FDA-MIMO Signal Processing for Mainlobe Jammer Suppression

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***Abstract*—Inspired by the fact that frequency diverse array (FDA) generates a time-variant transmit beampattern depending on both range and angle parameters, we propose joint utilization of the FDA and multiple-input multiple-output (MIMO) radar for counteracting mainlobe jamming signals. In doing so, the advantages of FDA and MIMO in range-time-dependent beampattern and increased degrees-of-freedom, respectively, are combined. The proposed algorithm development is based on eigenvalue projection and blocking matrix processing. Theoretical analysis and simulation results for evaluation of the FDA-MIMO framework are provided.**

Keywords*—Frequency diverse array (FDA), FDA-MIMO radar, mainlobe jammer, jammer suppression, blocking matrix, eigenvalue projection.

I. Introduction

Phased-array is an effective technique to deal with sidelobe interferences, but it could less efficiently handle mainlobe jamming interferences due to its only angular-dependent array factor and limited degrees-of-freedom (DoFs). When the jamming signals are correlated with the true target returns and even overlapped with the returns in both time and frequency domains, deceptive targets will be generated by the jammers in conventional processing algorithms [1]. It is even more demanding in modern electronic warfare, especially in the presence of deceptive jamming in the radar mainlobe region [2], [3]. More seriously, the jammer may have the same direction angle with the true target. In this paper, we study an emerging array, named frequency diverse array (FDA) [4], for mainlobe jammer suppression. FDA radar is different from orthogonal frequency division multiplexing (OFDM) radar that emits multi-carrier signals together by a single antenna, and stepped-frequency radar that radiates the signals in a one-by-one way.

Current FDA-related researches focus on range-dependent beampattern synthesis [5], joint range and angle estimation [6], [7], range-dependent interference suppression [8] and range-ambiguous clutter suppression [9]. These applications are attributed to the frequency offsets applied across the array antennas, which generate a range-, angle- and time-dependent beampattern and enable new space-time range adaptive processing [10], not just the traditional space-time adaptive processing in phased-array and multiple-input multiple-output (MIMO) radars.

Additionally, FDA radar receiver signal processing also has received attention in the literature [11]–[13]. Jones and Rigling [11] proposed the band-limited coherent, full-band pseudo-coherent and full-band coherent FDA receiver configurations, which regard the full-band coherent FDA receiver as the most efficient receiver. This receiver architecture was further modified in [12]. Subsequently, multi-carrier matched filtering-based FDA receiver was developed to receive the coherent pulsed-FDA signal [13] and was further proved optimal under the maximum likelihood criterion for general FDA signal in [14].

In [15], we initially analyzed the functionality of FDA in counteracting mainlobe jamming signals, where FDA subarrays are adopted to increase the DOFs. Nevertheless, in this paper, we propose joint utilization of FDA and MIMO for counteracting mainlobe jammers. In doing so, the advantage of FDA in providing range-time-dependent beampattern and coherent array gain and the advantage of MIMO in increasing DOFs are jointly exploited. Moreover, eigenvalue projection and blocking matrix processing are applied in the mainlobe jammer suppression algorithm.

The rest of this paper is organized as follows: Section II presents the blocking matrix processing to suppress FDA radar non-mainlobe jamming signals, while Section III proposes the eigenvalue projection-based algorithm to suppress FDA-MIMO radar mainlobe jamming signals. Finally, simulation results and concluding remarks are provided in Sections IV and V, respectively.

II. Blocking Matrix Processing for FDA Radar Non-Mainlobe Jammer Suppression

According to the FDA radar scheme detailed in [16], we consider a uniform linear FDA radar with N colocated transmit elements spaced by d_t and M colocated receive elements spaced by d_r. Suppose the transmitter and receiver arrays are closely located so that a far-field target can be seen by both of them at the same direction angle.

