in the rows of such a Hadamard matrix the Walsh functions $w_0[t_k]$, $w_4[t_k]$, $w_2[t_k]$, $\ldots$, $w_3[t_k]$, $w_7[t_k]$, with $k \in \Psi$.

Hence, by considering a periodic (T_0) function $f(t)$ absolutely integrable in $[0, T_0)$, by arranging $M = 2^p$ equally spaced samples of $f(t)$ in a column vector $\bar{f} = [f(t_1), \ldots, f(t_M)]^T$, we can express –by virtue of Eq. (5)– the corresponding DWT of $f(t)$ through the following matrix equation

$$\bar{L} = \frac{1}{M} H_M \cdot \bar{f} \tag{7}$$

being $\bar{L}$ a column vector with the Walsh coefficients (sorted according to a bit-reversal permutation of the row indices) and H_M the Hadamard matrix with order M.

In the next sections, we will apply the DWT matrix expression in Eq. (7) to approximate the variable time-delayed sine waveforms which time-modulate the excitations of an antenna array.

III. Time-varying Array Factor Controlled by Walsh Functions

In this section, we apply Walsh synthesis to the design of TMAs with beamsteering (BS) capabilities. The idea is to approximate the functions which time-modulate the array excitations (sine waveforms) by means of linear combinations of the Walsh functions (easily implemented by SPDT switches and fixed components). Hence, the time-varying array factor will be expressed as a function of the Walsh coefficients of such modulating waveforms.

We consider a linear TMA with N isotropic elements having unitary static excitations $I_n = 1$, $n \in \{0, 1, \ldots, N-1\}$ to design a feeding architecture for the n-th TMA element consisting of SPDT switches, fixed attenuators, and fixed delay lines (see Fig. 1). In such a feeding network, the excitation of the n-th antenna element is time-modulated by the periodic (T_0) pulse $u_n(t) + ju_n(t-\tau)$, being $u_n(t)$ an approximation of a sine waveform with fundamental frequency ($\omega_0 = 2\pi/T_0$) and τ a time delay defined beforehand.

The novelty of this work lies in the way to synthesize $u_n(t)$, which is done through the linear combination

$$u_n(t) = \sum_{i=0}^{M-1} C_{ni} w_{ni}(t) \tag{8}$$

where $w_{ni}(t)$ is the periodic (T_0) Walsh function $w_i(t)$ with order i (see Fig. 3) and a time delay D_n, i.e., $w_{ni}(t) = w_i(t - D_n)$, being C_{ni} the corresponding Walsh coefficients computed considering $f(t) = u_n(t)$. Hence, $\bar{f} = \bar{u}_n$ in Eq. (7). On the other hand, we consider the Fourier expansion of each Walsh function

$$w_{ni}(t) = \sum_{q=-\infty}^{\infty} W_{ni}^q \mathrm{e}^{jq\omega_0 t} \tag{9}$$

being W_{ni}^q the exponential Fourier series coefficients. By substituting Eq. (9) into Eq. (8), we have

$$u_n(t) = \sum_{q=-\infty}^{\infty} \left[\sum_{i=0}^{M-1} W_{ni}^q C_{ni} \right] \mathrm{e}^{jq\omega_0 t}. \tag{10}$$

If we select a delay τ verifying that $\omega_0 \tau = \pi/2$, then $\mathrm{e}^{-jq\omega_0\tau} = (-j)^q$ and, by applying the time-shifting property of the Fourier coefficients in Eq. (10), we obtain

$$\begin{aligned} & u_n(t) + ju_n(t-\tau) = \\ & \sum_{q=-\infty}^{\infty} \left[1 - (-j)^{q+1}\right] \left[\sum_{i=0}^{M-1} W_{ni}^q C_{ni} \right] \mathrm{e}^{jq\omega_0 t}, \end{aligned} \tag{11}$$

