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Fast Direct Detection of Accelerating Radar Targets

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***Abstract*—This paper describes a group theoretic method for the detection of accelerating targets in both active and passive radar applications. The method directly produces a two dimensional *range-Doppler rate* map by utilizing multiple time and frequency shifts of the slow time data to structure the problem as one of detection of a multi-channel unknown rank-one component in noise. Our technique provides considerable computation saving when compared to the optimal method of computing and searching the three dimensional *range-Doppler-Doppler rate* map.**

***Index Terms*—Accelerating radar targets, range-acceleration processing, range-Doppler migration, accelerating target detection.**

I. Introduction

The detection and parameter estimation for accelerating radar targets has a long history. Kelly [1] analysed the problem of detection and parameter estimation of the range, Doppler and Doppler rate of an accelerating radar target, for a general waveform. The paper describes the GLR detector and maximum likelihood estimation and computes the range-Dopple-Doppler rate ambiguity function and what is essentially the Fisher information/Cramer-Rao Bound. As noted by Kelly, the special case of pulsed radar signals had been tackled a little earlier in [2], [3].

In the case of a pulse train in the delay-Doppler narrow band approximation, and where one can neglect range migration during a coherent processing interval, the problem becomes one of detecting a linear frequency modulated signal in the time series consisting of the slow time samples for each specific range. Computing the GLR statistic for an array of Doppler-Doppler rate pair constructs the so-called *chirpogram* or *chirp transform* [4], [5]. The range-Doppler-Doppler rate map, displays the GLR statistic, is a three dimensional function consisting of a two dimensional chirpogram for each range tested. This is a significant computational challenge, although work has been done on making this computation more efficient [6], [7], [8].

It should be noted that there are a large number of methods for the detection of accelerating target based estimation from and subsequent corrections to the range-Doppler map [9], [10], [11], [12]. Here we take a quite different approach.

Suppose that one is interested in detecting those targets in the radar scene which have significant acceleration. That is, to directly construct a range-Doppler rate map, in a direct and computationally efficient way, without the need to search through Doppler values. The purpose of this paper is to describe a method for achieving this. Before, describing our approach, we note that given a detection of an accelerating target at a given range one can de-chirp the slow time series for that range to get an accurate estimate of the targets Doppler.

As noted above, for a given hypothesised range the detection problem becomes one of LFM signal in noise and the statistical optimal detector of a chirp signal in Gaussian white noise is the *chirpogram*, which involves a two-dimensional search over the entire plane of frequencies and chirp rates. A well-known computationally efficient but statistically suboptimal method for detection/estimation of LFM signals is to use the discrete ambiguity function proposed by Peleg and Porat, [13], [14]. This technique has been applied to radar in [15]. The major drawback of this approach are the dominating cross terms which preclude it use at low SNR which is a major drawback in the radar context. Recently we [16] have proposed a technique for the detection of weak LFM signals which involves only a linear search over the range of chirp rates rather than two-dimensional search over the entire plane of frequencies and chirp rates involved in the use of the chirpogram. The technique is based on the fact that an LFM signal of any particular chirp rate is the mutual eigenvector of a set of commuting time and frequency shift operators. This method works at very low SNR and can handle multiple signals and interferers. Here this method is adapted to directly and efficiently compute a range-Doppler rate map for accelerating radar targets.

II. Signal Model

Consider a pulse Doppler radar which transmits a coherent modulated pulse train consisting N_p pulses,

$$s(t) = \sum_{n=0}^{N_p-1} p(t-nT)e^{i\omega_c t}$$

where $p(t)$ denotes the pulse waveform, T is the pulse repetition interval (PRI) and ω_c is the angular carrier frequency. For a point target with constant radial acceleration a, initial range r_0, and initial radial velocity v_0 the returned baseband signal is

$$x(t) = \alpha_0 \sum_{n=0}^{N_p-1} p(t-nT-2r(t)/c)e^{-i\omega_c 2r(t)/c} \quad (1)$$

where the distance from the radar to the target, $r(t)$ is given by

$$r(t) = r_0 + v_0 t + \frac{a}{2}t^2.$$

It is assumed that target motion is such that over a coherent processing interval (CPI),

