Separation of independent/dependent sources using copulas

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***Abstract*—In this paper, we introduce a new convolutive blind source separation approach for independent/dependent source components. The proposed approach represents an efficient tool for separating linear convolutive mixing models, especially, when the source components are statistically dependent. Its efficiency is illustrated by some simulation results.**

***Index Terms*—Blind source separation, Kullback-Leibler divergence, Copulas, Dependent source components.**

I. INTRODUCTION

Convolutive blind source separation (BSS) problem is a fundamental issue in applications of many different fields such as feature extraction [1], [2], radio communications [3], acoustical surveillance [10], etc. Convolutive (BSS) aims to recover unobserved signals from linear convolutive mixtures of them, where there is no, or very limited, information about the original source signals or the mixing system. In convolutive mixing models, the mixing process is quite complicated. Each observation is a sum of many different weighted and delayed source signals, i.e., we take into account the propagation of each source to different sensors in the medium. In other words, for arriving to the different sensors, each source passes through different transfer functions.

In literature, several methods have been proposed for separating this kind of mixtures such as [11], [12], [13], [14] which are based on independent component analysis (ICA), assuming that the source signals are statistically independent [4]. Under the standing boundedness condition of the source signals, authors in [15], [16] have developed some geometric methods replacing the statistical independence assumption with a geometric type condition on the source signal samples, see condition (A1) in [15], [16]. Frequency-domain methods have been also developed for separating convolutive mixtures using a short-time-Fourier transform (STFT), and applying standard tools of instantaneous blind source separation to each of the STFT channels, see e.g. [16], [17]. In [6], [18] Keziou et al proposed a new approach that enables separation of independent/dependent source components from their linear instantaneous mixtures (up to scale and permutation indeterminacies). In this approach, a statistical concept, namely copulas, was introduced to model the dependency structure of the source components. A solution to the separation problem was obtained by minimizing the Kullback-Leibler divergence between the copula density of the estimated source components and the copula density of the source components. Since in many applications, a model of linear instantaneous mixtures is unsuitable, we propose, in this paper, a new blind source separation approach that provides an efficient solution for the separation of convolutive mixtures for both independent and dependent source components (up to a permutation and filtering indeterminacies). This paper extends the instantaneous approach introduced in [6] for the convolutive BSS problem. We have extended the objective functions proposed in [6] to cover the more general case where the observed signals are convolutive mixtures of the sources. The proposed approach consists of minimizing a new estimated separation criterion based on the Kullback-Leibler divergence between copula densities. For clarity, we will treat separately the case of independent source components, then the case of dependent source components.

II. PRINCIPLE OF CONVOLUTIVE BLIND SOURCE SEPARATION

Suppose that we have p observed signals $x_1(t), \cdots, x_p(t)$ which are assumed to be linear convolutive mixtures of p source signals $s_1(t), \cdots, s_p(t)$. In this framework, the mixing system is composed of linear filters which can be mathematically modeled by

$$x_i(t) := \sum_{j=1}^{p} \boldsymbol{a}_{ij}(t) * s_j(t) + b_i(t), \quad \forall i = 1, \ldots, p, \qquad (1)$$

where "*" is the convolution operator, and $\boldsymbol{a}_{ij}(t)$ is the impulse response from the j-th source to the i-th sensor. The model (1) can be also written in its matrix form as

$$\boldsymbol{x}(t) = \boldsymbol{A}(t) * \boldsymbol{s}(t) + \boldsymbol{b}(t), \qquad (2)$$

where $\boldsymbol{A}(t)$ is the unknown matrix of linear filters that groups the impulse responses $\boldsymbol{a}_{ij}(t)$,
$\boldsymbol{s}(t) := (s_1(t), \cdots, s_p(t))^\top \in \mathbb{R}^p$ is the unknown vector of sources to be estimated, $\boldsymbol{x}(t) := (x_1(t), \cdots, x_p(t))^\top \in \mathbb{R}^p$ represents the observed vector signal at time t and $\boldsymbol{b}(t)$ is an additive noise vector. We assume that the noise is reduced by applying some form of preprocessing such as denoising the

observed signals through regularization approach, see e.g. [5]. We then write

$$\boldsymbol{x}(t) = \boldsymbol{A}(t) * \boldsymbol{s}(t). \tag{3}$$

Note that here the number of source components is assumed to be equal to the number of observed ones. Assuming that the filters of $\boldsymbol{A}(t)$ can be modeled by causal finite impulse response (FIR) ones, with maximum degree $L < \infty$, the transposition, into the discrete time domain, of model (2) gives