The signal emitted from the nth FDA element operating with carrier frequency f_0 and frequency offset Δf can be expressed as

$$s_n(t) = \phi(t)e^{j2\pi f_n t},\ 0 \leq t \leq T_p,\ n = 1, 2, \ldots, N \qquad (1)$$

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where $f_n = f_0 + (n-1)\Delta f$, $\phi(t)$ is the complex envelope of the baseband waveform, and T_p is the pulse duration. The received echo signal at the mth element can be written as

$$y_m(t) = \sum_{n=1}^{N} w_{T,n}^{*} \phi\left(t - \tau_{n,T} - \tau_{m,T}\right) e^{j2\pi f_n\left(t-\tau_{n,T}-\tau_{m,T}\right)} \tag{2}$$

where $w_{T,n}$ is the nth transmit element weight, * is the conjugate operator, $\tau_{n,T}$ and $\tau_{m,T}$ are the time delays from the nth transmit element to the target and from the target to the mth receive element, respectively. Under far-field narrow-band assumption and denoting $\tau_0 = \tau_{1,T} + \tau_{1,T}$, we then have

$$\begin{aligned} y_m(t) \approx& \sum_{n=1}^{N} w_{T,n}^{*} \phi\left(t-\tau_0\right) e^{j2\pi f_n\left(t-\tau_{n,T}-\tau_{m,T}\right)} \\ =&\phi\left(t-\tau_0\right) \\ &\times \sum_{n=1}^{N} w_{T,n}^{*} e^{j2\pi\left[f_0+(n-1)\Delta f\right]\left[t-\tau_0+\frac{1}{c_0}d_t(n-1)\sin\theta+\frac{1}{c_0}d_r(m-1)\sin\theta\right]} \\ =&\phi\left(t-\tau_0\right) e^{j2\pi f_0\left(t-\frac{2r}{c_0}\right)} e^{j2\pi f_0\frac{1}{c_0}d_r(m-1)\sin\theta} \\ &\times \sum_{n=1}^{N} w_{T,n}^{*} e^{j2\pi(n-1)\left[\frac{d_t}{\lambda}\sin\theta+\Delta f\left(t-\frac{2r}{c_0}\right)\right]} \end{aligned} \tag{3}$$

where c_0 is the speed of light, r and θ are the range and direction of the target, respectively.

The signals received by all M elements can then be grouped in a matrix form as:

$$\begin{aligned} \mathbf{Y}(t) =& \left[y_1(t), y_2(t), \ldots, y_M(t)\right]^T \\ =&\mathbf{w}_T^H \left[\mathbf{a}_r(r) \odot \mathbf{a}_\theta(\theta) \odot \mathbf{a}_t(t)\right] \mathbf{b}(\theta)\phi(t-\tau_0)e^{j2\pi f_0\left(t-\frac{2r}{c_0}\right)} \end{aligned} \tag{4}$$

where ^{T} is the transpose operator, $\odot$ is the Hadamard product operator, $\mathbf{w}_T = \left[w_{T,1}, w_{T,2}, \ldots, w_{T,N}\right]^T$, $\lambda = \frac{c_0}{f_0}$, and

$$\mathbf{a}_r(r) = \left[1, e^{j\pi\Delta f\frac{4r}{c_0}}, \ldots, e^{j\pi(N-1)\Delta f\frac{4r}{c_0}}\right]^T \tag{5a}$$

$$\mathbf{a}_\theta(\theta) = \left[1, e^{j2\pi f_0\frac{1}{c_0}d_t\sin\theta}, \ldots, e^{j2\pi f_0\frac{1}{c_0}(N-1)d_t\sin\theta}\right]^T \tag{5b}$$

$$\mathbf{a}_t(t) = \left[1, e^{j2\pi\Delta f t}, \ldots, e^{j2\pi(N-1)\Delta f t}\right]^T \tag{5c}$$

$$\mathbf{b}(\theta) = \left[1, e^{j2\pi f_0\frac{1}{c_0}d_r\sin\theta}, \ldots, e^{j2\pi f_0\frac{1}{c_0}(N-1)d_r\sin\theta}\right]^T. \tag{5d}$$

