Fractional Programming for Energy Efficient Power Control in Uplink Massive MIMO Systems

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***Abstract*—Recently, massive multiple input multiple output (MIMO) is considered as a promising technology to significantly improve the spectral efficiency (SE) and energy efficiency (EE) of fifth generation (5G) networks. Effective control of the transmit power and other network resources helps to maximize energy efficiency of massive MIMO systems. In this work, an energy efficient power control algorithm is proposed for uplink massive MIMO systems with zero forcing (ZF) detection and imperfect channel state information (CSI) at a base station (BS). By using large system analysis, we first derive closed-form lower bound spectral efficiency expression. Then, by utilizing methods from fractional programming theory, an energy efficient power control algorithm is derived. Numerical results validate the effectiveness of the proposed power control algorithm and show the impacts of maximum transmitter power and minimum rate constraints on energy efficiency maximization.**

***Index Terms*—Massive MIMO, Spectral Efficiency, Energy Efficiency, Fractional Programming Theory.**

I. Introduction

Emergence of various mobile broadband networks and services is causing a substantial increment in mobile data traffic [1]. In order to support this mobile data traffic, improving the spectral efficiency is one of the major recommendations of the forthcoming 5G networks [1], [1], [3]. However, given the dramatic growth of the number of connected devices and networks, improving the spectral efficiency by increasing the transmit power seems not sustainable due to economical, operational and environmental concerns [4]. Hence, along with spectral efficiency, energy efficiency is considered as another critical design criterion for 5G networks [1], [4].

By adopting a very large number of antennas at the BS, which is called massive MIMO [3], 5G networks can greatly improve the spectral efficiency and energy efficiency [3]. Massive MIMO is expected to increase the spectral efficiency $10\times$ and simultaneously improve energy efficiency in the order of $100\times$ as compared to current mobile networks [3].

Recent studies show that optimal allocation of transmit power and other network resources helps to maximize the energy efficiency of massive MIMO systems [4], [6], [7], [11]. In [6], optimal resource allocation is proposed to maximize the energy efficiency in massive MIMO systems. An iterative algorithm is formulated under a realistic power consumption model. Assuming uniform rates for each user, the algorithm aims to achieve optimal energy efficiency in the system. A power control algorithm for energy efficiency maximization in 5G systems is proposed in [9]. The authors develop a unified framework for energy efficiency maximization under transmit power and quality of service (QoS) constraints. A more general energy efficiency optimization algorithm that investigates the hidden monotonic structure of energy efficiency maximization problem is proposed in [10]. By combining fractional programming theory and sequential convex optimization, the authors develop sub-optimal and sequential energy efficient power control algorithms. The results show that an interplay between fractional programming and sequential optimization helps to develop effective energy efficient power control algorithms.

In this work, an energy efficient power control algorithm is proposed in uplink massive MIMO systems with ZF detection and imperfect CSI at the BS. Minimum mean square error (MMSE) based channel estimation is considered at the BS. By using large system analysis, a closed-form lower bound spectral efficiency expression is formulated. Then, by utilizing methods from fractional programming theory, an energy efficient power control algorithm is derived under maximum transmit power and minimum rate constraints at each user. Simulation results are provided to validate and consolidate the theoretical analysis.

The rest of the paper is organized as follows. In Section II, the system model for uplink massive MIMO is presented. Energy efficiency is formulated in Section III. Energy efficient power control algorithm is derived in Section IV. Numerical results are discussed in Section V and conclusions are drawn in Section VI.

II. The Massive MIMO System Model

We consider a single cell uplink massive MIMO system where the BS is equipped with M antennas to serve K single antenna users in the same time frequency resource. Let $\mathbf{x} = \sqrt{\mathbf{p}}\,\mathbf{s}$ denote the complex valued $K \times 1$ transmitted signal from the K users. Then, an $M \times 1$ received signal $\mathbf{y}$ at the BS is give by [5]

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{G}$ represents an $M \times K$ Rayleigh fading channel matrix between the BS and the K users with $\mathrm{g}_{mk} \triangleq [\mathbf{G}]_{mk}$ being the channel coefficient between the mth antenna of the BS and the kth user; $\mathbf{s} = (s_1, s_2, \cdots, s_K)^T$ is the information bearing vector with $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_K$ and $\mathbf{p} = [p_1, p_2, \cdots, p_K] \in \mathbb{R}_+^K$ is the power allocation vector applied to all users. The vector $\mathbf{n}$ represents additive white Gaussian noise at the BS antennas with zero mean and variance σ^2 [8]. In the subsequent section, we further describe the uplink channel model.