Therefore, the architecture shown in Fig. 1 leads to the following time-varying array factor (with the term $\mathrm{e}^{j\omega_c t}$ explicitly included) as a function of the modulating Walsh functions parameters

$$\begin{aligned} F(\theta, t) &= \mathrm{e}^{jq\omega_c t} \sum_{n=0}^{N-1} \left[u_n(t) + ju_n(t-\tau)\right] \mathrm{e}^{jkz_n \cos\theta} \\ &= \sum_{q=-\infty}^{\infty} \sum_{n=0}^{N-1} \sum_{i=0}^{M-1} \frac{1-(-j)^{q+1}}{\sqrt{2}} W_{ni}^q C_{ni} \mathrm{e}^{jkz_n \cos\theta} \mathrm{e}^{j(\omega_c + q\omega_0)t} \\ &= \sum_{q=-\infty}^{\infty} F_q(\theta) \mathrm{e}^{j(\omega_c + q\omega_0)t} \end{aligned} \tag{12}$$

where z_n is the n-th array element position on the z axis, θ is the angle with respect to such a main axis, $k = 2\pi/\lambda$ represents the wavenumber for a carrier wavelength $\lambda = 2\pi c/\omega_c$, and ω_c is the carrier frequency. Notice that

$$\begin{aligned} F_q(\theta) &= \sum_{n=0}^{N-1} \sum_{i=0}^{M-1} \frac{1-(-j)^{q+1}}{\sqrt{2}} W_{ni}^q C_{ni} \mathrm{e}^{jkz_n \cos\theta} \\ &= \sum_{n=0}^{N-1} I_{nq} \mathrm{e}^{jkz_n \cos\theta} \end{aligned} \tag{13}$$

is the spatial array factor at the frequency $\omega_c + q\omega_0$ and

$$I_{nq} = \sum_{i=0}^{M-1} \frac{1-(-j)^{q+1}}{\sqrt{2}} W_{ni}^q C_{ni}, \ n \in \{0, 1, \ldots, N-1\} \tag{14}$$

are the corresponding dynamic excitations that synthesize the radiated pattern at such a frequency.

IV. Numerical Simulations

For our TMA BS design we assume that all the array elements are modulated by identical sinusoidal waves (or better said, the same stair step approximations of such sine waves) but subjected to different time delays. We will select the delays so that the TMA dynamical excitations have progressive phases, hence allowing for BS. Therefore, we only need to carry out the Walsh synthesis of a single non-delayed sine waveform with period T_0 since when we consider the synthesis of a sine replica with a delay D_n, it will consists of the same set of Walsh functions as the non delayed version, but individually subjected to D_n.

Firstly, we will perform the discrete Walsh synthesis of $f(t) = \sin(2\pi/T_0 t)$. Without loss of generality, we assume a normalized period $T_0 = 1$. Bearing in mind Eq. (7), by evaluating $f(t)$ for $M = 8$ equally spaced points in the interval $(0, 2\pi]$, we have the column vector $\bar{f} = [f(\pi/8), f(3\pi/8), \ldots, f(15\pi/8)]^T$, and by considering the Hadamard matrix W_8 we arrive at

$$\bar{L}_8 = (1/8) \cdot H_8 \cdot \bar{f} = [0, 0, 0, 0, 0.653, 0, 0, -0.271] \quad (15)$$

being only L_5 and L_8 different from zero. Taking into account the bit-reversal relationship between the index $k-1$ of L_k and the corresponding Walsh functions, the only possible functions involved in the synthesis are $w_1(t)$ and $w_7(t)$ (Fig. 3). Hence, we approximate

$$f(t) = \sin(2\pi t) \approx 0.653\, w_1(t) - 0.271\, w_7(t) \quad (16)$$

whereas $u_n(t)$ in Eq. (8) can be expressed as

$$\begin{aligned} u_n(t) &= 0.653\, w_1(t - D_n) - 0.271\, w_7(t - D_n) \\ &= 0.653\, w_{n1}(t) - 0.271\, w_{n7}(t) \end{aligned} \quad (17)$$

being $C_{n1} = 0.653$ and $C_{n7} = -0.271$. Consequently, it is possible to perform BS (see Fig. 1) with four SPDT switches per antenna element. On the other hand, since the Fourier coefficients W_{n1}^q and W_{n7}^q (Eq. (9)) of $w_{n1}(t)$ and $w_{n7}(t)$, respectively, are zero for q even, it is satisfied that