$$p(t - 2r(t)/c) \approx p(t - 2r_0/c)$$

so the returned signal, (1) can be represented as

$$x(t) = \alpha \sum_{n=0}^{N_p-1} p(t - nT - \frac{2r_0}{c})e^{-i(\omega_d t + \frac{1}{2}\beta_d t^2)} + \nu(t)$$

where $\omega_d = 2\omega_c v_0/c$ is the instantaneous Doppler frequency and $\beta_d = 2\omega_c a/c$ is the Doppler chirp rate. The complex number $\alpha = \alpha_0 e^{2i\omega_c r_0/c}$ represents the amplitude and phase of the target return and $\nu(t)$ is additive Gaussian white noise with zero mean and variance σ^2 per real dimension. With standard Doppler processing the output of the matched filtered baseband return, is sampled for each hypothesized initial range, at delays $2r_0/c, 2r_0/c+T, \cdots, 2r_0/c+N_pT$ to give the slow time series

$$y_n = \alpha e^{i(\omega_d n + \frac{1}{2}\beta_d^2)} + \nu_n,\ n = 0, 1, \ldots, N_p - 1. \quad (2)$$

Thus, for each hypothesised range the problem reduces to detection of a linear frequency modulated signal in noise. The usual Doppler processing assumes that a is negligible and the problem reduces to the detection of a single complex tone in noise. The corresponding GLR detection statistic

$$\gamma(r_0, \omega_d)^2 = \left| \sum_{n=0}^{N_p-1} y_n(r_0)e^{-i\omega T_d n} \right|^2$$

and a plot of this statistic over a suitably chosen discrete set of values for r_0 and ω_d is referred to as a range-Doppler plot. For a fixed value of r_0, $\gamma(r_0, \omega_d)^2$ is the periodogram of the time series $y_n(r_0)$, $n = 0, \cdots, N_p$. If a target at r_0 has an appreciable radial acceleration then the energy of the signal is spread over a range of Doppler frequencies. This reduces the detectability of the target.

When the targets have non-negligible radial accelerations then it is necessary to work with the model (2). In this case the GLR detection statistic is

$$\gamma(r_0, \omega_d, \beta_d)^2 = \left| \sum_{j=0}^{N_p-1} y_n(r_0)e^{-i(\omega_d T n + \frac{1}{2}\beta_d T^2 n^2)} \right|^2.$$

Now a fixed value of r_0, $\gamma(r_0, \omega_d, \beta_d)^2$ is the so-called chirpogram of the slow time series $y_n(r_0)$ and overall the range-Doppler is replaced by the range-Doppler-Doppler chirp rate plot. This is a three dimensional function. and takes significantly more computation than the corresponding range-Doppler plot, even if one puts significant effort into efficient computation [7]

III. Range-Acceleration Processing

Recently Sirianunpiboon et al. [16] have proposed a technique for the detection of weak LFM signals which involves only a linear search over the range of chirp rates rather than a two-dimensional search over the entire plane of frequencies and chirp rate involved in the use of the chirpogram. The technique is based on the fact that an LFM signal of any particular chirp rate is the mutual eigenvector of a set of commuting time and frequency shift operators. This method works at very low SNR and can handle multiple signals and interferers.

This section proposes the application of this technique to enable range-acceleration processing, i.e., for each range bin, the technique is applied to (2) to detect the presence of accelerating targets at that range, bypassing the need to search in target velocity. The details of the technique is given in [16], however for completeness, we first briefly describe the technique in order to provide a background for this application.

A. A Group Invariance Approach to LFM Signal Detection

Consider an LFM signal in noise,

$$y(t) = \alpha s^{\omega,\beta}(t) + \nu(t) \quad (3)$$

where $s^{\omega,\beta}(t) = e^{i(\omega t + \frac{\beta}{2}t^2)}$ represents an LFM signal at time t. The signal model here is to be compared with (2). Consider the time-shift operator $\mathcal{T}(\tau)$ and the demodulation operator $\mathcal{D}(\omega)$ which are respectively defined by

$$(\mathcal{T}(\tau)\psi)(t) = \psi(t + \tau)$$
$$(\mathcal{D}(\omega)\psi)(t) = \psi(t)e^{-i\omega t}.$$

