On Room Impulse Response Measurement Using Orthogonal Periodic Sequences

Alberto Carini
Dept. of Engineering and Architecture
University of Trieste
Trieste, Italy
acarini@units.it

Simone Orcioni
Dept. of Information Engineering
Polytechnic University of Marche
Ancona, Italy
s.orcioni@univpm.it

Stefania Cecchi
Dept. of Information Engineering
Polytechnic University of Marche
Ancona, Italy
s.cecchi@univpm.it

***Abstract*—The paper discusses a measurement method for the room impulse response (RIR) which is robust towards the nonlinearities affecting the power amplifier or the loudspeaker used in the measurement. In the proposed approach, the measurement system is modeled with a Volterra filter. The first order kernel of the Volterra filter, i.e., the linear part, is efficiently determined using orthogonal periodic sequences (OPSs) and the cross-correlation method. The approach shares many similarities with RIR measurements based on perfect periodic sequences (PPSs). In contrast to PPSs, the proposed approach is able to directly measure the impulse response for small signals of the measurement system. Moreover, the input signal can be any periodic persistently exciting sequence and can also be a quantized sequence. Measurements performed on an emulated scenario compare the proposed approach with other competing RIR measurement methods.**

I. Introduction

The knowledge of the room impulse response (RIR) is a key issue in acoustic and audio signal processing. It can be used for analyzing and characterizing the acoustic response of a room, estimating parameters like reverberation time, early decay time, clarity, definition, interaural cross-correlation, lateral energy fraction, etc. [1]. It is also used in the first step of many audio applications, like room response equalization [2], spatial audio rendering [3], virtual sound [4], room geometry inference [5], and others.

In the literature many different approaches for measuring the RIR can be found: from the use of impulsive signals and time stretched pulses, to maximal length sequences (MLSs) [6], perfect periodic sequences (PPSs) for linear systems [7], linear sweeps, exponential sweeps (ESs) [8, 9], perfect sweeps [10], and many others.

The nonlinearities present in the measurement chain can severely affect the performance of many of these approaches. While the acoustic path can be considered as a linear system, the volume used to guarantee a high SNR of the used measurement signal versus noise floor often causes the appearance of nonlinear effects in the power amplifier or in the loudspeaker of the measurement system.

These nonlinear effects are often the cause of artifacts in the obtained response, such as the spikes in the measured RIR using MLSs approach [11]. ESs [8, 9] and synchronized ESs [12] are often used to counteract the effect of measurement system nonlinearities. Indeed, these techniques can be made immune to nonlinearities, provided that the measurement system can be modeled as a memoryless nonlinearity followed by a linear filter, i.e., as a Hammerstein filter [13]. Unfortunately, for nonlinearities with memory, also the measurement with ES technique is affected by artifacts caused by nonlinear distortions [14, 15].

An approach for RIR estimation opposing the effect of nonlinearities was proposed in [16, 17], where the entire measurement chain (power amplifier, loudspeaker, acoustic path and microphone) is modeled as a Legendre nonlinear (LN) filter. These filters are linear combinations of polynomial basis functions orthogonal for white uniform inputs [18] that admit PPSs, i.e., periodic sequences that guarantee the perfect orthogonality of the basis functions over a sequence period. Using a PPS input, the coefficients of the LN filter can be estimated computing the cross-correlation between the system output and the basis functions. In [16, 17], the RIR is estimated extracting with PPSs the first-order kernel (the set of linear term coefficients) of the LN filter modelling the measurement chain. In [19] this approach was extended to Wiener nonlinear (WN) filters, polynomial filters with orthogonal basis functions for white Gaussian inputs, which derive from the truncation of the Wiener nonlinear series.

PPSs are obtained by imposing the orthogonality of the basis functions and solving a system of nonlinear equations with an iterative approach. In [20], a novel family of sequences called orthogonal periodic sequences (OPSs) is introduced. Their main purpose, as for the PPSs, is the identification of functional link polynomial (FLiP) filters, a filter class that includes LN, WN, Volterra, and many other filters. As the PPSs, they allow the perfect estimation of a FLiP filter on a finite time interval with the cross-correlation method. In contrast to PPSs, OPSs can identify also non-orthogonal FLiP filters, as the Volterra filters. With OPSs, the input sequence does not need to be perfect periodic, it can have any distribution, and can also be a quantized sequence. OPSs can be derived solving linear systems and can often identify FLiP filters with a sequence period and a computational complexity much smaller than that of PPSs.

