A New Signal-Selective Wide-Band Ambiguity Function

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***Abstract*—A new signal-selective wide-band cross-ambiguity function is introduced. It performs a sinusoidally weighted correlation of a signal with a time-scaled and delayed version of a reference signal. If the reference signal is almost cyclostationary and the frequency of the weighting sinusoid is one of its cycle frequencies properly scaled, the new function, referred to as the wide-band cyclic cross-correlation function, exhibits the signal selectivity properties that are typical of cyclostationarity-based techniques. The new function is exploited for the localization and speed estimation of a wide-band moving source whose signal impinges on two sensors in a severe noise and interference environment.**

***Index Terms*—cyclostationarity, wide-band ambiguity function, moving source localization**

I. Introduction

In radar/sonar problems, the detection and the range and velocity estimation of a moving source or target lead to consider the narrow-band cross-ambiguity function (NB-CAF) of the received signal and a replica of the transmitted waveform available at the receiver [11, Chap. 10]. The NB-CAF also plays a central role in passive radar/sonar problems when signals are received on two separate sensors in order to estimate source direction and speed [5].

The NB-CAF is a function of delay and frequency shift. It appears in the detector/estimator structure provided that the so called *narrow-band condition* is satisfied. That is, if the product of signal bandwidth and length of the observation interval is much smaller than the ratio of the medium propagation speed and the relative radial speed between the receiver and the target or source [8, Sec. 7.5], [11, Sec. 9.1]. In such a case, the Doppler effect can be modeled just as a frequency shift of the carrier and does not influence the complex envelope of the received signal.

If the narrow-band condition is not satisfied, then a time-stretch in the complex envelope of the received signal must be accounted for and the wide-band cross-ambiguity function (WB-CAF) must be considered for detection/estimation [13]. The WB-CAF is a function of delay and time-scale factor.

In the case of signals received on two sensors, assuming the validity of the narrow-band condition, the time-difference-of-arrival (TDOA) and frequency-difference-of-arrival (FDOA) are estimated by locating the peak of the magnitude of the NB-CAF. Then, from these estimates, the source direction and speed are derived [10]. If the narrow-band condition is not satisfied, then source direction and speed are obtained from the TDOA and the time-scale-ratio (TSR) that are estimated by locating the peak of the magnitude of the WB-CAF [7].

The detector/estimator structures based on the NB-CAF and the WB-CAF are derived by a maximum likelihood (ML) criterion assuming a stationary Gaussian noise model for the additive disturbances. If strong nonstationary interfering signals contaminate the received useful signals, NB-CAF and WB-CAF based estimators perform poorly. In such a case, interference tolerant estimation techniques exploit the signal selectivity properties of cyclostationarity-based algorithms [4], [9]. In passive radar and sonar problems, several techniques have been proposed in [3] for TDOA estimation and in [6] for TDOA and FDOA estimation. In all these techniques the narrow-band condition is assumed to be valid in modeling the received signals.

In this paper, a new signal selective wide-band cross-ambiguity function is introduced. It is called the *wide-band cyclic cross-correlation function* (WB-CCCF) and performs a sinusoidally weighted cross-correlation of one signal and a stretched version of another signal which is called the reference signal.

The WB-CCCF is *wide-band* since a possible wide-band Doppler effect (that is, a time-scale stretch) in the first signal with respect to the second one can be appreciated. The WB-CCCF is *signal selective* since the frequency of the sinusoidal weighting function can be chosen to be a properly scaled cycle frequency of the reference signal if it is almost-cyclostationary (ACS). In such case, the contribution of possible disturbance signals added to the two signals of interest are canceled, even if these disturbance signals are correlated, provided that they do not exhibit joint cyclostationarity at the considered cycle frequency.

As an example of application, the WB-CCCF is exploited to estimate TDOA and TSR of the signal transmitted by a low Earth orbit (LEO) satellite and received by two sensors far apart separated, in the presence of strong possibly intentional interference impinging on both sensors. At low values of the signal-to-interference ratio (SIR), the proposed method significantly outperforms the estimation methods based on the NB-CAF and WB-CAF and a competitive cyclostationarity-based technique.