$$1 - (-j)^{q+1} = \begin{cases} 2 & q \in \Upsilon \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

where Υ is defined as $\Upsilon = \{q = 4k - 3; k \in \mathbb{Z}\} = \{\ldots, -7, -3, 1, 5, 9, 13, \ldots\}$. Hence, the frequency-mirrored unwanted harmonics are removed (single sideband (SSB) features) and we can express the dynamic excitations for each harmonic pattern at $\omega_c + q\omega_0$ (see Eq. (14)) as

$$I_{nq} = \sqrt{2}(0.653\, W_{n1}^q - 0.271\, W_{n7}^q), n \in \{0, \ldots, N-1\} \quad (19)$$

Since the signals $u_n(t)$ applied to each antenna are delayed replicas of each other, their Fourier coefficients will be equal in modulus but different in phase. Hence, in view of Eq. (14), since $w_{n1}(t)$ and $w_{n7}(t)$ have the same delay for a given q, $|I_{nq}|$ is the same for all n, yielding a uniform linear array for each frequency $\omega_c + q\omega_0$, being $q \in \Upsilon$.

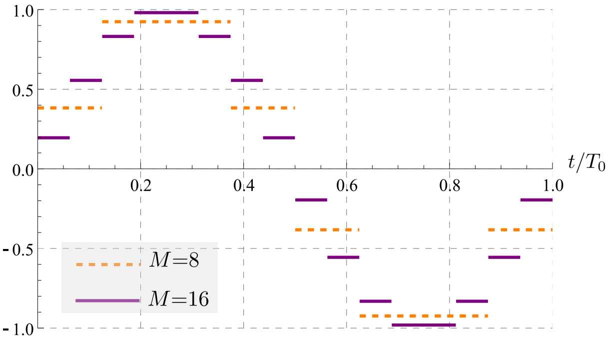

Fig. 4. Stair-step approximations of $\sin(2\pi t)$ considering the DWT with $M = 8$ and $M = 16$ equally spaced points, respectively.

If we consider a larger number of equally spaced points $M = 16$, i.e., $\bar{f} = [f(\pi/16), f(3\pi/16), \cdots, f(31\pi/16)]^T$, and the Hadamard matrix W_{16}, we have

$$\begin{aligned} \bar{L}_{16} =& (1/16) \cdot H_{16} \cdot \bar{f} = [0, 0, 0, 0, 0, 0, 0, 0, \\ & 0.641, 0, 0, -0.053, 0, -0.127, -0.265, 0]^T, \end{aligned} \quad (20)$$

Hence, using the aforementioned bit-reversal permutation, we have that the functions involved in the sine synthesis are $w_1(t)$, $w_7(t)$, $w_{11}(t)$, and $w_{13}(t)$ (see Fig. 3), arriving at

$$\begin{aligned} u_n(t) =& 0.641\, w_{n1}(t) - 0.053\, w_{n7}(t) \\ & - 0.127\, w_{n11}(t) - 0.265\, w_{n13}(t). \end{aligned} \quad (21)$$

Given that the Walsh functions in Eq. (21) also satisfy Eq. (18),

$$\begin{aligned} I_{nq} =& \sqrt{2}(0.641\, W_{n1}^q - 0.053\, W_{n7}^q - 0.127 W_{n11}^q \\ & - 0.265\, W_{n13}^q), \ n \in \{0, 1, \ldots, N-1\}. \end{aligned} \quad (22)$$

Fig. 4 shows the stair-step modulating pulses $u_n(t)$ for the cases with $M = 8$ (Eq. (17)) and $M = 16$ (Eq. (21)) considering $D_n = 0$. Fig. 5 illustrates the relative power level (expressed in dB) of $|I_{nq}|$ (see Eq. (19) and Eq. (22)), taking $|I_{n1}|$ as a reference for each case. We observe that the level of the first unwanted harmonic for $M = 8$ at $q = -7$ is -17 dB, whereas for the case with $M = 16$ decreases to -23.5 dB at $q = -15$ (see also the corresponding radiated patterns in Fig. 6). Notice that the level of rejection of the side band radiation is significantly better than that obtained in [5] for rectangular pulses (-13.98 dB). Hence, the use of more points yields a 40 % improvement in the rejection of the first unwanted harmonic at the expense of employing twice the number of Walsh functions and, consequently, twice the number of switches per antenna (trade-off between complexity and performance).