$$\boldsymbol{x}(n) = [\boldsymbol{A}(z)]\boldsymbol{s}(n) = \sum_{k=0}^{L} \boldsymbol{A}_k\, \boldsymbol{s}(n-k),\ \forall n = 1, \ldots, N, \tag{4}$$

where $\boldsymbol{A} = \{\boldsymbol{A}_0, \ldots, \boldsymbol{A}_L\}$ are finite impulse response (FIR) filters with maximum degree L. Notice that $\boldsymbol{A}_k$ are $p \times p$ matrices for all $k = 0, \ldots, L$. Assume additionally that $\boldsymbol{A}(z)$ is minimum phase, see e.g. [24]. We have then the existence of unique inverse causal filter $\boldsymbol{A}(z)^{-1}$ of $\boldsymbol{A}(z)$. We suppose also that $\boldsymbol{A}(z)^{-1}$ is FIR filter, or can be approximated by a FIR filter, with degree L. Under the above assumptions, and due to the form of mixtures (4), the estimated sources can be obtained by applying a linear filtering on the observed mixtures (4), taking the form

$$\boldsymbol{y}(n) := [\boldsymbol{B}(z)]\boldsymbol{x}(n) = \sum_{k=0}^{L} \boldsymbol{B}_k\, \boldsymbol{x}(n-k),\ \forall n = 1, \ldots, N, \tag{5}$$

where $\boldsymbol{B} = \{\boldsymbol{B}_0, \ldots, \boldsymbol{B}_L\}$ are finite impulse response (FIR) filters with the same degree L, and $\boldsymbol{B}_k$ are $p \times p$ matrices for all $k = 0, \ldots, L$. If the source components are statistically independent, [12] proved that the filter matrix $\widehat{\boldsymbol{B}}$, making the components of $[\widehat{\boldsymbol{B}}(z)]\boldsymbol{x}(n)$ independent (in terms of stochastic process to be specified in subsection IV-A below), leads to the source separation (accurate estimates $\widehat{\boldsymbol{y}}(n)$ of the source signals up to a permutation and a filtering indeterminacies), i.e.,

$$\widehat{\boldsymbol{y}}(n) := [\widehat{\boldsymbol{B}}(z)]\boldsymbol{x}(n) = [\widehat{\boldsymbol{B}}(z)\boldsymbol{A}(z)]\boldsymbol{s}(n), \tag{6}$$

where the filter matrix $\widehat{\boldsymbol{B}}(z)$ satisfies $[\widehat{\boldsymbol{B}}(z)\boldsymbol{A}(z)] = \boldsymbol{P}\boldsymbol{H}(z)$, with $\boldsymbol{P}$ a permutation and $\boldsymbol{H}$ a filtering operator. In the following, we will deal with the convolutive blind source separation problem in order to separate, using "copulas", not only independent source components but also dependent ones. For clarity, we will study separately, the case where the source components are independent in Section IV-A, and the case where they are dependent in Section IV-B.

III. Brief recall on Copulas

We give the following brief recall on copulas. The concept of copula was introduced by Sklar in [7] as a function which couples a joint distribution function with its univariate margins. Consider a random vector $\boldsymbol{Y} := (Y_1, \ldots, Y_p)^\top \in \mathbb{R}^p$, $p \geq 2$, with joint distribution function (d.f.)

$$\begin{aligned} F_{\boldsymbol{Y}}(\cdot) :\ \boldsymbol{y} \in \mathbb{R}^p \mapsto F_{\boldsymbol{Y}}(\boldsymbol{y}) &:= F_{\boldsymbol{Y}}(y_1, \ldots, y_p) \\ &:= \mathbb{P}(Y_1 \leq y_1, \ldots, Y_p \leq y_p), \end{aligned} \tag{7}$$

and continuous marginal d.f.'s

$$F_{Y_i}(\cdot) :\ y_i \in \mathbb{R} \mapsto F_{Y_i}(y_i) := \mathbb{P}(Y_i \leq y_i),\ \forall i = 1, \ldots, p. \tag{8}$$

Sklar characterization theorem [7] ensures the existence of a unique function $\mathbb{C}_{\boldsymbol{Y}}(\cdot) : [0,1]^p \to [0,1]$ such that $\forall \boldsymbol{y} := (y_1, \ldots, y_p)^\top \in \mathbb{R}^p$

$$F_{\boldsymbol{Y}}(\boldsymbol{y}) = \mathbb{C}_{\boldsymbol{Y}}(F_{Y_1}(y_1), \ldots, F_{Y_p}(y_p)). \tag{9}$$