Let the target position be (θ_0, r_0), while the jammer position is (θ_j, r_j). Then (2) can be rewritten as:

$$\begin{aligned} y_m(t) =&\phi(t-\tau_0)e^{j2\pi f_0\left(t-\frac{r_0}{c_0}\right)} e^{j2\pi f_0\frac{1}{c_0}d_r(m-1)\sin\theta_0} \\ &\times \mathbf{w}_T^H \left[\mathbf{a}_r(r_0) \odot \mathbf{a}_\theta(\theta_0) \odot \mathbf{a}_t(t)\right] \\ &+ \phi(t-\tau_j)e^{j2\pi f_0\left(t-\frac{r_j}{c_0}\right)} e^{j2\pi f_0\frac{1}{c_0}d_r(m-1)\sin\theta_j} \\ &\times \mathbf{w}_T^H \left[\mathbf{a}_r(r_j) \odot \mathbf{a}_\theta(\theta_j) \odot \mathbf{a}_t(t)\right] \end{aligned} \tag{6}$$

where ^{H} is the conjugate transpose, τ_j is the corresponding time delay for the jammer defined similarly as τ_0. According to the blocking matrix processing principle [17], we let

$$p = -e^{-j2\pi\frac{f_0}{c_0}d_r\sin\theta_j} \tag{7}$$

to achieve the following jammer cancellation:

$$y_m(t) + p \cdot y_{m+1}(t) = 0. \tag{8}$$

In this case, it is seen that the jamming signals are, in theory, totally cancelled, while the desired target signals are still preserved if $\theta_0 \neq \theta_j$. If both jammer and target are in the mainlobe, the target signal will also be removed in the blocking matrix processing.

Therefore, we design the blocking matrix:

$$\mathbf{B} = \begin{bmatrix} 1 & -e^{-j2\pi\frac{f_0}{c_0}d_r\sin\theta_j} & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & -e^{-j2\pi\frac{f_0}{c_0}d_r\sin\theta_j} \end{bmatrix}. \tag{9}$$

The jamming signals can then be effectively suppressed by the following operation:

$$\mathbf{Y}_b = \mathbf{B}\mathbf{Y}. \tag{10}$$

At this step, traditional beamforming algorithm, minimum variance distortionless response (MVDR) and subspace algorithm, multiple signal classification (MUSIC) can then be applied to estimate the direction-of-arrival (DOA) and other target parameters [18]. Note that, if the jammer and target are located at different range positions, the jamming signal can still be effectively suppressed by designing a range-related blocking matrix, even if the jammer and target are in the mainlobe.

III. Eigenvalue Projection-Based FDA-MIMO Radar Mainlobe Jammer Suppression

The blocking matrix processing method is only suitable for the case of $\theta_0 \neq \theta_j$. Otherwise, the target signals may also be cancelled. To handle this problem, we further propose the eigenvalue projection-based algorithm for FDA-MIMO radar to suppress mainlobe jamming signals.

According to the FDA radar scheme detailed in [8], the emitted signal at the nth element is

$$s_n(t) = \phi_n(t)e^{j2\pi f_n t},\ 0 \leq t \leq T_p,\ n = 1, 2, \ldots, N \tag{11}$$

where $\phi_n(t)$ is the baseband waveform for the nth element. In the receiver, the signals are first down-converted to baseband, and then followed by separately matched filtering. For orthogonal waveforms, under narrow-band assumption, the FDA-MIMO radar received baseband signals can be represented as [19]

$$\mathbf{Y} = \mathbf{a}(\theta_0, r_0) \otimes \mathbf{b}(\theta_0) + \mathbf{a}(\theta_j, r_j) \otimes \mathbf{b}(\theta_j) \tag{12}$$

where $\otimes$ denotes the Kronecker product operator, and

$$\mathbf{a}(\theta, r) = \left[1, e^{j\varphi_1}, e^{j2\varphi_1}, \ldots, e^{j(M-1)\varphi_1}\right]^T \tag{13}$$

with φ_1 being $2\pi\left(\frac{d}{\lambda}\sin\theta - \frac{\Delta f}{c_0}2r\right)$. Note that, here, $d_t = d_r = d$ is assumed in the above equation.