The paper is organized as follows. The model for the signal

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received from a moving source is presented in Section II. The new WB-CCCF is introduced in Section III and its exploitation for time-scale factor and delay estimation is described in Section IV. The problem of passive wide-band moving source location is addressed in Section V. Numerical results are presented in Section VI and conclusions are drawn in Section VII.

II. Received Signal from a Wide-Band Moving Source

In this section, the model for the received signals coming from a wide-band moving source and impinging on two sensors is described. This model constitutes the motivation to introduce the WB-CCCF in the next section.

Let $\widetilde{x}(t)$ be the complex envelope signal emitted by a wide-band source with bandwidth B and carrier frequency f_c in relative motion with respect to two fixed sensors and let $\widetilde{r}_1(t)$ and $\widetilde{r}_2(t)$ be the received complex envelope signals on the two sensors (Fig. 1). If the relative radial speeds $\mathrm{v}_i, i = 1, 2,$ between the moving source and each sensor can be considered constant within the observation interval, we have [8, Sec. 7.3.3]

$$\widetilde{r}_i(t) = \widetilde{y}_i(t) + \widetilde{n}_i(t) \qquad i = 1, 2 \tag{1}$$

with

$$\widetilde{y}_i(t) = a_i\, \widetilde{x}(s_i(t - \tau_i))\, e^{j2\pi\nu_i t}\,. \tag{2}$$

In (1), $\widetilde{y}_i(t)$ is the useful signal and $\widetilde{n}_i(t)$ is the disturbance signal. In (2), the time-scale factor $s_i = 1 - \mathrm{v}_i/c$ and the frequency shift $\nu_i = (s_i - 1)f_c$ describe the Doppler effect, τ_i is the propagation delay, and a_i is a complex gain that accounts for propagation attenuation and phase shift.

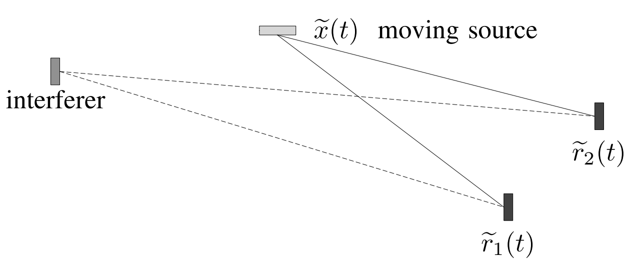

Fig. 1. A moving source transmits the signal $\widetilde{x}(t)$. Two sensors receive the signals $\widetilde{r}_i(t)$, $i = 1, 2,$ containing also correlated interference.

If the so called *narrow-band condition*

$$BT \ll \frac{c}{|\mathrm{v}_i|} = \frac{1}{|1 - s_i|} \tag{3}$$

holds, where c is the medium propagation speed and T the duration of the observation interval, then in (2) the time-scale factors s_i can be considered equal to 1 in the arguments of the complex envelopes (but not in the complex exponents) and the Doppler effect reduces to a frequency shift of the carrier, [8, Sec. 7.5] which is a commonly adopted model [11, Sec. 9.1].

If $\widetilde{y}_1(t)$ is assumed as reference signal, then $\widetilde{y}_2(t)$ can be expressed as

$$\widetilde{y}_2(t) = a_\Delta\, \widetilde{y}_1(s_\Delta(t - \tau_\Delta))\, e^{j2\pi\nu_\Delta t} \tag{4}$$

where $s_\Delta \triangleq s_2/s_1$ is the TSR, $\nu_\Delta \triangleq \nu_2 - s_\Delta\nu_1$ is the FDOA, $\tau_\Delta \triangleq \tau_2 - \tau_1/s_\Delta$ is the TDOA, and $a_\Delta \triangleq (a_2/a_1)\, e^{j2\pi\nu_1(s_\Delta\tau_2 - \tau_1)}$ is the complex-gain ratio (CGR).

Starting from estimates of TDOAs τ_Δ taken from several pairs of sensors, parameters to locate the source can be obtained [10]. In addition, starting from estimates of TSRs s_Δ or FDOAs ν_Δ, the source velocity can be estimated [11, Chap. 10], [13].