In this paper we propose a methodology to estimate the RIR, robust towards the nonlinearities affecting the measurement

978-9-0827-9703-9/19/$31.00 ©2019 IEEE

chain, using OPSs. In contrast to PPS and exponential sweeps [13], OPSs allow the direct estimate of the first order kernel of the Volterra model, i.e., of the measurement system impulse response for small signals. In Section II, the Volterra filter, used as model of the measurement chain, is presented. It will be identified using a cross-correlation method based on OPS. The RIR estimation methodology is presented in Section III, while experimental results are shown in Section IV. Finally, conclusion is reported in Section V.

II. Volterra filters and OPSs

In the proposed approach the measurement system is modeled as a discrete-time Volterra filter. Volterra filters derive from the double truncation, with respect to order and memory, of the Volterra series [21]. They can arbitrarily well approximate any discrete-time, time invariant, finite memory, continuous nonlinear system. In triangular form, the discrete time Volterra filter, of order K, memory N, has the following input-output relationship:

$$y(n) = h_0 + \sum_{r=1}^{K} \sum_{n_1}^{N-1} \sum_{n_2=n_1}^{N-1} \cdots \sum_{n_r=n_{r-1}}^{N-1} h_{r,n_1,\ldots n_r} \cdot \\ \cdot x(n-n_1)x(n-n_2)...x(n-n_r). \quad (1)$$

Volterra filters are linear combinations of basis functions. Each basis function is a product of delayed input samples and can be written in the following form

$$x(n-n_1)x(n-n_2)\ldots x(n-n_r), \quad (2)$$

where without loss of generality $0 \le n_1 \le n_2 \le \ldots < n_r$, and $r \in \mathbb{N}$. The order of the basis function is r, and its "diagonal number" is, by definition, the maximum time difference between the involved input samples, i.e., $n_r - n_1$. Since natural systems typically have the most relevant coefficients in correspondence to low diagonal numbers, the maximum diagonal number is often conveniently limited. A Volterra filter of order K, memory N, diagonal number D, is the linear combination of all basis functions in (2) with $n_r \le N-1$, of order up to K, and diagonal number up to D.

By properly arranging the basis function according to the so-called diagonal representation [22], Volterra filters can be implemented in the form of a filter bank as follows:

$$y(n) = \sum_{p=0}^{R-1} \sum_{m=0}^{N_p-1} h_p(m) f_p(n-m), \quad (3)$$

where $f_p(m)$ are the zero-lag basis functions: $f_0(n) = 1$, $f_1(n) = x(n)$, and all other basis functions are products of delayed input samples with $x(n)$ as a factor, e.g., $f_2(n) = x^2(n)$, $f_3(n) = x(n)x(n-1)$, ...; R is the number of zero-lag basis functions; N_p is the memory length of the basis function $f_p(n)$, which is N minus the diagonal number of $f_p(n)$.

We are interested in measuring the first order kernel of the Volterra filter, i.e., $h_1(j)$ for $j = 0, ..., N-1$, using OPSs and the cross-correlation method. Let us consider any persistently exciting periodic input sequence $x(n)$ of period L. The sequence is assumed persistently exciting to guarantee the invertibility of the input data matrix introduced in the following. The condition is satisfied when the samples of $x(n)$ have, for example, a white Gaussian, a white uniform, or a pink noise distribution. The sequence $x(n)$ could also be quantized.

Given the periodic input sequence $x(n)$, we want to obtain the corresponding OPS $z(n)$ of period L, suitable for the first order kernel estimation, such that

$$h_1(j) =< y(n)z(n-j) >_L, \quad (4)$$

for all $j = 0, .., N-1$, where $< a(n) >_L$ is the sum of $a(n)$ over a period of L consecutive samples. Thus, the OPS allows the estimation of h_1 simply computing the cross-correlation between the system output $y(n)$ and $z(n)$.