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A. Uplink Rayleigh Fading Channel Model

The physical channel between each user and the BS antennas is subjected to pathloss, shadowing and multipath fading effects. Considering all these propagation effects, the channel model for uplink massive MIMO is expressed as [2]

$$\mathbf{G} = \mathbf{H}\mathbf{D}^{\frac{1}{2}} \tag{2}$$

where $\mathbf{D} = \mathrm{diag}\{\beta_1, \beta_2, \cdots, \beta_K\} \in \mathbb{R}^{K\times K}$ is a diagonal matrix and represents the large scale fading (LSF) that show pathloss and shadowing effects with elements $\beta_k = \mathrm{d}_k^{-v}\psi_{_k}$, d_k is the Euclidean distance between the BS antenna and the kth user, v is the pathloss exponent and $\psi_{_k}$ represents a log-normal shadowing [4]. $\mathbf{H} \in \mathbb{C}^{\mathrm{M}\times\mathrm{K}}$ represents the multipath fading effect. We consider a Rayleigh fading channel model in which the elements of $\mathbf{H}$ are assumed to be independent and identically distributed random variables with $\mathcal{CN}(0,1)$ elements [1], [8]. We assume $\mathbf{G}$ to be unknown and MMSE based channel estimation is considered at the BS as reported in Section II-B.

B. MMSE-based Channel Estimation

In a real-world scenario, the true channel matrix $\mathbf{G}$ is unknown and estimated at the BS. To simplify the analysis, we assume that the LSF component is perfectly known both at the BS and the user. Thus, estimation is done only for $\mathbf{H}$. With this assumption, the MMSE based channel estimate of the kth user channel, $\mathbf{g}_k \in \mathbb{C}^{M\times 1}$, is given by [2]

$$\mathbf{g}_k = \hat{\mathbf{g}}_k - \tilde{\mathbf{g}}_k \tag{3}$$

where $\hat{\mathbf{g}}_k$ represents the channel estimate and $\tilde{\mathbf{g}}_k$ represents the channel estimation error. For large MIMO systems, the elements of $\hat{\mathbf{g}}_k$ and $\tilde{\mathbf{g}}_k$ are also modeled to have complex Gaussian distribution as [2]

$$\begin{aligned} \hat{\mathbf{g}}_k &\sim \mathcal{CN}(0, \hat{\beta}_k\mathbf{I}) \\ \tilde{\mathbf{g}}_k &\sim \mathcal{CN}(0, \tilde{\beta}_k\mathbf{I}) \end{aligned} \tag{4}$$

where $\hat{\beta}_k = \frac{\tau_p\rho_p\beta_k^2}{1+\tau_p\rho_p\beta_k}$ is the variance of the channel estimate, $\tilde{\beta}_k = \frac{\beta_k}{1+\tau_p\rho_p\beta_k}$ is the variance of the channel estimation error, $\tau_p \geq K$ is the pilot symbol length of the users per coherence interval [2], ρ_p represents the normalized transmit signal-to-noise ratio(SNR) of the pilot symbol and $\mathbf{I}$ is the identity matrix.

C. Spectral Efficiency in Massive MIMO Systems

Let an $M \times K$ matrix $\mathbf{W}$ be the model for a ZF detector which depends on the channel estimate, $\hat{\mathbf{G}}$, and is given by $\mathbf{W} = \hat{\mathbf{G}}(\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1}$, then the BS processes the received vector by premultiplying (1) with $\mathbf{W}^{\mathrm{H}}$ as [8]

$$\hat{\mathbf{x}} = \mathbf{W}^{\mathrm{H}}\mathbf{y} = \mathbf{W}^{\mathrm{H}}\mathbf{G}\mathbf{x} + \mathbf{W}^{\mathrm{H}}\mathbf{n}. \tag{5}$$