The function $\mathbb{C}_{\boldsymbol{Y}}(\cdot)$ is called a copula and it is a joint d.f. on $[0,1]^p$ with uniform marginals. We have $\forall \boldsymbol{u} = (u_1, \ldots, u_p)^\top \in [0,1]^p$,

$$\mathbb{C}_{\boldsymbol{Y}}(\boldsymbol{u}) = \mathbb{P}(F_{Y_1}(Y_1) \leq u_1, \ldots, F_{Y_p}(Y_p) \leq u_p).$$

Conversely, for any marginal d.f.'s $F_1(\cdot), \ldots, F_p(\cdot)$, and any copula function $\mathbb{C}(\cdot)$, the function $\mathbb{C}(F_1(\cdot), \ldots, F_p(\cdot))$ is a multivariate d.f. on $\mathbb{R}^p$. On the other hand, since the marginal d.f.'s $F_{Y_j}(\cdot)$, $j = 1, \ldots, p$, are assumed to be continuous, then the random variables $F_{Y_1}(Y_1), \ldots, F_{Y_p}(Y_p)$ are uniformly distributed on the interval $[0,1]$. Therefore, if the components $Y_1, \ldots, Y_p$ are statistically independent, then the corresponding copula is

$$\mathbb{C}_{\boldsymbol{Y}}(\boldsymbol{u}) = \prod_{i=1}^{p} u_i =: \mathbb{C}_0(\boldsymbol{u}),\ \forall \boldsymbol{u} \in [0,1]^p. \tag{10}$$

It is called the copula of independence. Define, if it exists, the copula density of the random vector $\boldsymbol{Y}$ by

$$\boldsymbol{c}_{\boldsymbol{Y}}(\boldsymbol{u}) := \frac{\partial^p \mathbb{C}_{\boldsymbol{Y}}(\boldsymbol{u})}{\partial u_1 \cdots \partial u_p},\ \forall \boldsymbol{u} \in [0,1]^p. \tag{11}$$

Then, the copula density of independence, denote it by $\boldsymbol{c}_0(\cdot)$, is the function taking the value 1 on $[0,1]^p$ and zero otherwise, namely,

$$\boldsymbol{c}_0(\boldsymbol{u}) := \mathbb{1}_{[0,1]^p}(\boldsymbol{u}),\ \forall \boldsymbol{u} \in [0,1]^p. \tag{12}$$

Let $f_{\boldsymbol{Y}}(\cdot)$, if it exists, be the probability density on $\mathbb{R}^p$ of the random vector $\boldsymbol{Y} := (Y_1, \ldots, Y_p)^\top$, and, respectively, $f_{Y_1}(\cdot), \ldots, f_{Y_p}(\cdot)$, the associated marginal probability densities. Then, for all $\boldsymbol{y} := (y_1, \ldots, y_p)^\top \in \mathbb{R}^p$, we have

$$f_{\boldsymbol{Y}}(\boldsymbol{y}) = \left(\prod_{i=1}^{p} f_{Y_i}(y_i) \right) \boldsymbol{c}_{\boldsymbol{Y}}(F_{Y_1}(y_1), \ldots, F_{Y_p}(y_p)). \tag{13}$$

Combining the relations (10-13), one can show that

$$\boldsymbol{c}_{\boldsymbol{Y}}(\boldsymbol{u}) = \boldsymbol{c}_0(\boldsymbol{u}),\ \forall \boldsymbol{u} \in [0,1]^p$$

if and only if (iff) the components of the vector $\boldsymbol{Y}$ are independent. We can refer to [8], [19] for more details on copula theory.

IV. Convolutive BSS for independent/dependent sources via copulas

Let $\boldsymbol{Y} = (Y_1, \ldots, Y_p)^\top \in \mathbb{R}^p$ be any random vector with continuous marginal distribution functions $F_{Y_1}(\cdot), \ldots, F_{Y_p}(\cdot)$. It has been shown in [6] that the Mutual Information (MI) of $\boldsymbol{Y}$ can be written as the Kullback-Leibler divergence between the copula density $\boldsymbol{c}_{\boldsymbol{Y}}$ of the vector $\boldsymbol{Y}$ and the copula density