Inserting (3) in (4), it can be proved that the OPS $z(n)$ must satisfy the linear equation system

$$< z(n) >_L = 0, \quad (5)$$
$$< x(n)z(n) >_L = 1, \quad (6)$$
$$< x(n-u)z(n) >_L = 0, \quad (7)$$
$$< f_p(n-v)z(n) >_L = 0, \quad (8)$$

for all $-(N-1) < u \le N-1$ with $u \ne 0$, $-(N-1) < v \le N_p - 1$, and $2 \le p \le R$. The system in (5)–(8) has Q equations and L variables (the samples of $z(n)$), with

$$Q = N_D + (R-1)(N-1), \quad (9)$$

where $R = \binom{D+K}{D+1} + 1$, and total number of coefficients N_D is given by $N_D = \binom{D+K+1}{D+1} + \binom{D+K}{D+1}(N-1-D)$ [23].

For $L > Q$, if the input is persistently exciting the linear system always admits a solution. The system can be written in matrix form,

$$\mathbf{Sz} = \mathbf{d}, \quad (10)$$

where $\mathbf{z}$ is the length L vector formed by the samples of $z(n)$, $\mathbf{d}$ is the length Q vector $[0, 1, 0, ..., 0]^T$, and $\mathbf{S}$ is a $Q \times L$ matrix formed by products of delayed input samples. The system has minimum norm solution

$$\mathbf{z} = \mathbf{S}(\mathbf{SS}^T)^{-1}\mathbf{d}, \quad (11)$$

where the matrix $\mathbf{SS}^T$ elements are cross-correlations of basis functions with different time delays (moments of the input samples over a period). $\mathbf{SS}^T$ can be put in block Toeplitz form by properly sorting the rows of the matrix $\mathbf{S}$. Thus, efficient algorithms exist for the solution of (5)–(8), as for example the algorithm in [24]. Given the nonlinear dependence of the elements of $\mathbf{SS}^T$ from $x(n)$, for some input signal $\mathbf{SS}^T$ could have a bad conditioning. Anyway, working with double precision arithmetic, for sufficiently large L a solution with sufficient accuracy has always been found.

In output noise absence, an input-OPS pair for a Volterra filter of order K, memory N, diagonal number D allows the identification of the first order kernel of any Volterra system up to the order K, memory N, diagonal number D. When the memory of the system to be modeled is $N + \Delta > N$, the first Δ estimated samples of $h_1(j)$ will be affected by an aliasing error. When the order of system if larger than K, or the diagonal number is larger than D, all estimated samples of $h_1(j)$ will be affected by an aliasing error, which will depend on the value of the coefficients of order larger than K, or diagonal number larger than D, and will depend also on the specific input-OPS pair.

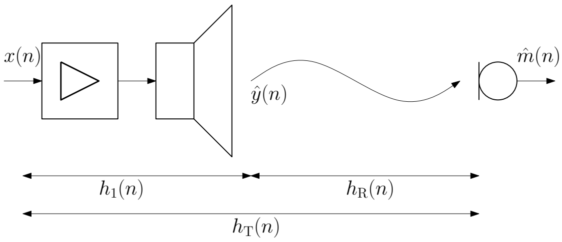

Fig. 1. Typical measurement system.

An error is also present when the output of the Volterra system is corrupted by an additive noise $\nu(n)$. We can estimate the mean-square deviation MSD_j of the coefficient $h_1(j)$ caused by the noise, where

$$\mathrm{MSD}_j = E[(h_1(j) - \tilde{h}_1(j))^2], \tag{12}$$

with $h_1(j)$ the coefficient measured with (4) and $\tilde{h}_1(j)$ its true value. From (4),

$$\mathrm{MSD}_j = E[(< \nu(n) z(n-j) >_L)^2]. \tag{13}$$

MSD_j is proportional to the power of $\nu(n)$, σ_ν^2, and according to (6) is inversely proportional to $< x^2(n) >_L$. To compare different OPSs on equal terms, the noise gain G_ν has been introduced in [20], with

$$G_\nu = \frac{\mathrm{MSD}_j}{\sigma_\nu^2} < x^2(n) >_L . \tag{14}$$

It can be proved that

$$G_\nu =< z^2(n) >_L \cdot < x^2(n) >_L , \tag{15}$$

and is independent from the coefficient index j.