Then, the detected signal for user k is given by [8]

$$\begin{aligned} \hat{x}_{_k} = &\sqrt{p_k}\,\mathbb{E}\{\mathbf{w}_k^H\mathbf{g}_k\}s_k + \sum_{i\neq k}^{K} p_i\mathbf{w}_k^H\mathbf{g}_i s_i \\ &+ \sqrt{p_k}\big(\mathbf{w}_k^H\mathbf{g}_k - \mathbb{E}\{\mathbf{w}_k^H\mathbf{g}_k\}\big)s_k + \mathbf{w}_k^H\mathbf{n} \end{aligned} \tag{6}$$

where $\mathbf{w}_k$ is the detector for user k. The signal to interference plus noise ratio (SINR) of the kth user is expressed as [8]

$$\gamma_{_k}(\mathbf{p}) = \frac{p_k|\mathbb{E}\{\mathbf{w}_k^H\mathbf{g}_k\}|^2}{\sum_{i=1}^{K} p_i\mathbb{E}\{|\mathbf{w}_k^H\mathbf{g}_i|^2\} - p_k|\mathbb{E}\{\mathbf{w}_k^H\mathbf{g}_k\}|^2 + \sigma^2\mathbb{E}||\mathbf{w}_k||^2}. \tag{7}$$

With SINR in (7), the uplink achievable sum rate of the system is given by [8]

$$R_s = B\big(1 - \frac{\tau_p}{T}\big)\sum_{k=1}^{K}\log_2(1 + \gamma_{_k}(\mathbf{p})) \tag{8}$$

where B is the bandwidth of the system, $(1 - \frac{\tau_p}{T})$ accounts the pilot overhead and T is the coherence interval [6]. The achievable sum rate in (8) is calculated by approximating the overall interference as additive Gaussian noise. Since the overall interference is the sum of a Gaussian distributed terms that characterizes the channel estimation error and a number of independent multiuser interference terms, the central limit theorem guarantees the accuracy of this approximation, especially for systems with large number of BS antennas [8]. Thus, (8) is expected to be sufficiently tight and tractable to derive analytical closed-form spectral efficiency expression [8].

D. Asymptotic Spectral Efficiency Formulation

Assuming large number of BS antennas and users, an asymptotic achievable sum rate is derived from (7) and formulated in Theorem 2.1.

Theorem 2.1: When both the number of BS antennas and the users become very large and satisfy $M \geq K + 1$, a closed-form lower bound expression for the uplink achievable sum rate is given by

$$R_s \triangleq B\big(1 - \frac{\tau_p}{T}\big)\sum_{k=1}^{K}\log_2\left(1 + \frac{p_k(M-K)\hat{\beta}_k}{\sum_{j=1}^{K} p_j\tilde{\beta}_j + \sigma^2}\right) \tag{9}$$

where $\hat{\beta}_k$ and $\tilde{\beta}_j$ are the variance of the channel estimate and the channel estimation error, respectively that are given in (4). *The proof is omitted due to space limitation.*

The result in Theorem 2.1 shows that the spectral efficiency depends only on the LSF channel coefficients and the system parameters. As a result, complicated signal processing that involves large-dimensional matrices from small scale fading channel coefficients is avoided. For easy analysis, the SINR in (9) is represented in compact form as

$$\gamma_{_k}(\mathbf{p}) = \frac{p_k b_k}{\sigma^2 + \sum_{j=1}^{K} p_j w_j} \tag{10}$$

where $b_k = (M-K)\hat{\beta}_k$, $w_j = \tilde{\beta}_j$, and $\mathbf{p} = (p_1, p_2, \cdots, p_{_K})^T$ is the power allocation vector of the users. We use (9) and (10) to formulate the proposed energy efficiency optimization algorithm.