Additionally, by defining the overall time-modulation efficiency as $\eta = \eta_{\text{TMA}} \cdot \eta_s$ [9], where $\eta_{\text{TMA}} = P_U^{\text{TM}}/P_R^{\text{TM}}$ (P_U^{TM} and P_R^{TM} are the useful and total mean power values radiated by the TMA, respectively) accounts for the ability of the TMA to filter out and radiate only over the useful harmonics, whereas $\eta_s = P_R^{\text{TM}}/P_R^{\text{ST}}$ (P_R^{ST} is the total mean power radiated by a uniform static array with N elements) accounts for the reduction of the total mean power radiated

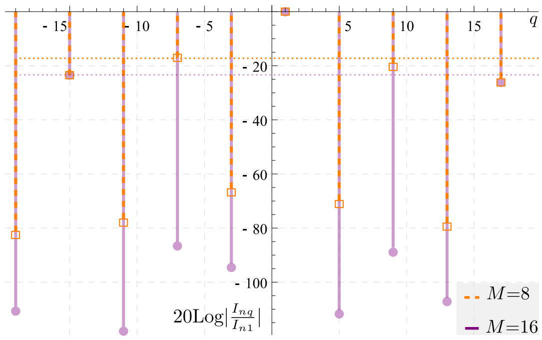

Fig. 5. Relative power level of the TMA excitations for the different orders q compared to that of the useful harmonic at $q = 1$ for the cases with $M = 8$ and $M = 16$.

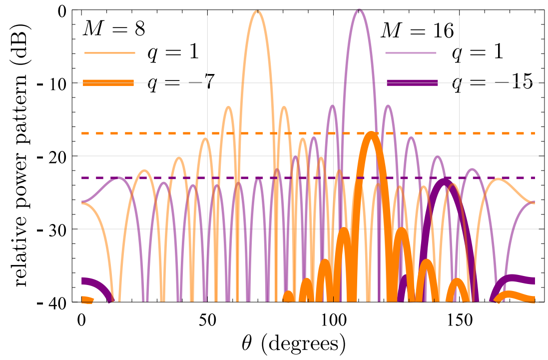

Fig. 6. Relative power radiated pattern of TMAs designed by means of the DWT for the cases with $M = 8$ and $M = 16$.

by a uniform static array caused by the insertion of the TMA switched feeding network. Notice that Walsh functions are especially suited for the approximation of a sine function by means of bipolar sequences and hence allow for strongly attenuate those harmonics with a lower order ($q = 3, 5$).

Considering I_{nq} in Eq. (19) and Eq. (22) in the formulas given in [9], we have that $\eta_{\text{TMA}} = 94.96\%$ for $M = 8$ and $\eta_{\text{TMA}} = 98.72\%$ for $M = 16$. Therefore, the use of more SPDT switches provides a 4% improvement, arriving at $\eta_s \approx 50.00\%$ in both cases, hence leading to an overall efficiency $\eta(\text{dB}) = 10\log_{10}(\eta) = -3.23\,\text{dB}$ for $M = 8$ and $\eta(\text{dB}) = -3.07\,\text{dB}$ for $M = 16$.

In summary, we observe that even if we employ more switches, we have an upper limit in the efficiency imposed by η_s. In other words, a reduction of the transmitted signal power is caused by the effect of time-modulating the excitations with a stair-step waveform with respect to the signal power transmitted by a uniform static array (notice that the latter has no intrinsic BS ability). In spite of such a limitation with respect to static arrays, it is remarkable that the proposed technique is competitive when compared to standard BS networks based on VPSs which exhibit insertion losses and costs directly dependent on the carrier frequency (recall that TMAs are limited by the signal bandwidth rather than the carrier frequency [2]). Indeed, if we calculate the total insertion losses of the proposed BS network by adding to the sideband radiation (SR) losses $10\log_{10}(\eta_{\text{TMA}})$ and the modulation efficiency losses, $10\log_{10}(\eta_s)$, those losses corresponding to the switching hardware (typically $< 0.5\,\text{dB}$, e.g. [14]) and to the fixed broadband phase shifters (also $< 0.5\,\text{dB}$ [15]), we arrive at values significantly lower than those found in commercial VPSs. As an orientation, insertion losses in the millimeter wave band vary from 6 to 9 dB [16].