$\boldsymbol{c}_0$ of independence. In fact, using the relation (13), one can write

$$\begin{aligned} MI(\boldsymbol{Y}) &:= \int_{\mathbb{R}^p} \log\left(\frac{f_{\boldsymbol{Y}}(\boldsymbol{y})}{\prod_{i=1}^{p} f_{Y_i}(y_i)}\right) f_{\boldsymbol{Y}}(\boldsymbol{y})\, d\boldsymbol{y} \\ &= \int_{[0,1]^p} \log\left(\frac{\boldsymbol{c}_{\boldsymbol{Y}}(\boldsymbol{u})}{1}\right) \boldsymbol{c}_{\boldsymbol{Y}}(\boldsymbol{u})\, d\boldsymbol{u} \\ &= \int_{[0,1]^p} \log\left(\frac{\boldsymbol{c}_{\boldsymbol{Y}}(\boldsymbol{u})}{\boldsymbol{c}_0(\boldsymbol{u})}\right) \boldsymbol{c}_{\boldsymbol{Y}}(\boldsymbol{u})\, \mathrm{d}\boldsymbol{u} \qquad (14) \\ &=: KL\left(\boldsymbol{c}_{\boldsymbol{Y}}, \boldsymbol{c}_0\right) \\ &= \mathbb{E}\left[\log\left(\boldsymbol{c}_{\boldsymbol{Y}}(F_{Y_1}(Y_1), \ldots, F_{Y_p}(Y_p))\right)\right], \end{aligned}$$

where $\mathbb{E}[\cdot]$ denotes the mathematical expectation. The integral in (14) is, by definition, the Kullback-Leibler divergence between $\boldsymbol{c}_{\boldsymbol{Y}}$ and $\boldsymbol{c}_0$. Recall that $KL(\boldsymbol{c}_{\boldsymbol{Y}}, \boldsymbol{c}_0)$ is nonnegative, and

$$KL(\boldsymbol{c}_{\boldsymbol{Y}}, \boldsymbol{c}_0) = 0 \quad \text{iff} \quad \boldsymbol{c}_{\boldsymbol{Y}}(\boldsymbol{u}) = \boldsymbol{c}_0(\boldsymbol{u}),\ \forall \boldsymbol{u} \in [0,1]^p, \quad (15)$$

which is equivalent with the fact that the components of the vector $\boldsymbol{Y}$ are independent.

A. Independent source components

In this subsection, we give a solution to the BSS problem for convolutive mixtures of independent source components in the context of copulas. We will use the following statistical approach. For any delay-integer-vector $\boldsymbol{q} := (q_1 = 0, q_2, \ldots, q_p)^\top \in \{0\} \times \{-2L, \ldots, 2L\}^{p-1}$, we will consider that the delayed source signal $\boldsymbol{s}^{\boldsymbol{q}}(n) := (s_1^{q_1}(n), \ldots, s_p^{q_p}(n))^\top := (s_1(n-q_1), \ldots, s_p(n-q_p))^\top, n = 1, \ldots, N$, are N i.i.d realizations of a random vector $\boldsymbol{S}^{\boldsymbol{q}} := (S_1^{q_1}, \ldots, S_p^{q_p})^\top \in \mathbb{R}^p$. Likewise, we consider that the delayed estimated signal $\boldsymbol{y}^{\boldsymbol{q}}(n) := (y_1^{q_1}(n), \ldots, y_p^{q_p}(n))^\top := (y_1(n-q_1), \ldots, y_p(n-q_p))^\top, n = 1, \ldots, N$, are N realizations of random vector $\boldsymbol{Y}^{\boldsymbol{q}} := (Y_1^{q_1}, \ldots, Y_p^{q_p})^\top$ for all $\boldsymbol{q} := (q_1 = 0, q_2, \ldots, q_p)^\top \in \{0\} \times \{-2L, \ldots, 2L\}^{p-1}$. Since we deal with convolutive mixtures, it is easy to show that the independence between two scalar random sources $y_1(n)$ and $y_2(n)$ for all n is not sufficient to separate the signals. The independence of the components $y_1(n_1)$ and $y_2(n_2)$ is needed for all n_1 and n_2. In other words, the independence of $y_1(n)$ and $y_2(n-k)$ for all n and at all delay k, is necessary to achieve separation, see e.g. [20]. We propose then the following separation criterion

$$\begin{aligned} \boldsymbol{B} &\mapsto \mathcal{J}_{ind}(\boldsymbol{Y}^{\boldsymbol{q}}) := \sum_{\boldsymbol{q}} KL\left(\boldsymbol{c}_{\boldsymbol{Y}^{\boldsymbol{q}}}, \boldsymbol{c}_0\right) \\ &= \sum_{\boldsymbol{q}} \mathbb{E}\left[\log\left(\boldsymbol{c}_{\boldsymbol{Y}^{\boldsymbol{q}}}(F_{Y_1^{q_1}}(Y_1^{q_1}), \ldots, F_{Y_p^{q_p}}(Y_p^{q_p}))\right)\right], \qquad (16) \end{aligned}$$