E. Power Consumption Model in Massive MIMO Systems

Accurate modeling of the system power consumption is required to formulate the energy efficiency and to obtain reliable guidelines on energy efficiency optimization with respect to the system parameters [6]. The total power consumption of the proposed uplink massive MIMO system is given by the sum of transmitted power and circuit power (CP) consumption as

$$P_{\text{tot}} = P_{\text{tx}} + P_{\text{cp}} \tag{11}$$

where P_{tx} is the power consumed by the power amplifier.It accounts for the power used for uplink pilot and data transmission which is expressed as

$$P_{\text{tx}} = \big(1 - \frac{\tau_p}{T}\big) \sum_{k=1}^{K} \frac{1}{\eta_k} p_k + \frac{\tau_p}{T} \frac{1}{\eta_k} K p_p \tag{12}$$

where $\eta_k \in (0,1)$ is the power amplifier efficiency of user k. P_{cp} represents the circuit power consumption of the system given by

$$P_{\text{cp}} = \rho_a M + \sigma_{\text{sc}} \bar{R}_0 + \Theta_0 \tag{13}$$

where $\rho_a \triangleq \chi(P_{\text{TC}}+P_{\text{CE}}+P_{\text{LP}})$ denotes the circuit power consumption per BS antenna, χ represents the impact of cooling and other effects at the BS, P_{LP} is the power consumption for linear processing, P_{TC} accounts for the power consumption of transceiver chains and P_{CE} denotes the power consumption for channel estimation [6]. $\sigma_{\text{sc}}\bar{R}_0$ accounts for the power consumption that increases with the uplink data rate with scaling factor σ_{sc}. To simplify our analysis, the rate dependent power consumption is assumed to be fixed [6]. Θ_0 shows a static circuit power consumption and it is mostly assumed as fixed [6]. Finally, plugging (12) and (13) into (11), the total system power consumption is expressed as [9]

$$P_{\text{tot}}(\mathbf{p}) = \sum_{k=1}^{K} \mu_k p_k + P_0 \tag{14}$$

where $\mu_k = (1 - \frac{\tau_p}{T})\frac{1}{\eta_k}$ and P_0 is the sum of the pilot power consumption and circuit power consumption of the system.

III. Energy Efficiency in Massive MIMO Systems

The energy efficiency (in bits/Joule) of a wireless system is commonly defined as a benefit-cost ratio, where the achievable rate is compared with the associated energy consumption of the system [8]. One of the well known established metrics to measure this benefit-cost ratio is the global energy efficiency (GEE) which is given by [6], [11]

$$\text{GEE}(\mathbf{p}) = \frac{B\big(1 - \frac{\tau_p}{T}\big) \sum_{k=1}^{K} \log_2(1 + \gamma_k(\mathbf{p}))}{\sum_{k=1}^{K} \mu_k p_k + P_0}. \tag{15}$$

Based on this expression, the energy efficient power optimization problem is formulated as [9]

$$\begin{aligned} \max_{\mathbf{p}} \ \text{GEE}(\mathbf{p}) &= \frac{B\big(1 - \frac{\tau_p}{T}\big) \sum_{k=1}^{K} \log_2(1 + \gamma_k(\mathbf{p}))}{\sum_{k=1}^{K} \mu_k p_k + P_0} \\ \text{subject to:} &\quad 0 \le p_k \le P_{\max,k} \qquad \forall k \\ &\quad \log_2(1 + \gamma_k(\mathbf{p})) \ge R_{0,k} \ \forall k \end{aligned} \tag{16}$$

where $P_{\max,k}$ is the maximum transmit power constraint and $\log_2(1 + \gamma_k(\mathbf{p})) \ge R_{0,k}$ is the minimum rate requirement (or the QoS constraint) at each user. Due to the nonconcave objective function and the nonconvex QoS constraint, (16) is a nonlinear fractional programming problem and intractable to solve analytically [9], [11]. Therefore, to tackle this challenge, we utilized methods from fractional programming theory such as the Dinkelbach algorithm [12] to solve the problem. The Dinkelbach algorithm is a tool that helps to solve concave-convex fractional programming (CCFP) problems by solving a sequence of an easier problem which converges to the global solution [12].