Eqs. (1) and (2) can be equivalently written in terms of analytic signals as

$$r_i(t) = y_i(t) + n_i(t) \tag{5}$$

where

$$y_i(t) \triangleq \widetilde{y}_i(t)\, e^{j2\pi f_c t} = b_i\, x(s_i(t - \tau_i)) \tag{6}$$

is the analytic signal corresponding to the complex envelope $\widetilde{y}_i(t)$, and similarly $r_i(t)$, $x_i(t)$, and $n_i(t)$. In (6), $b_i \triangleq a_i\, e^{j2\pi f_c s_i \tau_i}$.

If $y_1(t)$ is assumed as reference signal, then $y_2(t)$ can be expressed as

$$y_2(t) = b_\Delta\, y_1(s_\Delta(t - \tau_\Delta)) \tag{7}$$

where $b_\Delta \triangleq a_\Delta\, e^{j2\pi f_c s_\Delta \tau_\Delta}$.

III. The Wide-Band Cyclic Cross-Correlation Function

Let $x(t)$ and $y(t)$ be two complex-valued finite-power signals. The new signal-selective wide-band cross-ambiguity function, dubbed here as the *(conjugate) wide-band cyclic cross-correlation function (WB-CCCF)* is defined as

$$\chi_{yx^{(*)}}(\sigma, \alpha, \tau) \triangleq \left\langle \mathrm{E}\Big\{ y(t)\, x^{(*)}(\sigma(t - \tau)) \Big\}\, e^{-j2\pi\alpha\sigma t} \right\rangle_t \tag{8}$$

where $(*)$ is an optional complex conjugation and $\langle\cdot\rangle_t \triangleq \lim_{T\to\infty}(1/T)\int_{-T/2}^{T/2}(\cdot)\,\mathrm{d}t$. The WB-CCCF is the (conjugate) cyclic cross-correlation function at cycle frequency $\sigma\alpha$ of the signal $y(t)$ ad a time-scaled version of $x(t)$ with time-scale factor σ. Both presence and absence of complex conjugation in (8) are of interest for complex signals.

Let us consider the input and output analytic signals, $x(t)$ and $y(t)$, respectively, of the Doppler channel between transmitter and receiver in relative motion with constant relative radial speed (see (6)):

$$y(t) = b_0\, x(\sigma_0(t - \tau_0))\,. \tag{9}$$

By replacing (9) into (8), we have

$$\chi_{yx^{(*)}}(\sigma, \alpha, \tau) = b_0\, \chi_{xx^{(*)}}\Big(\frac{\sigma}{\sigma_0}, \alpha, \sigma_0(\tau - \tau_0)\Big)\, e^{-j2\pi\alpha\sigma\tau_0} \tag{10}$$

where $\chi_{xx^{(*)}}(\sigma, \alpha, \tau)$ is defined according to (8) with $y \equiv x$ and is referred to as (conjugate) wide-band cyclic autocorrelation function.

If $x(t)$ is ACS, we have

$$\mathrm{E}\Big\{x(t_1)\, x^{(*)}(t_2)\Big\} = \sum_{\alpha\in A} R^{\alpha}_{xx^{(*)}}(t_1 - t_2)\, e^{j2\pi\alpha t_2} \tag{11}$$

where A is the countable set of (conjugate) cycle frequencies and

$$R^{\alpha}_{xx^{(*)}}(\tau) \triangleq \left\langle \mathrm{E}\Big\{x(t+\tau)\, x^{(*)}(t)\Big\}\, e^{-j2\pi\alpha t}\right\rangle_t \tag{12}$$

is the (conjugate) cyclic autocorrelation function of $x(t)$ at (conjugate) cycle frequency α [4]. In such a case, (10) specializes into

$$\chi_{yx^{(*)}}(\sigma,\alpha,\tau) = b_0 R^{\alpha}_{xx^{(*)}}(\sigma_0(\tau-\tau_0))e^{-j2\pi\alpha\sigma_0\tau}\,\delta_{\sigma-\sigma_0} \tag{13}$$

which is nonzero only for $\alpha \in A$ and $\sigma = \sigma_0$. By taking $y \equiv x$, and hence $b_0 = 1$, $\sigma_0 = 1$, and $\tau_0 = 0$ in (9), we have that (13) specializes into

$$\chi_{xx^{(*)}}(\sigma,\alpha,\tau) = R^{\alpha}_{xx^{(*)}}(\tau)\, e^{-j2\pi\alpha\tau}\, \delta_{\sigma-1}\,. \tag{14}$$

A function similar to the WB-CCCF (8) is defined in [12]. The function in [12] however, contains a complex exponential with frequency shift due to the Doppler effect on the carrier frequency of the analyzed signal and not related to a signal cycle frequency in order to obtain signal selectivity.