The corresponding transmit and receive spatial frequencies can be expressed, respectively, as

$$f_t = \frac{\varphi_1}{2\pi} = \frac{d}{\lambda}\sin\theta - \frac{2\Delta f}{c_0} r \tag{14a}$$

$$f_r = \frac{\varphi_2}{2\pi} = \frac{d}{\lambda}\sin\theta. \tag{14b}$$

In traditional MIMO radar, the targets will be diagonally distributed in the transmit-receive spectral distribution because of the same transmit and receive spatial frequencies. In contrast, for FDA-MIMO radar, the targets may be arbitrarily distributed in the transmit-receive spectra due to the spatial frequency offset $\frac{2\Delta f}{c_0} r$. This property can be exploited to suppress mainlobe jamming signals.

For simplicity, considering only the jamming signal components, we can formulate the overall receiving jamming signals for the M-element FDA-MIMO receiver as

$$\mathbf{Y}_j = \mathbf{b}(\theta_j) \otimes \mathbf{a}(\theta_j, r_j) = \begin{bmatrix} b_1(\theta_j)\mathbf{a}(\theta_j, r_j) \\ b_2(\theta_j)\mathbf{a}(\theta_j, r_j) \\ \vdots \\ b_M(\theta_j)\mathbf{a}(\theta_j, r_j) \end{bmatrix}. \tag{15}$$

Using the projection matrix

$$\bar{\mathbf{P}} = \mathbf{I} - \frac{\mathbf{a}(\theta_j, r_j)\mathbf{a}^H(\theta_j, r_j)}{\mathbf{a}^H(\theta_j, r_j)\mathbf{a}(\theta_j, r_j)} \tag{16}$$

and defining

$$\mathbf{P} = \begin{bmatrix} \bar{\mathbf{P}} & \bar{\mathbf{P}} & \ldots \bar{\mathbf{P}} \end{bmatrix} \tag{17}$$

with $\mathbf{I}$ being the $M \times M$ identity matrix, we then have

$$\mathbf{P}\mathbf{Y}_j = \begin{bmatrix} \bar{\mathbf{P}} & \bar{\mathbf{P}} & \ldots \bar{\mathbf{P}} \end{bmatrix} \begin{bmatrix} b_1(\theta_j)\mathbf{a}(\theta_j, r_j) \\ b_2(\theta_j)\mathbf{a}(\theta_j, r_j) \\ \vdots \\ b_M(\theta_j)\mathbf{a}(\theta_j, r_j) \end{bmatrix} = \mathbf{0}_{N\times 1} \tag{18a}$$

where $\mathbf{0}_N$ denotes the zero vector. This implies that the jammers are successfully suppressed.

Specifically, if $\varphi_1(t) = \varphi_2(t) = \ldots = \varphi_N(t) = \varphi(t)$, i.e., the same waveform is used for all transmit elements, the FDA-MIMO radar simplifies to the standard FDA radar. In this case, the received signal is modeled as [13]

$$\mathbf{Y} = \xi_0 \left[\mathbf{w}_T^H \mathbf{a}_\theta(\theta)\right] \mathbf{a}_r(r) \otimes \mathbf{b}(\theta) + \mathbf{a}_r(r) \otimes \mathbf{n} \tag{19}$$

where ξ_0 is the target reflection coefficient, and $\mathbf{n}$ denotes the noise vector. Accordingly, the jamming signal can be modeled as

$$\mathbf{Y}_j = \xi_j \left[\mathbf{w}_T^H \mathbf{a}_\theta(\theta_j)\right] \mathbf{a}_r(r) \otimes \mathbf{b}(\theta_j) + \mathbf{a}_r(r_j) \otimes \mathbf{n} \tag{20}$$

where ξ_j denotes the jamming signal strength. Similarly, the projection matrix is

$$\mathbf{P} = \mathbf{I} - \frac{\mathbf{a}_r(r_j)\mathbf{a}_r^H(r_j)}{\mathbf{a}_r^H(r_j)\mathbf{a}_r(r_j)}. \tag{21}$$