A. Fractional Programming: Proof of Concept

The idea of the Dinkelbach algorithm is built on the relation between a fractional program

$$\max_{\boldsymbol{x} \in \mathcal{S}} \frac{f(\boldsymbol{x})}{g(\boldsymbol{x})} \tag{17}$$

and an equivalent substructive function

$$F(\lambda) = \max_{\boldsymbol{x} \in \mathcal{S}} \big(f(\boldsymbol{x}) - \lambda g(\boldsymbol{x})\big). \tag{18}$$

where $\mathcal{S}$ denotes the set defined by the constraints and $F(\lambda)$ is an auxiliary function with parameter λ [12]. If we assume that $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are continuous, $g(\boldsymbol{x})$ is positive and $\mathcal{S}$ is compact, then $F(\lambda)$ is existed and continuous. Besides, $F(\lambda)$ is strictly decreasing and has a unique root at λ^*. If we consider $\boldsymbol{x}^* \in \mathcal{S}$ and $\lambda^* = \frac{f(\boldsymbol{x}^*)}{g(\boldsymbol{x}^*)}$, then $\boldsymbol{x}^*$ is a solution of (17) if and only if

$$\boldsymbol{x}^* = \arg\max_{\boldsymbol{x} \in \mathcal{S}} \big(f(\boldsymbol{x}) - \lambda^* g(\boldsymbol{x})\big). \tag{19}$$

As a result, solving a fractional programming problem is equivalent to finding the unique zero of the auxiliary function $F(\lambda)$ which can be done by using the Dinkelbach algorithm formulated in [12].

IV. Energy Efficient Power Control Algorithm Formulation

As stated in Section III-A, fractional programming provides efficient tools to maximize a fractional function when the numerator is a concave function, the denominator is a convex function and the constraint set is convex [12]. But, due to the multiuser interference term in (9), the objective function in (16) does not have a concave numerator. Therefore, finding the global solution of (16) is computationally intensive. To tackle this issue, we first employ a lower-bound on logarithm to approximate the objective function; then we apply the Dinkelbach algorithm to find the optimal solution [7]. Specifically, for all $\gamma_k(\mathbf{p}), \bar{\gamma} \ge 0$, we get the following logarithmic inequality

$$\log_2(1 + \gamma_k(\mathbf{p})) \ge \alpha_k \log_2(\gamma_k(\mathbf{p})) + \beta_k \tag{20}$$

that is tight at $\gamma_k(\mathbf{p}) = \bar{\gamma}$ when α_k and β_k are adaptively calculated as [7]

$$\begin{aligned} \alpha_k &= \frac{\bar{\gamma}}{\bar{\gamma}+1} \\ \beta_k &= \log_2(1+\bar{\gamma}) - \frac{\bar{\gamma}}{\bar{\gamma}+1} \log_2 \bar{\gamma}. \end{aligned} \tag{21}$$

By using the approximation in (21), the lower-bound for the objective function in (16) is reformulated as

$$\text{GEE}(\mathbf{p}) \geq \frac{B\left(1-\frac{\tau_p}{T}\right)\sum_{k=1}^{K}[\alpha_k \log_2(\gamma_k(\mathbf{p}))+\beta_k]}{\sum_{k=1}^{K}\mu_k p_k + P_0}. \quad (22)$$

Assuming $p_k = 2^{q_k}$, where $\mathbf{q} = (q_1, q_2, \cdots, q_K)^T \in \mathbb{R}$, (22) is further simplified as

$$\begin{aligned} h(\mathbf{q}) \triangleq\ & \frac{B\left(1-\frac{\tau_p}{T}\right)\sum_{k=1}^{K}[\alpha_k \log_2(b_k)+\alpha_k q_k+\beta_k]}{\sum_{k=1}^{K}\mu_k 2^{q_k}+P_0} \\ & - \frac{B\left(1-\frac{\tau_p}{T}\right)\sum_{k=1}^{K}[\alpha_k \log_2(\sigma^2+\sum_{i=1}^{K} w_i 2^{q_i})]}{\sum_{k=1}^{K}\mu_k 2^{q_k}+P_0}. \end{aligned} \quad (23)$$

By using (23), the optimization problem in (16) is reformulated as

$$\begin{aligned} & \max_{\mathbf{q}}\ h(\mathbf{q}) \\ & \text{subject to:}\quad 0 \leq 2^{q_k} \leq P_{\max,k} \qquad \forall k \\ & \qquad \log_2(1+\gamma_k(\mathbf{p})) \geq R_{0,k}\ \ \forall k. \end{aligned} \quad (24)$$

It is shown in [7] that for any given α_k and β_k, both the numerator and denominator of (23) are differentiable, and concave and convex in q_k, respectively. Besides, the minimum rate constraint can be reformulated as

$$2^{q_k} b_k + (1-2^{R_{0,k}})\left(\sigma^2 + \sum_{i=1}^{K} w_i 2^{q_i}\right) \geq 0 \quad (25)$$

which is convex in q_k [14]. As a result, (24) is a fractional programming optimization problem which can be solved by means of fractional programming tools [12] such as the Dinkelbach algorithm [12]. Finally, the complete iterative procedure for the proposed energy efficient power optimization is summarized in Algorithm 1.