The set of operators

$$A_\beta = \{\mathcal{D}(\beta\tau)\mathcal{T}(\tau) | \tau \in \mathbb{R}\}$$

is a subgroup of the Heisenberg-Weyl group, for which all of the operators commute. This implies that the entire subgroup of operators have common eigenvectors. The action of this subgroup on the LFM signal $s^{\omega,\beta}(t)$ is given by

$$\mathcal{D}(\beta\tau)\mathcal{T}(\tau)s^{\omega,\beta}(t) = e^{i(\omega\tau + \frac{\beta}{2}\tau^2)}s^{\omega,\beta}(t).$$

Thus $s^{\omega,\tau}(t)$ is an eigenvector of the operator $\mathcal{D}(\beta\tau)\mathcal{T}(\tau)$ with the eigenvalue $e^{i(\omega\tau + \frac{\beta}{2}\tau^2)}$. Choose a set

$$\{\mathcal{D}(\beta\tau_j)\mathcal{T}(\tau_j) | j = 1, \ldots, M\} \quad (4)$$

of M operators in A_β. The eigenvector property implies that the set of operators (4) preserves the signal $s^{\omega,\beta}$ merely multiplying it by a constant phase. Note that this property will only be true for LFM signals with chirp rate β. Applying the set of operators (4) to a noisy LFM signal as given in (3), we obtain

$$\mathcal{D}(\beta\tau_j)\mathcal{T}(\tau_j)y(t) = \alpha e^{i(\omega\tau_j + \frac{\beta}{2}\tau_j^2)}s^{\omega,\beta}(t) + \nu_j(t).$$

In discrete time this becomes

$$\boldsymbol{y}_j = A_j\boldsymbol{s} + \boldsymbol{\nu}_j,\ 0 \le j \le M - 1$$

where $\boldsymbol{y}_j = y(n + \tau_j)e^{-i\beta\tau_j n}, 0 \le n \le N - 1$, $A_j = \alpha e^{i(\omega\tau_j + \frac{\beta}{2}\tau_j^2)}$, $\boldsymbol{\nu}_j = \nu(n + \tau_j)e^{-i\beta\tau_j n}$ is the new noise term and $\boldsymbol{s} \in \mathbb{C}^N$ is a vector of sampled chirp signal $s^{\omega,\beta}(t)$. Collecting our newly created channels together we have,

$$\boldsymbol{Y} = A\boldsymbol{s} + \boldsymbol{\nu}$$

where $\boldsymbol{Y} \in \mathbb{C}^{M\times N}$ is the data matrix with each row being $\boldsymbol{y}_j$ and $A \in \mathbb{C}^M$ denotes a vector of unknown complex amplitude with element A_j. The problem now can be formulated as a binary hypothesis test, i.e., for each range bin the detection problem is to test between the following:

$$H_0 : \boldsymbol{Y} = \boldsymbol{\nu}$$
$$H_1 : \boldsymbol{Y} = A\boldsymbol{s} + \boldsymbol{\nu}.$$

The generalised likelihood ratio test (GLRT) for this problem is

$$\lambda_{\max} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

where $\lambda_{\max}$ is the normalised largest eigenvalue of $\boldsymbol{Y}\boldsymbol{Y}^\dagger$.

B. Application to Range-Acceleration Detection

This section summarises a proposed range-acceleration processing for detection of accelerating targets. Consider the matched filter output as given in (2), for each range bin, denoting $\boldsymbol{y} = \{y_n\}_{n=0}^{N_p-1}$.

1) Let $\tau_j, j = 1, \ldots, M$ be a set of chosen time delay, shifting $\boldsymbol{y}$ by τ_j and demodulating with a Doppler(chirp) rate, β to form a data matrix $\boldsymbol{Y}$ of size $M \times N_p$, where each row is

$$\boldsymbol{y}_j = y(n + \tau_j)e^{-i\beta\tau_j n},\ 0 \le n \le N - 1$$

where the vector samples $\boldsymbol{y}_i$ are clipped so that the vectors are all the same length N for all j.