IV. Time-Stretch Factor and Delay Estimation

In this Section, a procedure for TSR and TDOA estimation is proposed. Note that this procedure is not obtained by an optimum criterion and is an ad hoc estimator. However, it generalizes to the case of finite-power ACS signals the approximately optimum technique proposed in [7] for finite-energy signals.

For every fixed value of σ, the function $\chi_{yx^{(*)}}(\sigma,\alpha,\tau)$ is a (conjugate) cyclic cross-correlation function. If $y(t)$ and $x(\sigma t)$ are jointly ACS, under mild assumptions expressed in terms of summability of cross-cumlants of the processes $y(t)$ and $x(\sigma t)$, a consistent estimate $\widehat{\chi}_{yx^{(*)}}(\sigma,\alpha,\tau)$ of the (conjugate) WB-CCCF (8), obtained by a measurement over a finite observation interval, is given by the (conjugate) cyclic cross-correlogram [8, Sec. 2.4]

$$\widehat{\chi}_{yx^{(*)}}(\sigma,\alpha,\tau) \triangleq \frac{1}{T}\int_{-T/2}^{T/2} y(t)\, x^{(*)}(\sigma(t-\tau))\, e^{-j2\pi\alpha\sigma t}\, \mathrm{d}t\,. \tag{15}$$

Assume that $\sigma_0 \in \mathcal{S}$, where $\mathcal{S} \triangleq [\sigma_{\mathrm{d,min}}, \sigma_{\mathrm{d,max}}] \cup [\sigma_{\mathrm{u,min}}, \sigma_{\mathrm{u,max}}]$ is the double interval of values of σ in which the source time-scale factor has to be searched. The set $\mathcal{S}$ should not contain the point $\sigma = 1$ if signals coming from fixed sources must be filtered out.

Let $\alpha = \alpha_0$ be a known or estimated (conjugate) cycle frequency of $x(t)$ such that the magnitude of the (conjugate) cyclic autocorrelation function $|R^{\alpha_0}_{xx^{(*)}}(\tau)|$ peaks at $\tau = 0$. This condition is verified for at least one cycle frequency by most communication signals [2, Chap. 12].

Accounting for (13), it follows that the parameters σ_0 and τ_0 can be estimated as those that maximize the magnitude of $\widehat{\chi}_{yx^{(*)}}(\sigma,\alpha,\tau)$ when (σ,τ) ranges in the scale-range cell $\mathcal{C} \triangleq \mathcal{S} \times [\tau_{\min}, \tau_{\max}]$. For $\alpha_0 = 0$, such an estimator reduces to that proposed in [7] which is based on the maximization of the magnitude of the WB-CAF.

V. Passive Wide-Band Moving Source Location

With reference to model (5)–(7) for the analytic signals received on the two sensors, and accounting for (10), the WB-CCCF of $r_1(t)$ and $r_2(t)$ with $(*) = *$ is given by

$$\begin{aligned}
&\chi_{r_2 r_1^*}(\sigma,\alpha,\tau)\\
&= \left\langle \mathrm{E}\left\{r_2(t)\, r_1^*(\sigma(t-\tau))\right\}\, e^{-j2\pi\alpha\sigma t}\right\rangle_t\\
&= b_\Delta\, \chi_{y_1 y_1^*}\Big(\frac{\sigma}{s_\Delta}, \alpha, s_\Delta(\tau-\tau_\Delta)\Big)\, e^{-j2\pi\alpha\sigma\tau_\Delta}\\
&\qquad + \chi_{n_2 y_1^*}(\sigma,\alpha,\tau)\\
&\qquad + b_\Delta\, \chi_{y_1 n_1^*}\Big(\frac{\sigma}{s_\Delta}, \alpha, s_\Delta(\tau-\tau_\Delta)\Big)\, e^{-j2\pi\alpha\sigma\tau_\Delta}\\
&\qquad + \chi_{n_2 n_1^*}(\sigma,\alpha,\tau)\,.
\end{aligned} \tag{16}$$