Then, we can get

$$\mathbf{P}\mathbf{Y}_j = \xi_j \left[\mathbf{w}_T^H \mathbf{a}_\theta(\theta_j)\right] \mathbf{P}\mathbf{a}_r(r) \otimes \mathbf{b}(\theta_j) + \mathbf{P}\mathbf{a}_r(r_j) \otimes \mathbf{n} = \mathbf{0}_N. \tag{22}$$

Obviously, the jamming signal is also successfully suppressed. Moreover, the target has the range position different from that of the jammer, even if they have the same direction angle (i.e., both jammer and target are in the mainlobe, but they have distinct ranges), the target signal can be successfully preserved in the suppression process.

IV. Simulation Results

In this section, simulation results are provided to compare the FDA-MIMO radar with conventional MIMO radar in mainlobe jammer suppression. We consider an X-band FDA-MIMO radar with carrier frequency $f_0 = 10$ GHz and frequency offset $\Delta f = 20$ kHz. A uniform linear array (ULA) with 10 elements spaced by half wavelength is used for both transmitting and receive functionalities, i.e., $N = M = 10$. The additive noise is modeled as complex Gaussian zero-mean spatially and temporally white random sequence. The signal-to-jammer ratio is assumed to be 0 dB.

Suppose the target and jammer are located at $(10°, 10\text{km})$ and $(10°, 15\text{km})$, respectively. That is, they have the same direction angle but distinct slant ranges. Fig.1 compares the jammer suppression results between the FDA-MIMO radar and MIMO radar. It is seen that, if the target and jammer have the same direction angle but distinct ranges, the FDA-MIMO radar is able to suppress the jammer and localized the target successfully through MVDR beamforming, while the MIMO radar cannot detect the target due to its range-independent target response.

We also test the scenario when the target and jammer have the same range of 10 km and small direction angle difference, namely, 10° and 12°, and the corresponding FDA-MIMO radar results under MVDR beamforming are given in Fig. 2. Although the FDA-MIMO radar still can localize the target and suppress the jammer, mainlobe distortion appears at the target position. This problem can be effectively handled by the eigenvalue projection algorithm, as shown in Fig. 3. It is necessary to note that, the jammer range is required for the eigenvalue algorithm, which is its disadvantage.

V. Conclusion

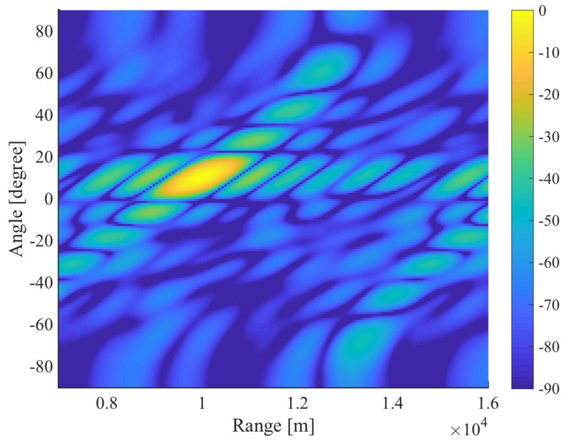
This paper proposed the use of FDA-MIMO radar for mainlobe jammer suppression by jointly utilizing blocking matrix processing and eigenvalue projection algorithms, which are verified by simulation results. Note that prior knowledge of the direction angle to the jammer is assumed and clutter is not considered in this paper. Engineering implementation issues will be further investigated in our future work.