We have found that G_ν can greatly change with L, since the choice of L influences the power of OPS $z(n)$. When $L = Q$, the minimum possible period of the OPS, G_ν often assumes very large values: the effect of noise is highly amplified and OPS identification is useless. On the contrary when $L \gg Q$, G_ν assumes reasonable values and we can take advantage of OPS identification.

III. Robust RIR measurement

Consider a typical scheme of the RIR measurement system, reported in Fig. 1. The system is composed of a power amplifier, a loudspeaker, a room acoustic path, and a microphone. The measurement aims at estimating its impulse response $h_\mathrm{R}(n)$, which is assumed to have length M. The power amplifier and the loudspeaker system at high volumes are often the source of nonlinear effects. The microphone can be considered as a linear system, due to the low level of the acquired signals. Its effect will be neglected in the following or better included in the model of the power amplifier and loudspeaker system. In these conditions, the measurement system of Fig. 1 can be modeled as a Volterra filter of order K, memory $N_\mathrm{T} = N + M - 1$ and diagonal number D. In fact, modeling the amplifier and loudspeaker system with (3) and neglecting $f_0(n) = 1$, the measurement system has the following input-output relationship,

$$\hat{m}(n) = h_\mathrm{R}(n) * \hat{y}(n) = h_\mathrm{R}(n) * \sum_{p=1}^{R-1} h_p(n) * f_p(n) =$$

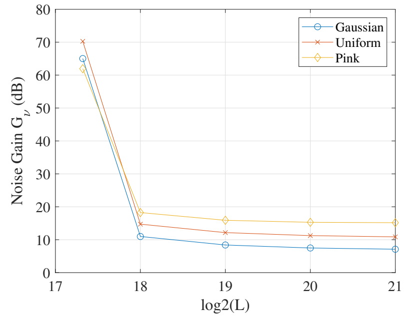

Fig. 2. Noise gain of the OPSs for different period L.

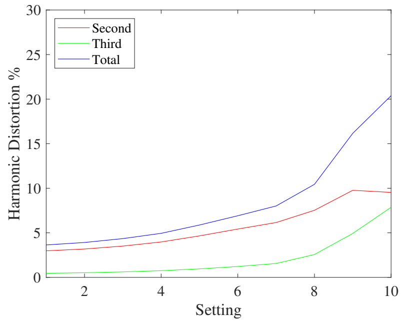

Fig. 3. Seconds, third, and total harmonic distortion.

$$= h_\mathrm{R}(n) * h_1(n) * x(n) + \sum_{p=2}^{R-1} h_\mathrm{R}(n) * h_p(n) * f_p(n). \tag{16}$$

Exploiting an input-OPS pair for Volterra filters of order K, memory N_T, and diagonal number D, the first order kernel of (16), $h_\mathrm{T}(n) = h_\mathrm{R}(n) * h_1(n)$, can be estimated with the cross-correlation method. Using the same input-OPS pair and the same reproduction volume, the first order kernel of the power amplifier and loudspeaker system $h_1(n)$ can be measured and characterized in an anechoic chamber.

As it was proposed for linear systems [25], $h_\mathrm{R}(n)$ can be obtained by equalizing $h_\mathrm{T}(n)$ with the inverse response of $h_1(n)$, exploiting the Kirkeby algorithm, as follows:

$$h_\mathrm{R}(n) = \mathrm{IFFT}\left[\frac{\mathrm{FFT}[h_\mathrm{T}(n)] \cdot \mathrm{FFT}[h_1(n)]^*}{\mathrm{FFT}[h_1(n)] \cdot \mathrm{FFT}[h_1(n)]^* + \epsilon(\omega)}\right], \tag{17}$$