where $KL(\boldsymbol{c}_{\boldsymbol{Y}^{\boldsymbol{q}}}, \boldsymbol{c}_0)$ is the Kullback-Leibler divergence between the copula density of the vector $\boldsymbol{Y}^{\boldsymbol{q}}$ and the copula density of independence $\boldsymbol{c}_0$. In view of property (15), $KL(\boldsymbol{c}_{\boldsymbol{Y}^{\boldsymbol{q}}}, \boldsymbol{c}_0)$ is nonnegative and achieves its minimum value zero iff the components of $\boldsymbol{Y}^{\boldsymbol{q}}$ are independent. To estimate the $p \times p$ dimension matrices $\boldsymbol{B}_k, \forall k = 0, \ldots, L$, we propose to minimize with respect to $\boldsymbol{B}$ a statistical estimate $\widehat{\mathcal{J}_{ind}}(\boldsymbol{Y}^{\boldsymbol{q}})$ of $\mathcal{J}_{ind}(\boldsymbol{Y}^{\boldsymbol{q}})$ constructed from the data $\boldsymbol{y}^{\boldsymbol{q}}(1), \ldots, \boldsymbol{y}^{\boldsymbol{q}}(N)$. We obtain then the following separation filter

$$\widehat{\boldsymbol{B}} := \arg\inf_{\boldsymbol{B}} \widehat{\mathcal{J}_{ind}}(\boldsymbol{Y}^{\boldsymbol{q}}), \qquad (17)$$

which leads to the estimated sources

$$\widehat{\boldsymbol{y}}(n) = \sum_{k=0}^{L} \widehat{\boldsymbol{B}}_k\, \boldsymbol{x}(n-k), \quad \forall n = 1, \ldots, N. \qquad (18)$$

Using (16), $\mathcal{J}_{ind}(\boldsymbol{Y}^{\boldsymbol{q}})$ can be estimated via a “plug-in” type procedure, as follows

$$\begin{aligned} \widehat{\mathcal{J}_{ind}}(\boldsymbol{Y}^{\boldsymbol{q}}) &:= \sum_{\boldsymbol{q}} \frac{1}{N} \sum_{n=1}^{N} \log \widehat{\boldsymbol{c}}_{\boldsymbol{Y}^{\boldsymbol{q}}}(\widehat{F}_{Y_1^{q_1}}(y_1(n-q_1)), \ldots, \\ &\quad \widehat{F}_{Y_p^{q_p}}(y_p(n-q_p))), \qquad (19) \end{aligned}$$

where $\forall \boldsymbol{u} \in [0,1]^p$,

$$\widehat{\boldsymbol{c}}_{\boldsymbol{Y}^{\boldsymbol{q}}}(\boldsymbol{u}) := \frac{1}{NH_1 \cdots H_p} \sum_{m=1}^{N} \prod_{j=1}^{p} k\left(\frac{\widehat{F}_{Y_j^{q_j}}(y_j^{q_j}(m)) - u_j}{H_j}\right), \qquad (20)$$

is the kernel estimate of the copula density $\boldsymbol{c}_{\boldsymbol{Y}^{\boldsymbol{q}}}(\cdot)$ and

$$\widehat{F}_{Y_j^{q_j}}(x) := \frac{1}{N} \sum_{m=1}^{N} K\left(\frac{y_j^{q_j}(m) - x}{h_j}\right), \ j = 1, \ldots, p,$$

is the smoothed estimate of the marginal distribution function $F_{Y_j^{q_j}}(\cdot)$ of the random variable $Y_j^{q_j}$, at any real value $x \in \mathbb{R}$, $K(\cdot)$ is the primitive of a kernel $k(\cdot)$, a symmetric centered probability density. In our forthcoming simulation study, we will use the triangular kernel $k(x) = (1-|x|)\mathbb{1}_{[-1,1]}(x), \forall x \in \mathbb{R}$. We choose the parameters $H_1, \ldots, H_p$ and $h_1, \ldots, h_p$ according to Silverman's rule of thumb [9], i.e., for all $j = 1, \ldots, p$, we take

$$H_j = \left(\frac{4}{p+2}\right)^{\frac{1}{p+4}} N^{\frac{-1}{p+4}}\, \widehat{\Sigma}_j, \quad h_j = \left(\frac{4}{3}\right)^{\frac{1}{5}} N^{\frac{-1}{5}}\, \widehat{\sigma}_j,$$

$\widehat{\Sigma}_j$ and $\widehat{\sigma}_j$ are, respectively, the empirical standard deviation of the data $\widehat{F}_{Y_j^{q_j}}(y_j^{q_j}(1)), \ldots, \widehat{F}_{Y_j^{q_j}}(y_j^{q_j}(N))$ and $y_j^{q_j}(1), \ldots, y_j^{q_j}(N)$. To compute the minimizer in $\boldsymbol{B}$ of (17), we use a gradient descent algorithm.