Under the assumption that the disturbance signals $n_1(t)$ and $n_2(t)$ are zero mean and independent of $x(t)$, we have

$$\chi_{n_2 y_1^*}(\sigma,\alpha,\tau) = \chi_{y_1 n_1^*}(\sigma,\alpha,\tau) = 0\,. \tag{17}$$

Under the assumption that $n_1(\sigma t)$ and $n_2(t)$ do not exhibit joint (conjugate) cyclostationarity with (conjugate) cycle frequency $\sigma\alpha$, we have

$$\chi_{n_2 n_1^*}(\sigma,\alpha,\tau) = 0\,. \tag{18}$$

The mild condition (18) means that the Loève bifrequency cross-spectrum [8, Sec. 1.1.2] of $n_1(t)$ and $n_2(t)$ does not have spectral masses concentrated on a line with slope $1/\sigma$ and intercept α in the bifrequency plane (f_1, f_2). In particular, $n_1(t)$ and $n_2(t)$ are not jointly spectrally correlated [8, Chap. 4] such that one of the support curves of the Loève bifrequency cross-spectrum is $f_2 = f_1/\sigma - \alpha$.

Let $x(t)$ be ACS. If conditions (17) and (18) are satisfied, thus from (16), accounting for (13) and (14), we have

$$\begin{aligned}
\chi_{r_2 r_1^*}(\sigma,\alpha,\tau) = b_\Delta\, |b_1|^2\, R^{\alpha/s_1}_{xx^*}\Big(s_1\, s_\Delta(\tau-\tau_\Delta)\Big)\\
e^{-j2\pi\alpha(s_\Delta\tau+\tau_1)}\, \delta_{(\sigma/s_\Delta)-1}
\end{aligned} \tag{19}$$

which is nonzero for $\sigma = s_\Delta$ and $\alpha/s_1 = \alpha_0$, where α_0 is a cycle frequency of $x(t)$.

If the cycle frequency α_0 is such that $|R^{\alpha_0}_{xx^*}(\tau)|$ peaks at $\tau = 0$ we have that

$$(\widehat{\sigma}_\Delta, \widehat{\tau}_\Delta) = \arg\max_{(\sigma,\tau)\in\mathcal{C}} \Big|\widehat{\chi}_{r_2 r_1^*}(\sigma,\alpha,\tau)\Big| \tag{20}$$

provides a TSR and TDOA estimation technique based on noisy measurements on the two sensors, which is inherently immune to the effects of noise and interference, regardless of the temporal and spectral overlap of signal-of-interest and disturbance. The proposed technique is highly tolerant to noise and interference in practice, providing satisfactory performance at arbitrarily low levels of signal-to-noise ratios (SNR) and SIR, provided that a sufficiently large observation interval is considered for the WB-CCCF estimate, the performance being degraded only by the cycle leakage effect [9, Sec. 9]. The smaller are SNR and SIR, the larger data-record length is needed.

Note that classical cyclostationarity-based techniques [3], [6] whose signal models are based on the validity of the narrow-band condition (3) have a lower bound for the minimum SNR and SIR for which satisfactory performance can be achieved. In fact, even if these techniques are inherently tolerant to the effects of disturbances, the data-record length cannot exceed a maximum value which is determined by (3).

VI. Numerical Results

Numerical experiments are carried out to corroborate the effectiveness of the TSR and TDOA estimation technique based on the WB-CCCF in a highly corruptive noise and interference environment and in a moving scenario such that the narrow-band condition (3) is not satisfied.

The moving source is a LEO satellite. Its altitude is $h = 200$ km and the orbital speed is $\mathrm{v}_o = 28061.5$ km h^{-1}.

The transmitted signal is a long-code binary direct-sequence spread-spectrum (DSSS) signal with number of chip per bit $N_c = 64$, chip period $T_c = 0.12\,\mu$s, bit period $T_p = N_c T_c$, and carrier frequency $f_c = 2$ GHz. The chip pulse is rectangular with 50% duty cycle and the approximate bandwidth is $B \simeq 2/T_c = 16.5$ MHz. The complex-envelope received signals are uniformly sampled with sampling frequency $f_s = 4B = 66$ MHz. The data-record length is $T = N_b T_p$, where N_b is the number of processed bits. The distance between sensors is $L = 150$ km and the projection of the satellite on the line determined by the two receivers lies between the two sensors. The values of TSR, FDOA, and TDOA are $s_\Delta = 1 + 1.7922 \cdot 10^{-5}$, $\nu_\Delta = 35.84$ kHz $= 0.0005431/T_s$, and $\tau_\Delta = -0.058068$ ms $= -3832.49 T_s$, where $T_s = 1/f_s$ is the sampling period.