where $\mathrm{FFT}[\cdot]$ and $\mathrm{IFFT}[\cdot]$ are direct and inverse FFT operators, respectively, $\epsilon(\omega)$ is a frequency-dependent regularization parameter. Very often, since the amplifier and loudspeaker affect the measurement in a known, mild manner, it is common to approximate directly $h_\mathrm{R}(n)$ with $h_\mathrm{T}(n)$, which is $h_\mathrm{R}(n) * h_1(n)$. In any case, for the orthogonality properties of OPSs, the measurements of $h_\mathrm{T}(n)$ and $h_\mathrm{R}(n)$ are not affected by the nonlinear kernels of the amplifier and loudspeaker systems, i.e., by $h_p(n)$ for all $p > 1$, provided that an OPS of sufficient order and memory is used. Thus, the proposed RIR measurement system is immune to the nonlinearities of the amplifier and loudspeaker, even when they have memory.

IV. Experimental results

In order to test the robustness towards nonlinearities of OPSs and compare the proposed method with competing approaches, we have considered an emulated scenario. Periodic input sequences with uniform, Gaussian, and pink distribution, having different periods $L = [163820, 2^{18}, 2^{19}, 2^{20}, 2^{21}]$ and quantized at 10 bits, have been applied to a real device, a Behringer MIC 100 vacuum tube preamplifier, at a sampling frequency of 44.1 kHz. For each input sequence, OPSs for the identification of the first order kernel of a Volterra filter with $K = 3$, $N = 8192$, $D = 2$, have been developed. The number of equations of the system in (5)–(8) in these conditions is $Q = 163820$. Figure 2 shows the noise gain in dB for the different OPSs versus the binary logarithm of L. Clearly, when $L = Q$ the noise gain assumes unacceptably large values that makes OPS measurement useless. On the contrary, for larger values of L the noise gain reduces and assumes reasonable values that allow profitable use of the OPSs.

For comparison, PPSs for WN and LN filters (of order 3, $K = 3$, $N = 8192$, D from 0 to 4, $\log_2(L)$ from 17 till 21), MLSs and ESs (with $\log_2(L)$ ranging from 17 to 21) have also been applied to the preamplifier in the same conditions. All input sequences had the same peak amplitude.

In the emulated scenario, the preamplifier implements the nonlinearities introduced by power amplifier and loudspeaker. It has a potentiometer that allows to control the amount of nonlinear distortion introduced. Ten different settings have been considered and Fig. 3 shows the second, third, and total harmonic distortion in percent on a 1 kHz tone at the maximum amplitude of the sequences. Clearly, many of the harmonic distortions of Fig. 3 are larger than those expected in a measurement system, but they have been selected so large to stress the robustness of the proposed approach. The recorded output of the preamplifier has been convolved with a known RIR and a white Gaussian noise has been added to the output to have a signal to noise ratio of 40 dB. The known RIR allows us to calculate the log-spectral distance (LSD) [26, 27] between the measured RIR and its actual value. The LSD is defined in the band $B = [k_1 \frac{F_S}{T}, k_2 \frac{F_S}{T}]$, with k_1 and $k_2 \in \mathbb{N}$, F_S the sampling frequency and T the number of the samples of the discrete Fourier transform (DFT), as follows:

$$\text{LSD} = \sqrt{\frac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} \left[10 \log_{10} \frac{|H_R(k)|^2}{|\hat{H}_R(k)|^2} \right]^2}, \quad (18)$$

where $|H_R(k)|$ is the actual room magnitude response and $|\hat{H}_R(k)|$ is the measured room magnitude response.

Figure 4 shows the LSD in dB, computed in the band $[100, 18000]$ Hz, of the measured RIR without any compensation of the pre-amplifier. The Figure reports results obtained with WN filter and PPS in panel (a), LN filter with PPS in panel (b), MLS based method in panel (c), ES based method in panel (d). The results of the proposed OPS methodology are presented in the panels (e–g) for a Volterra filter and uniform, Gaussian and pink input distribution, respectively. Figure 4 shows comparable results for all methods except the MLS that always performs worse than others. The numerical results of OPS based method, especially with Gaussian or pink inputs, are comparable with PPS, making it an interesting candidate for RIR estimation.