V. Conclusions

We have presented a novel approach to TMA design based on the DWT. The method allows TMA architectures to offer arbitrary angle BS by adjusting the switch-on instants, excellent levels of rejection of the undesired harmonics, and competitive insertion losses when compared to standard BS networks based on VPS.

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References

[1] H. E. Shanks and R. W. Bickmore, "Four-dimensional electromagnetic radiators," *Canadian Journal of Physics*, vol. 37, no. 3, pp. 263–275, 1959.

[2] R. Maneiro-Catoira, J. Brégains, J. A. Garcia-Naya, L. Castedo, P. Rocca, and L. Poli, "Performance analysis of time-modulated arrays for the angle diversity reception of digital linear modulated signals," *IEEE J. Sel. Topics Signal Process.*, vol. 11, no. 2, pp. 247–258, Mar. 2017.

[3] Y. Tong and A. Tennant, "Simultaneous control of sidelobe level and harmonic beam steering in time-modulated linear arrays," *Electronics Letters*, vol. 46, no. 3, pp. 201–202, Feb 2010.

[4] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Harmonic beamforming in time-modulated linear arrays," *IEEE Trans. Antennas Propag.*, vol. 59, no. 7, pp. 2538–2545, Jul. 2011.

[5] A. M. Yao, W. Wu, and D. G. Fang, "Single-sideband time-modulated phased array," *IEEE Trans. Antennas Propag.*, vol. 63, no. 5, pp. 1957–1968, May 2015.

[6] Q. Zhu, S. Yang, R. Yao, and Z. Nie, "Gain improvement in time-modulated linear arrays using spdt switches," *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 994–997, 2012.

[7] S. Yang, Y. Gan, and P. Tan, "Evaluation of directivity and gain for time-modulated linear antenna arrays," *Microw. Opt. Technol. Lett.*, vol. 42, p. 167171, 2004.

[8] R. Maneiro-Catoira, J. Bregains, J. A. Garcia-Naya, and L. Castedo, "Analog beamforming with single-sideband sub-time-modulated arrays," in *Proc. of IEEE International Symposium on Antennas and Propagation USNC/URSI National Radio Science Meeting*, July 2018, pp. 9–10.

[9] ——, "Time-modulated phased array controlled with non-ideal bipolar squared periodic sequences," *IEEE Antennas Wireless Propag. Lett.*, pp. 1–1, 2019.

[10] J. Walsh, "A closed set of orthogonal functions," *Ann. Journ. Math*, no. 55, pp. 5–24, 1923.

[11] A. C. Grove, "An introduction to walsh functions and their applications," *International Journal of Mathematical Education in Science and Technology*, vol. 14, no. 1, pp. 43–53, 1983. [Online]. Available: https://doi.org/10.1080/0020739830140107

[12] C. F. Chen and C. H. Hsiao, "Time-domain synthesis via walsh functions," *Proceedings of the Institution of Electrical Engineers*, vol. 122, no. 5, pp. 565–570, May 1975.

[13] K. Beauchamp, *Walsh Functions and their application*. Academic Press Inc. London, 1975.

[14] "Analog devices," http://www.analog.com, reference: HMC545. Accessed: 2019-02-01.

[15] A. M. Abbosh, "Broadband fixed phase shifters," *IEEE Microw. Wireless Compon. Lett.*, vol. 21, no. 1, pp. 22–24, Jan 2011.

[16] "Qorvo," http://www.qorvo.com, references: QPC1000, TGP2100, TGP2102, TGP2104, TGL2767. Accessed: 2019-02-01.