Remark 1: We notice that the computational load of the proposed algorithm increases with L and p. However, we can use the following “stochastic” implementation of our proposed algorithm: at each iteration, we randomly choose a delay $\boldsymbol{q}$ from the set $\{0\} \times \{-2L, \ldots, 2L\}^{p-1}$ and we take $KL(\boldsymbol{c}_{\boldsymbol{Y}^{\boldsymbol{q}}}, \boldsymbol{c}_0)$ as the current criterion instead of $\sum_{\boldsymbol{q}} KL(\boldsymbol{c}_{\boldsymbol{Y}^{\boldsymbol{q}}}, \boldsymbol{c}_0)$.

B. Dependent source components

In this subsection, we describe our approach of BSS for possible dependent source components. When the source components are dependent, we assume that we have some prior information about the copula density of the random source vector $\boldsymbol{S}$. Note that this is possible for many practical problems. It can be done, from realizations of the random vector source $\boldsymbol{S}$, by a model selection procedure in semiparametric copula density models $\{\boldsymbol{c}_\theta(\cdot);\, \theta \in \Theta \subset \mathbb{R}\}$, typically indexed by a univariate parameter θ, see e.g. [21]. The parameter θ can be estimated using maximum semiparametric likelihood, see e.g. [22], [23]. Denote by $\bar{\theta}$ the obtained value of θ and $\boldsymbol{c}_{\bar{\theta}}(\cdot)$ the copula density describing the dependency structure of the source components. Let $\mathcal{Q} := \{\boldsymbol{q} \in \{-2L, \ldots, 2L\}^p \text{ s.t. } q_1 = 0,\, q_i \neq 0,\, \forall i = 2, \ldots, p, \text{ and } q_i \neq q_j,\, \forall i \neq j\}$. Since the components of the random vector $\boldsymbol{S^q} := (S_1^{q_1}, \ldots, S_p^{q_p})^\top$ are independent when $\boldsymbol{q} \in \mathcal{Q}$, and the components of $\boldsymbol{S}$ are dependent with copula density $\boldsymbol{c}_{\bar{\theta}}(\cdot)$, we propose the following separation criterion

$$\boldsymbol{B} \mapsto \mathcal{J}_{dep}(\boldsymbol{Y^q}) :=\ KL\left(\boldsymbol{c_Y}, \boldsymbol{c}_{\bar{\theta}}\right) + \sum_{\boldsymbol{q}\in\mathcal{Q}} KL\left(\boldsymbol{c_{Y^q}}, \boldsymbol{c}_0\right), \tag{21}$$

where $KL\left(\boldsymbol{c_Y}, \boldsymbol{c}_{\bar{\theta}}\right)$ is the Kullback-Leibler divergence, between the copula density of the vector $\boldsymbol{Y}$ and the copula density of the source components, given by

$$\begin{aligned} KL\left(\boldsymbol{c_Y}, \boldsymbol{c}_{\bar{\theta}}\right) &:= \int_{[0,1]^p} \log\left(\frac{\boldsymbol{c_Y}(\boldsymbol{u})}{\boldsymbol{c}_{\bar{\theta}}(\boldsymbol{u})}\right) \boldsymbol{c_Y}(\boldsymbol{u})\, \mathrm{d}\boldsymbol{u} \\ &= \mathbb{E}\left[\log\left(\frac{\boldsymbol{c_Y}(F_{Y_1}(Y_1), \ldots, F_{Y_p}(Y_p))}{\boldsymbol{c}_{\bar{\theta}}(F_{Y_1}(Y_1), \ldots, F_{Y_p}(Y_p))}\right)\right]. \end{aligned} \tag{22}$$