References

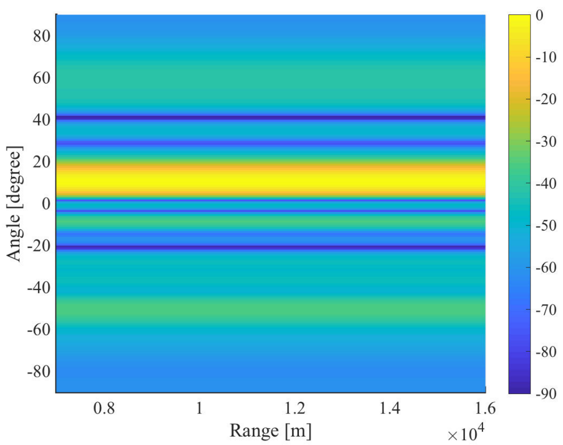
[1] A. Farina, *Electronic Counter-Countermeasure*, Mc-Graw-Hill, Inc, 2008, Chapter 24 of Radar Handbook, 3rd edition.

[2] J. H. Qian and Z. S. He, “Mainlobe interference suppression with eigenprojection algorithm and similarity constraints,” *Electronics Lett.*, vol. 52, no. 2, pp. 228–230, February 2016.

[3] X. P. Yang, Z. G. Zhang, T. Zeng, T. Long, and T. K. Sarkar, “Mainlobe interference suppression based on eigen-projection processing and covariance matrix reconstruction,” *IEEE Antennas Wireless Propag. Lett.*, vol. 13, no. 1, pp. 1369–1372, 2014.



(a)



(b)

Fig. 1: Jammer suppression results for target and jammer at $P_0 = (10°, 10\text{km})$ and $P_j = (10°, 15\text{km})$, respectively: (a) FDA-MIMO radar, (b) MIMO radar.

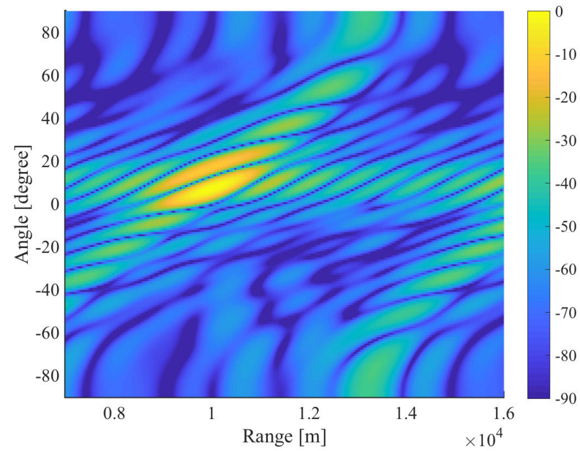


Fig. 2: Jammer suppression results for target and jammer at $P_0 = (10°, 10\text{km})$ and $P_j = (12°, 10\text{km})$, respectively.

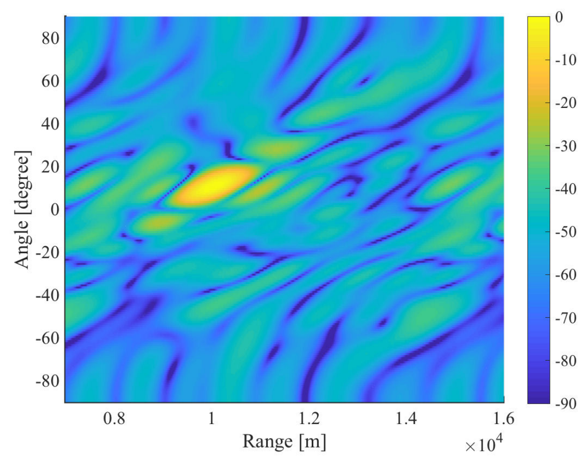


Fig. 3: Eigenvalue projection-based jammer suppression results for target and jammer at $P_0 = (10°, 10\text{km})$ and $P_j = (12°, 10\text{km})$, respectively.

[4] W.-Q. Wang, H. C. So, and A. Farina, "An overview on time/frequency modulated array processing," *IEEE J. Sel. Topics Signal Process.*, vol. 11, no. 2, pp. 228–246, March 2017.