Figure 5 shows the LSD in dB of the measured RIR after the nonlinearity compensation performed with the Kirkeby algorithm (17). Panels are organized as in the previous figure. The best results are obtained with a Gaussian input distribution, i.e., with the WN filter identified with PPSs of panel (a), and with the Volterra filter identified with OPSs of panel (f), demonstrating the effectiveness of the proposed methodology.

V. Conclusion

The paper discussed the application of the recently proposed OPSs [20] to RIR identification. The methodology is the same used in [19], that demonstrated to be robust towards nonlinearities affecting the measurement chain. The OPS based methodology offers similar performance as the PPS one, with all the advantages of OPSs over PPSs. Specifically, OPSs can directly estimate the measurement system impulse response for small signals, i.e., the first order kernel of the Volterra model. They can be derived solving a linear system. More importantly, the persistently exciting periodic input sequence can be arbitrarily chosen, e.g., can also be a pink noise, and can be a quantized sequence.

References

[1] M. Kleiner, *Acoustics and audio technology*. Fort Lauderdale: J. Ross Publishing, 2011.

[2] S. Cecchi, A. Carini, and S. Spors, "Room response equalization–a review," *Applied Sciences*, vol. 8, no. 1, article no. 16, 2018.

[3] H. Hacihabiboglu, E. De Sena, Z. Cvetkovic, J. Johnston, and J. O. Smith III, "Perceptual spatial audio recording, simulation, and rendering: An overview of spatial-audio techniques based on psychoacoustics," *IEEE Signal Processing Magazine*, vol. 34, no. 3, pp. 36–54, 2017.

[4] J. Garas, *Adaptive 3D Sound Systems*. New York: Kluwer Academic Publishers, 2000.

[5] F. Antonacci, J. Filos, M. Thomas, E. Habets, A. Sarti, P. Naylor, and S. Tubaro, "Inference of room geometry from acoustic impulse responses," *IEEE Trans. Audio, Speech and Language Processing*, vol. 20, no. 10, pp. 2683–2695, 2012.

[6] M. R. Schroeder, "Synthesis of low-peak-factor signals and binary sequences with low autocorrelation," *IEEE Trans. on Information Theory*, pp. 85–89, Jan. 1970.

[7] C. Antweiler, "Multi-channel system identification with perfect sequences," in *Advances in Digital Speech Transmission*, R. Martin, U. Heute, and C. Antweiler, Eds., SPIE, John Wiley & Sons, 2008, pp. 171–198.

[8] A. Farina, "Simultaneous measurement of impulse response and distortion with a swept-sine technique," in *Proc. 108 AES Convention*, Paris, France, Feb. 2000.

[9] S. Müller and P. Massarani, "Transfer-function measurement with sweeps," *J. Audio Eng. Soc.*, vol. 49, no. 6, pp. 443–471, 2001.

[10] C. Antweiler and G. Enzner, "Perfect sequence lms for rapid acquisition of continuous-azimuth head related impulse responses," in *Proc. of WASPAA '09*, 2009, pp. 281–284.

[11] M. Wright and J. Vanderkooy, "Comments on -Aspects of MLS measuring systems- and author's reply," *J. Audio Eng. Soc*, vol. 43, no. 1/2, pp. 48, 49, Feb. 1995.

[12] A. Novak, P. Lotton, and L. Simon, "Synchronized swept-sine: Theory, application, and implementation," *J. Audio Eng. Soc*, vol. 63, no. 10, pp. 786–798, 2015.

[13] T. Schmitz and J.-J. Embrechts, "Hammerstein kernels identification by means of a sine sweep technique applied to nonlinear audio devices emulation," *J. Audio Eng. Soc*, vol. 65, no. 9, pp. 696–710, 2017.

[14] D. G. Ćirić, M. Marković, M. Mijić, and D. Šumarac Pavlović, "On the effects of nonlinearities in room impulse response measurements with exponential sweeps," *Applied Acoustics*, vol. 74, no. 3, pp. 375 –382, Mar. 2013.