The criterion function $\boldsymbol{B} \mapsto\ \mathcal{J}_{dep}(\boldsymbol{Y^q})$ is nonnegative and achieves its minimum value zero iff $\boldsymbol{B} = \boldsymbol{A^{-1}}$ (up to filtering and permutation indeterminacies), i.e.,

$$\boldsymbol{A^{-1}} = \arg\inf_{\boldsymbol{B}}\ \mathcal{J}_{dep}(\boldsymbol{Y^q}),$$

provided that, $\boldsymbol{c}_{\bar{\theta}}(\cdot)$, the copula density of $\boldsymbol{S}$ satisfies the following assumption: for any regular filter matrix $\boldsymbol{M}(z)$, if the copula density of $[\boldsymbol{M}(z)]\boldsymbol{S}$ is equal to $\boldsymbol{c}_{\bar{\theta}}(\cdot)$, then $\boldsymbol{M}(z) = \boldsymbol{PH}(z)$, where $\boldsymbol{P}$ a permutation and $\boldsymbol{H}$ a filtering operator. Notice that for the second term in (21), we have $\sum_{\boldsymbol{q}\in\mathcal{Q}} KL\left(\boldsymbol{c_{Y^q}}, \boldsymbol{c}_0\right) = 0$ if $\boldsymbol{Y} = [\boldsymbol{PH}(z)]\boldsymbol{S}$. In fact, if $\boldsymbol{Y} = [\boldsymbol{PH}(z)]\boldsymbol{S}$, then $\boldsymbol{Y^q} = [\boldsymbol{PH}(z)]\boldsymbol{S^q}$, $\forall \boldsymbol{q} \in \mathcal{Q}$, because $\boldsymbol{PH}(z)$ is diagonal. Therefore, the components of $\boldsymbol{Y^q}$ are independent $\forall \boldsymbol{q} \in \mathcal{Q}$, which follows from the independence of the components of $\boldsymbol{S^q}$, $\forall \boldsymbol{q} \in \mathcal{Q}$. Therefore, we propose the following estimate of the separating filter

$$\widehat{\boldsymbol{B}} = \arg\inf_{\boldsymbol{B}}\ \widehat{\mathcal{J}_{dep}}(\boldsymbol{Y^q}), \tag{23}$$

where $\widehat{\mathcal{J}_{dep}}(\boldsymbol{Y^q})$ is the statistical estimate, of the criterion $\mathcal{J}_{dep}(\boldsymbol{Y^q})$, defined by

$$\begin{aligned} \widehat{\mathcal{J}_{dep}}(\boldsymbol{Y^q}) \ &:=\ \widehat{KL}\left(\boldsymbol{c_Y}, \boldsymbol{c}_{\bar{\theta}}\right) + \sum_{\boldsymbol{q}\in\mathcal{Q}} \widehat{KL}\left(\boldsymbol{c_{Y^q}}, \boldsymbol{c}_0\right) \\ &:=\ \frac{1}{N}\sum_{n=1}^{N} \log\left(\frac{\widehat{\boldsymbol{c}}_{\boldsymbol{Y}}(\widehat{F}_{Y_1}(y_1(n)), \ldots, \widehat{F}_{Y_p}(y_p(n)))}{\widehat{\boldsymbol{c}}_{\bar{\theta}}(\widehat{F}_{Y_1}(y_1(n)), \ldots, \widehat{F}_{Y_p}(y_p(n)))}\right) \\ &\quad + \sum_{\boldsymbol{q}\in\mathcal{Q}} \frac{1}{N}\sum_{n=1}^{N} \log(\widehat{\boldsymbol{c}}_{\boldsymbol{Y^q}}(\widehat{F}_{Y_1^{q_1}}(y_1(n-q_1)), \cdots, \\ &\quad \widehat{F}_{Y_p^{q_p}}(y_p(n-q_p)))), \end{aligned}$$

where the estimates of marginal distribution functions $\widehat{F}_{Y_i^{q_i}}(\cdot)$ and copula density $\widehat{\boldsymbol{c}}_{\boldsymbol{Y^q}}(\cdot)$ are defined as above. We obtain then the following estimated sources

$$\widehat{\boldsymbol{y}}(n) = \sum_{k=0}^{L} \widehat{\boldsymbol{B}}_k\, \boldsymbol{x}(n-k),\ \forall n = 1, \ldots, N.$$

We compute the solution $\widehat{\boldsymbol{B}}$ in (23), by a descent gradient algorithm.

Remark 2: The criterion function (21) supposes the knowledge of the copula density model of the source components (with known parameter θ) which is possible, from training samples of the random source vector $\boldsymbol{S}$, by a model selection procedure. However, in many real cases, we have not this information about the source copula density model. Notice that the proposed criterion (21) can be generalized to overcome the more general cases, where the copula density model of the source components and/or the associated parameter are unknown, in a similar way as in [6], [18] for the instantaneous mixtures.