Each disturbance term $n_1(t)$ and $n_2(t)$ contains circular white Gaussian noise (WGN) with SNR = 0 dB in the band $(-f_s/2, f_s/2)$ and interference with SIR = –3 dB. The interference on each sensor is given by the superposition of two pure tones at frequencies $2/T_p$ and $\sqrt{2}/T_p$ and a jamming binary phase-shift keying (BPSK) signal with carrier frequency f_c and symbol period $T_{pi} = T_p/2$. The same interference signal impinges on each sensor with different time-scale, frequency shift, and delay. On each sensor, the power spectrum of the interfering BPSK signal completely overlaps that of the useful DSSS signal. In addition, the interfering terms on the two sensors are correlated.

The cycle frequency $\alpha = s_1 \alpha_0$ is estimated on the first sensor with the technique of [1]. The received signals are sampled after frequency conversion at $f_c' = 0.25\, f_s$.

The proposed method is compared with the classical estimation methods based on the NB-CAF and WB-CAF. Specifically, the FDOA and TDOA are estimated by maximizing with respect to (ν, τ) the magnitude of the NB-CAF [11, Chap. 10]

$$\mathtt{NBCAF}(\nu, \tau) \triangleq \int_{-T/2}^{T/2} \widetilde{r}_2(t)\, \widetilde{r}_1^*(t - \tau)\, e^{-j2\pi\nu t}\, \mathrm{d}t\,. \tag{21}$$

The TSR and TDOA are estimated by maximizing with respect to (σ, τ) the magnitude of the WB-CAF [13]

$$\mathtt{WBCAF}(\sigma, \tau) \triangleq \int_{-T/2}^{T/2} r_2(t)\, r_1^*(\sigma(t - \tau))\, \mathrm{d}t\,. \tag{22}$$

In Figs. 2–4, results for a typical single realization are reported for $N_b = 2^{14}$ symbols of the DSSS signal. Due to the signal selectivity of the WB-CCCF, the magnitude of the WB-CCCF presents a peak centered in a point of the (σ, τ) plane close to the true TSR and TDOA values (Fig.2). In contrast, starting from the magnitudes of the NB-CAF and WB-CAF, wrong estimates are obtained due to the presence of the interfering terms (Figs. 3 and 4). In particular, the peak due to the BPSK interfering signal is higher than the peak of the useful DSSS signal and gives rise to a significant bias in the estimates.

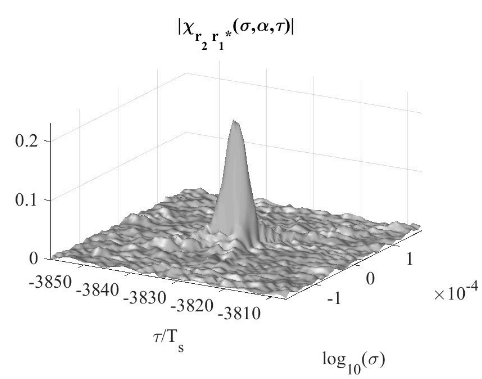

Fig. 2. Magnitude of the WB-CCCF as function of σ and τ/T_s.

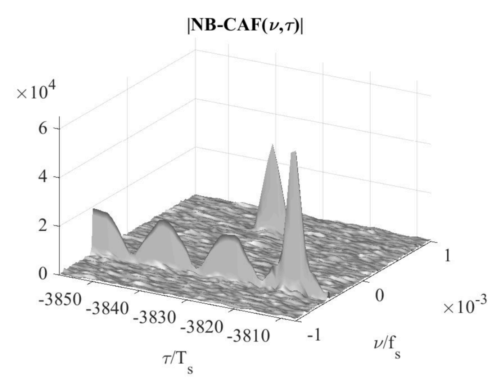

Fig. 3. Magnitude of the NB-CAF as function of ν/f_s and τ/T_s.