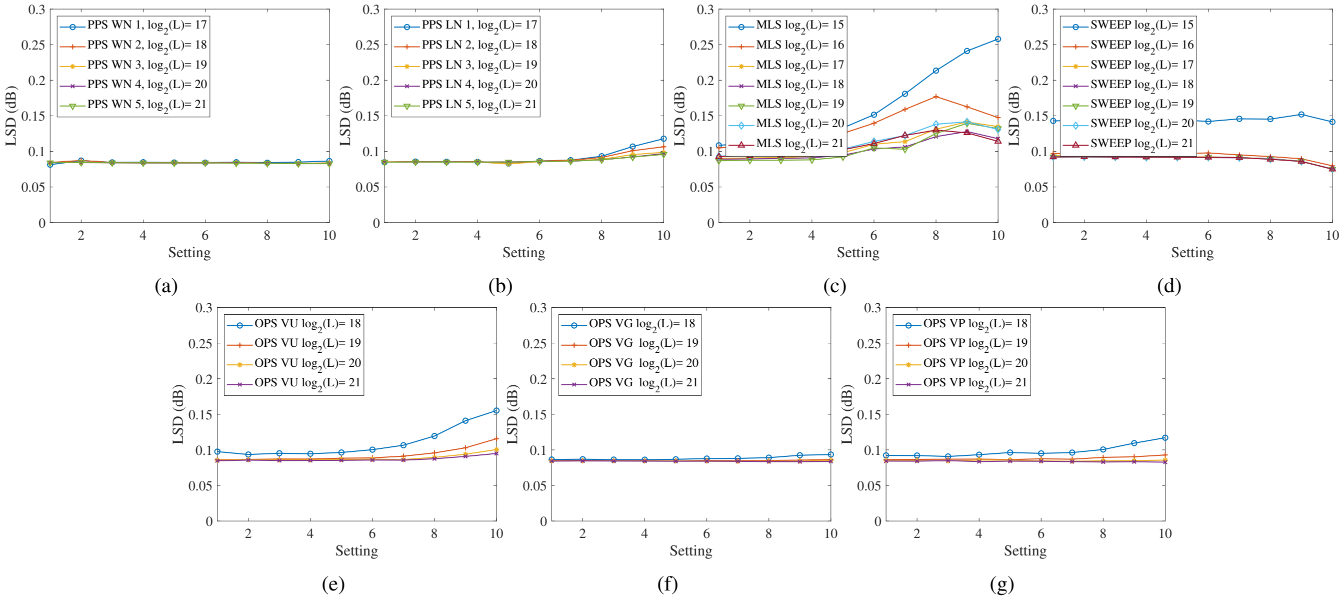

Fig. 4. LSD between measured and real RIRs in band $[100, 18000]$ Hz without pre-amplifier compensation: (a) PPSs for WN filter, (b) PPSs for LN filter, (c) MLSs, (d) ESs, (e) OPSs for Volterra with uniform distribution, (f) OPSs for Volterra with Gaussian distribution, (g) OPSs for Volterra with pink distribution.