Algorithm 1 Energy efficient power control algorithm.

A. Initialization:
1. Set maximum iterations N, tolerance ϵ and $n=0$.
2. Initialize the power allocation $\mathbf{p}^{(0)}$ with a feasible value.
3. Set $\bar{\gamma}_k^{(0)} = \gamma_k^{(0)}(\mathbf{p})$ and compute $\alpha_k^{(0)}$ and $\beta_k^{(0)}$ from (21).

B. Iterative Operation:
1: $n = n+1$
2: Solve (24) via Dinkelbach algorithm with $\alpha_k^{(n-1)}$, $\beta_k^{(n-1)}$
3: Set $\mathbf{q}^{(n)} = \arg\max h(\mathbf{q})$ and then $\mathbf{p}^{(n)} = 2^{\mathbf{q}^{(n)}}$.
4: Set $\bar{\gamma}_k^{(n)} = \gamma_k^{(n)}(\mathbf{p})$ and update $\alpha_k^{(n)}$and $\beta_k^{(n)}$from (21).
5: Until converegence of $\mathbf{p}^\star$ or $n = N$.

Output: $\mathbf{p}^\star$

V. Simulation Results and Analysis

We evaluate the performance of the proposed power control algorithm for EE maximization. First, we analyze the accuracy of the closed-form lower bound SE approximation in (9). Then, the impacts of maximum power and minimum rate constraints on EE maximization are analyzed. We assume the pilot is transmitted at maximum power and optimization is done to the power allocation for data transmission. For the simulation, we assume that the users are distributed uniformly in a circular cell of radius $250\,\text{m}$ except for an exclusion zone ($R_{\min} \leq 35\,\text{m}$) near the BS [2]. We use the standard system parameters shown in Table I [6]. The log-normal shadowing standard deviation is $\sigma_{\text{sh}} = 8\,\text{dB}$ and the path loss exponent is $v = 3.8$. All users are assumed to have the same maximum transmit power constraints ($P_{\max,k} = P_{\max}$) and the same minimum rate constraints ($R_{0,k} = R_0$) for all k. We deploy CVX with the MOSEK solver [14] to simulate the system.

TABLE I: Part of the simulation parameters.

Parameter	**Value**	**Parameter**	**Value**
T	200	ρ_a	0.002 W
B	20 MHz	Θ_0	0.8 W
(M, K)	(200 , 10)	$\sigma_{\text{sc}} R_0$	0.4 W
τ_p	10	η_{BS}	0.39
ρ_p	5 ,10	η_{UE}	0.3

Figure 1 compares the closed-form lower bound spectral efficiency in (9) with Monte-Carlo realization in (8). The result shows that the gap between analytical approximation and the simulated values is very small. Thus, it is reasonable to design the proposed energy efficient power optimization algorithm by using this closed-form spectral efficiency approximation.

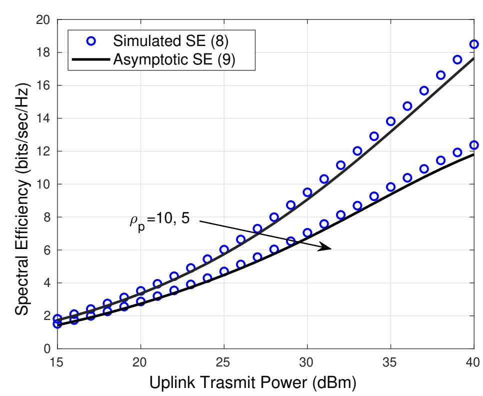

Fig. 1: Spectral efficiency versus uplink transmitter power.