[5] A.-M. Yao, P. Rocca, W. Wu, A. Massa, and D.-G. Fang, "Synthesis of time-modulated frequency diverse arrays for short-range multi-focusing," *IEEE J. Sel. Topics Signal Process.*, vol. 11, no. 2, pp. 282–294, March 2017.

[6] W.-Q. Wang and H. C. So, "Transmit subaperturing for range and angle estimation in frequency diverse array radar," *IEEE Trans. Signal Process.*, vol. 62, no. 8, pp. 2000–2011, April 2014.

[7] C. Cui, J. Xiong, W. Wu, and W.-Q. Wang, "Localization performance analysis of FDA radar receiver with two-stage estimator," *IEEE Trans. Aerosp. Electronic Syst.*, vol. 54, no. 6, pp. 2873–2887, December 2018.

[8] J. Xiong, W.-Q. Wang, and K. D. Gao, "FDA-MIMO radar range-angle estimation: CRLB, MSE, and resolution analysis," *IEEE Trans. Aerosp. Electronic Syst.*, vol. 54, no. 1, pp. 284–294, February 2018.

[9] J. W. Xu, S. Q. Zhu, and G. S. Liao, "Range ambiguous clutter suppression for airborne FDA-STAP radar," *IEEE J. Sel. Topics Signal Process.*, vol. 9, no. 8, pp. 1620–1631, August 2015.

[10] J. W. Xu, S. Q. Zhu, and G. S. Liao, "Space-time-range adaptive processing for airborne radar systems," *IEEE Sensors J.*, vol. 15, no. 3, pp. 1602–1610, March 2015.

[11] A. M. Jones and B. D. Rigling, "Frequency diverse array radar receiver atchitectures," in *Proc. Int. Waveform Diversity & Design Conf.*, Kauai, USA, January 2012, pp. 211–217.

[12] S. Han, C. Y. Fan, and X. T. Huang, "A novel receiver architecture for frequency diverse array radar," in *Proc. Progress in Electromagetic Research Symp.*, Shanghai, China, August 2016, pp. 2270–2274.

[13] R. H. Gui, W.-Q. Wang, C. Cui, and H. C. So, "Coherent pulsed-FDA radar receiver design with time-variance consideration: SINR and CRB analysis," *IEEE Trans. Signal Process.*, vol. 66, no. 1, pp. 200–214, January 2018.

[14] R. H. Gui, W.-Q. Wang, and H. Z. Shao, "General receiver design for FDA radar," in *Proc. IEEE Radar Conf.*, Oklahoma City, USA, April 2018, pp. 280–285.

[15] G. Liu, H. Huang, and W.-Q. Wang, "Frequency diverse aray radar in counteracting mainlobe jamming signals," in *Proc. IEEE Radar Conf.*, Seattle, DC, May 2017, pp. 1–5.

[16] W.-Q. Wang, "Overview of frequency diverse array in radar and navigation applications," *IET Radar, Sonar Navig.*, vol. 10, no. 6, pp. 1001–1012, July 2016.

[17] O. Hoshuyama, A. Sugiyama, and A. Hirano, "A robust adaptive

beamformer for microphone arrays with a blocking matrix using constrained adaptive filters," *IEEE Trans. Signal Process.*, vol. 47, no. 10, pp. 2677–2684, October 1999.

[18] C. Cui, J. Xu, R. H. Gui, W.-Q. Wang, and W. Wu, "Search-free DOD, DOA and range estimation for bistatic FDA-MIMO radar," *IEEE Access*, vol. 6, no. 1, pp. 15431–15445, April 2018.

[19] J. W. Xu, G. S. Liao, S. Q. Zhu, L. Huang, and H. C. So, "Joint range and angle estimation using MIMO radar with frequency diverse array," *IEEE Trans. Signal Process.*, vol. 63, no. 13, pp. 3396–3410, July 2015.