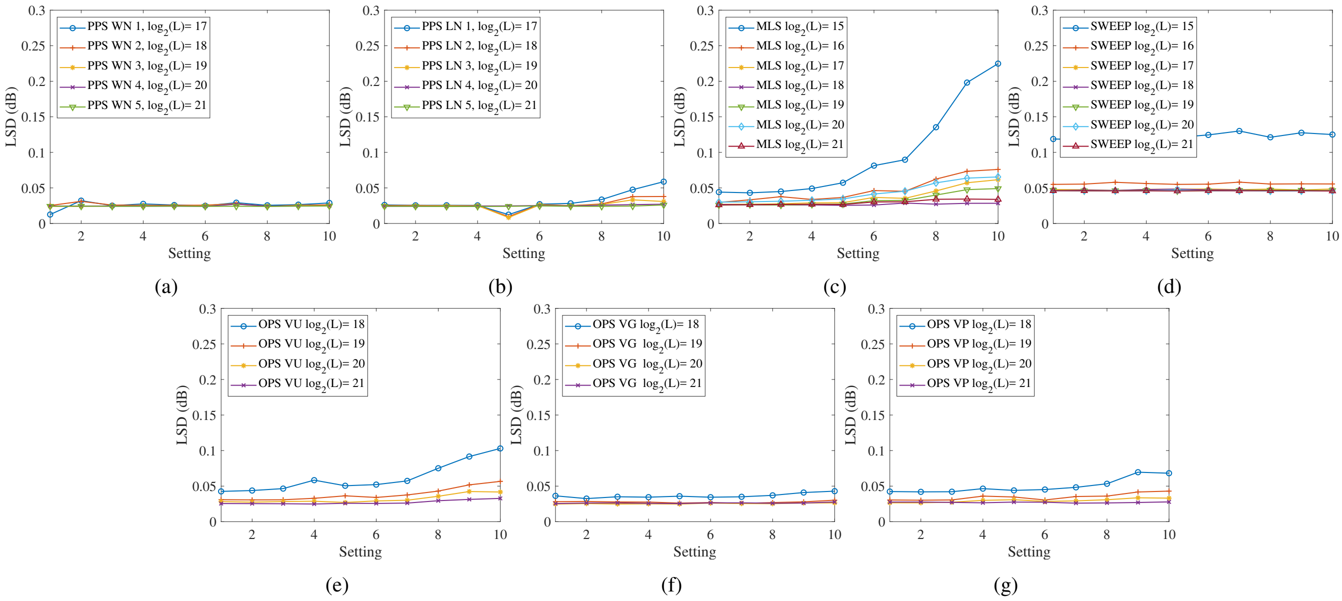

Fig. 5. LSD between measured and real RIRs in band $[100, 18000]$ Hz with pre-amplifier compensation: (a) PPSs for WN filter, (b) PPSs for LN filter, (c) MLSs, (d) ESs, (e) OPSs for Volterra with uniform distribution, (f) OPSs for Volterra with Gaussian distribution, (g) OPSs for Volterra with pink distribution.

[15] A. Torras-Rosell and F. Jacobsen, "A new interpretation of distortion artifacts in sweep measurements," *J. Audio Eng. Soc*, vol. 59, no. 5, pp. 283–289, May 2011.

[16] A. Carini, S. Cecchi, and L. Romoli, "Robust room impulse response measurement using perfect sequences for legendre nonlinear filters," *IEEE/ACM Trans. on Audio, Speech, and Language Processing*, vol. 24, no. 11, pp. 1969–1982, 2016.

[17] ——, "Room impulse response estimation using perfect sequences for Legendre nonlinear filters," in *Proc. of EUSIPCO 2015*, 2015, pp. 2541–2545.

[18] A. Carini, S. Cecchi, L. Romoli, and G. L. Sicuranza, "Legendre nonlinear filters," *Signal Processing*, vol. 109, pp. 84 –94, Apr. 2015.

[19] A. Carini, S. Cecchi, A. Terenzi, and S. Orcioni, "On room impulse response measurement using perfect sequences for wiener nonlinear filters," in *Proc. of EUSIPCO 2018*, 2018, pp. 987–991.

[20] ——, "Introducing orthogonal periodic sequences for the identification of functional link polynomial filters," in *Proc. of ICASSP 2019*, 2019.

[21] V. J. Mathews and G. L. Sicuranza, *Polynomial signal processing*. New York: Wiley, 2000.

[22] G. M. Raz and B. D. V. Veen, "Baseband volterra filters for implementing carrier based nonlinearities," *IEEE Trans. Signal Processing*, vol. 46, no. 1, pp. 103–114, Jan. 1998.

[23] G. L. Sicuranza and A. Carini, "Nonlinear system identification using quasi-perfect periodic sequences," *Signal Processing*, vol. 120, pp. 174–184, Mar. 2016.

[24] G.-O. Glentis and N. Kalouptsidis, "Efficient algorithms for the solution of block linear systems with Toeplitz entries," *Linear Algebra and its Applications*, vol. 179, pp. 85 –104, 1993.

[25] A. Farina, "Advancements in impulse response measurements by sine sweeps," in *Proc. 122 AES Convention*, Vienna, Austria, 2007.

[26] Y. Huang, J Benesty, and J. Chen, *Acoustic mimo signal processing*. Springer, 2006.

[27] S. Bharitkar and C. Kyriakakis, *Immersive audio signal processing*. New York: Springer, 2006.