Figure 2 shows the energy efficiency in (15) with the SE in Figure 1. As it is shown, the energy efficiency is a unimodal function [11] with the transmit power and this is the key feature which allows to save energy by energy efficient power allocation.

Figure 3 shows the impact of maximum power constraint on the global EE achieved by the proposed power control algorithm. Simulation results from the equal power allocation algorithm are included as a reference [13]. The result shows that in low transmit power regime, increasing the maximum available transmit power is energy efficient. Whereas, when the transmitter power grows large, the system EE saturates at a certain level. This is because once $P_{\max}$ is large enough to attain maximum EE, the excess fraction of the transmit power is no longer used and increasing the transmitter power further cannot improve the EE.

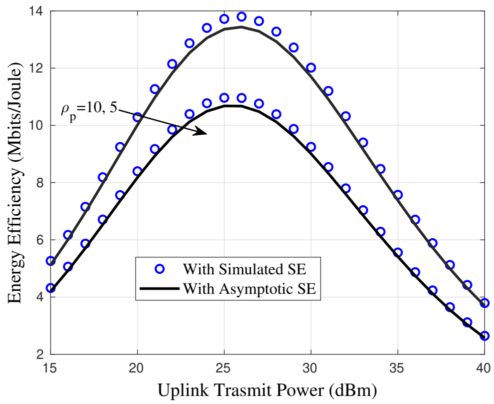

Fig. 2: Energy efficiency versus uplink transmitter power with spectral efficiency in Figure 1.

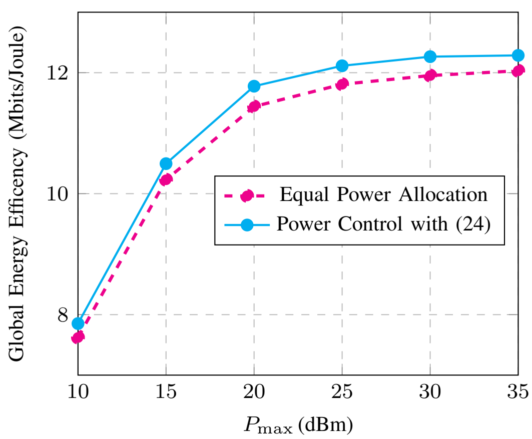

Fig. 3: Impact of maximum transmit power constraint on global energy efficiency.

Finally, Figure 4 shows the impact of the minimum rate constraint on the EE of the proposed power control algorithm. The result shows that when R_0 is small, the global EE remains unchanged. This is because when R_0 takes a small value, the power optimization solution that maximizes the EE can also satisfy the minimum rate requirements of each user. Meanwhile, when R_0 increases, the EE decreases. This is because when the minimum required rate of each user increases, an excess fraction of the power should be allocated to the users that have the worst links to achieve the required rate and which results in a lower EE in the system.

VI. Conclusion

In this work, we have investigated and analyzed an energy efficient power control algorithm in uplink massive MIMO system. To this end, by utilizing tools from fractional programming theory, an energy efficient power control algorithm has been derived. Numerical results have been done to validate the effectiveness of the proposed algorithm. The impacts of transmitter power and minimum rate constraints on global energy efficiency optimization have been analyzed. The results

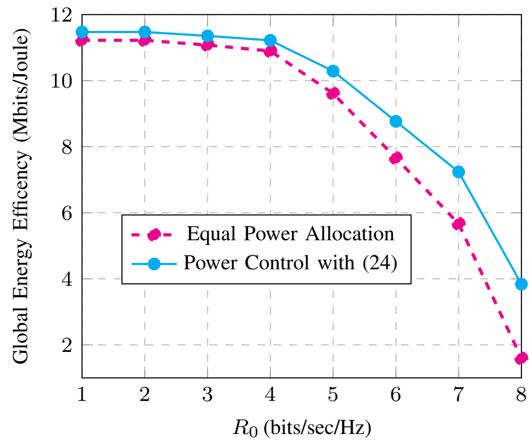

Fig. 4: Impact of minimum rate constraint on global energy efficiency. We assume $P_{\max}$ =20 dBm and $R_{0,k} = R_0 \, \forall k$.

show that the global energy efficiency increases with the maximum power constraint and decreases with the minimum rate constraint.

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