The performance analysis is made in the presence of WGN (SNR = 0 dB) and BPSK interference correlated on the two sensors for increasing values of SIR (Fig. 5). For comparison purpose, also the SPECCOA method with compensated frequency shift [6] is considered since it is one of the robustest cyclostationarity-based TDOA and FDOA estimation techniques. The normalized sample root mean-squared error (RMSE) is reported for (top) TSR and (bottom) TDOA estimates. 100 Monte Carlo trials have been carried

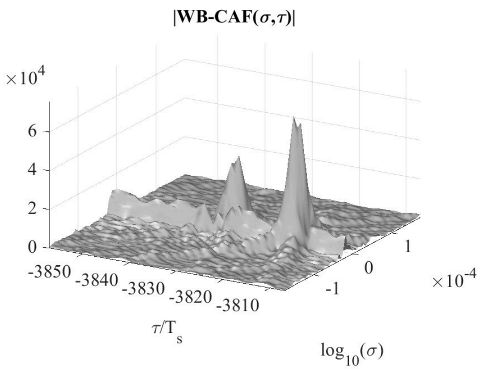

Fig. 4. Magnitude of the WB-CAF as function of σ and τ/T_s.

out. The TSR is considered only for the the WB-CCCF and WB-CAF based methods since only these methods model the Doppler effect in terms of the time-scale factor. The FDOA estimates obtained by the NB-CAF based technique and the SPECCOA with compensated frequency shift technique are not considered since the narrow-band condition is not satisfied. TDOA estimates are reported for all considered methods.

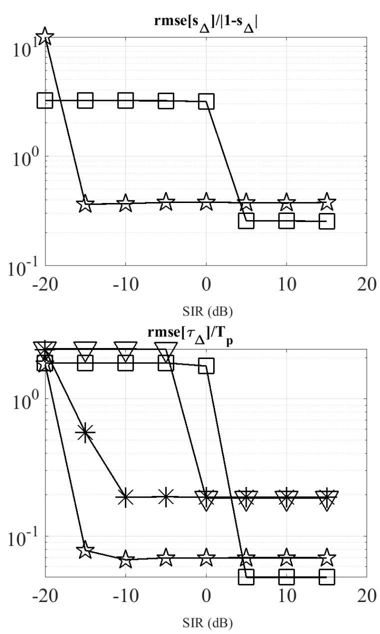

Fig. 5. Normalized RMSE as function of SIR for (top) TSR and (bottom) TDOA. ($\bigtriangledown$) NB-CAF; ($\square$) WB-CAF; ($*$) SPECCOA with compensated frequency shift; ($\star$) WB-CCCF.

At low values of SIR, the proposed WB-CCCF based estimation method has significantly better performance with respect to all competitors due to the signal selectivity of the method. In contrast, for high values of SIR the method based on the WB-CAF has better performance since it is an approximate maximum likelihood estimate in WGN [7]. For high values of SIR, the RMSE of all methods reaches a floor since the data-record length is finite and fixed. The RMSE of SPECCOA and NB-CAF methods is higher since, for the considered value of N_b, the narrow-band condition (3) is not satisfied ($BT = 32768$ and $1/|1 - s_\Delta| \simeq 55795$).

VII. Conclusion

A new signal-selective wide-band cross-ambiguity function is introduced. It is referred to as the wide-band cyclic cross-correlation function (WB-CCCF) since it is the cyclic cross-correlation of a signal and a time-scaled version of another signal named the reference signal. If the two signals are those on two sensors receiving the signal coming from a wide-band moving source, the WB-CCCF can be exploited to estimate the time-difference-of-arrival and the time-scale ratio of the two signals. Form these estimates, possibly obtained by more sensor pairs, the source can be localized and its velocity estimated. The estimator based on the WB-CCCF exhibits the typical signal selectivity properties of cyclostationarity-based signal processing techniques. It is suitable to be exploited when the so called narrow-band condition is not satisfied and, hence, in the presence of large observation intervals, large bandwidths and in high velocity scenarios. Simulation results show that the estimator based on the WB-CCCF outperforms classical estimation methods based on the narrow-band and the wide-band cross-ambiguity functions. In addition, it outperforms classical cyclostationarity-based methods for large data-record lengths when the narrow-band condition is not satisfied.

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