V. Simulation results

In this section, we give some simulation results for the proposed approach. We deal with convolutive mixtures (two mixtures of two sources) of two kinds of sample sources: uniform i.i.d. with independent components presented in Figure 1.a, i.i.d. (with uniform marginals) vector sources with dependent components generated from Frank copula with $\theta = 3.6$ in Figure 2.a. All signals are centered and normalized. For measuring the separation quality, we use the output SNRs defined by

$$SNR_i := 10\log_{10}\left(\frac{\mathbb{E}(y_i^2)}{\mathbb{E}(y_i^2|_{s_i=0})}\right),\ \forall i = 1, 2.$$

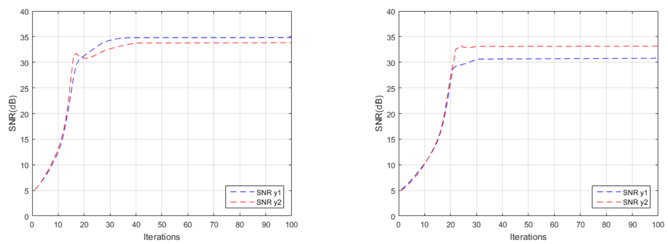
The obtained simulation results will be compared with those obtained in [11] (MI method) under the same conditions, see Figures 1-3. The used mixing system is

$$\boldsymbol{A}(z) = \begin{bmatrix} 1 + 0.2z^{-1} + 0.1z^{-2} & 0.5 + 0.3z^{-1} + 0.1z^{-2} \\ 0.5 + 0.3z^{-1} + 0.1z^{-2} & 1 + 0.2z^{-1} + 0.1z^{-2} \end{bmatrix}.$$

The gradient descent parameter is taken as $\mu = 0.08$ in all cases. We observe from Figures 1.a and 1.b that both methods, the proposed and the MI ones, give good results for the standard case of independent source components. Moreover, we see, from Figure 2.a that our proposed method is able to separate, with good performance, convolutive mixtures of dependent source components. Figure 3.a presents the separation criterion $\widehat{KL}\left(\boldsymbol{c_Y}, \boldsymbol{c}_{\bar{\theta}}\right)$ for Frank copula. We can see from this figure that $\widehat{KL}\left(\boldsymbol{c}_{\boldsymbol{Y}}, \boldsymbol{c}_{\bar{\theta}}\right)$ converge to 0 when the source components are dependent unlike the MI method in Figure 3.b. In summary, when the source components are independent, both methods give equivalent results. However, when the source components are dependent, the MI approach fails while the proposed one is still working with good accuracy.

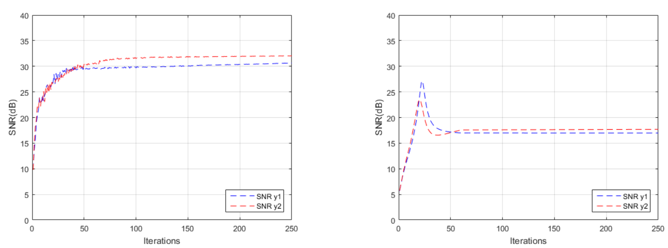
VI. Conclusion

We have proposed a new convolutive blind source separation approach, by minimizing an appropriate separation criterion on copulas, in order to separate linear convolutive mixtures of independent/dependent source components.



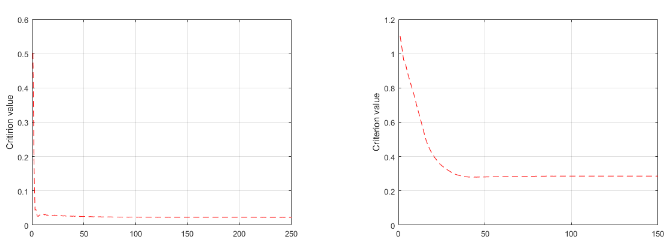
(a) (b)

Fig. 1. SNRs vs iterations with independent sources: (a) The proposed method, (b) The MI one.



(a) (b)

Fig. 2. Separation of dependent sources from Frank copula: (a) The proposed method, (b) The MI one.



(a) (b)

Fig. 3. Separation of dependent sources from Frank copula: (a) $\widehat{KL}\left(\boldsymbol{c}_{\boldsymbol{Y}}, \boldsymbol{c}_{\bar{\theta}}\right)$ (b) The MI method.

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