2) Compute the GLRT statistics, i.e., the normalised largest eigenvalue ($\lambda_{\max}$) of $\boldsymbol{Y}\boldsymbol{Y}^\dagger$.

For each range bin, we now obtain GLRT detection statistics as a function of Doppler rate β for data $y_n, n = 0, \ldots, N_p - 1$, that is the range-Doppler rate, or range-radial acceleration map.

C. Computational Aspects

For each hypothesised range one first computes the sequences

$$z_{j\ell}(n) = y(n + \tau_j)\overline{y(n + \tau_\ell)}$$

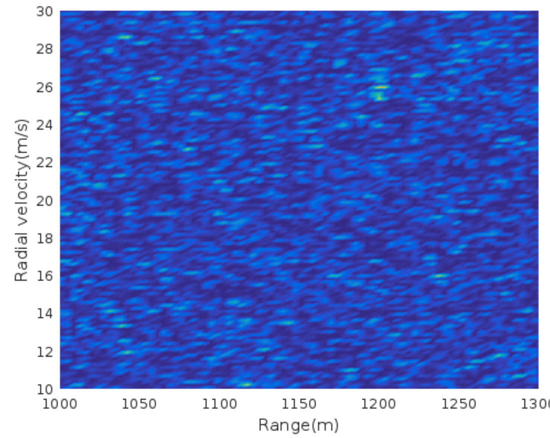
for each $j \ge \ell = 1, \ldots, M$. Choose the equispaced Doppler rates k/B to be considered. The j, ℓ element of the matrix $\boldsymbol{Y}\boldsymbol{Y}^\dagger$ can be computed for all these Doppler rates as

$$[\boldsymbol{Y}\boldsymbol{Y}^\dagger]_{j\ell} = \sum_{j=1}^{M} z_{j\ell}(n)e^{-i(\tau_j-\tau_\ell)kn/B}.$$

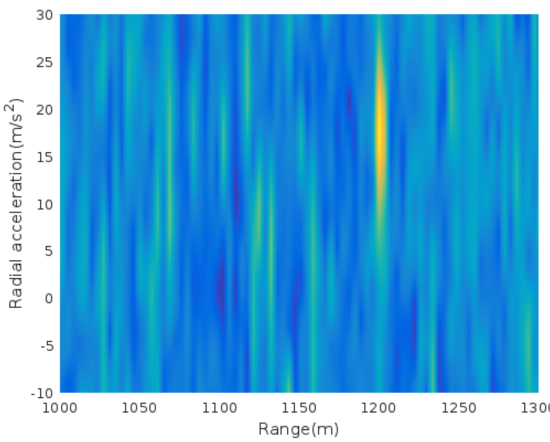
For appropriate B this can be computed efficiently for all Doppler rates at the same time using the chirp-Z transform.

IV. Simulation

This section provides simulation results to demonstrates the performance of the proposed range-acceleration detector. The radar parameters used in the simulations are as follows, the carrier frequency, $f_c = 9.4$ GHz and the bandwidth is 20 MHz. A CPI consist of $N = 512$ pulses at a PRI of 200 μs and pulse width is 5 μs. A set of delays used in the simulation is



(a) Range-Radial Velocity Map



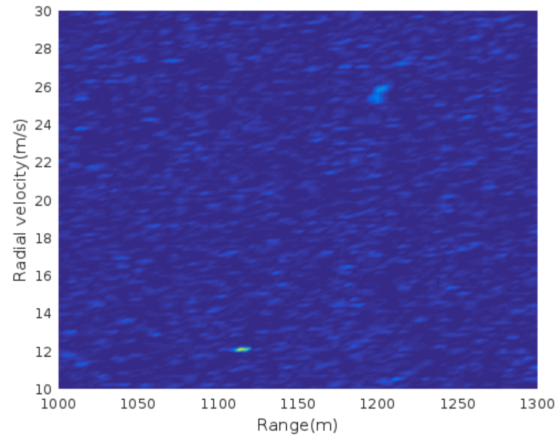
(b) Range-Radial Acceleration Map

Fig. 1: A moving target is at initial range of 1200 m, moving with radial velocity 25 m/s and accelerating at 18 m/s^2.

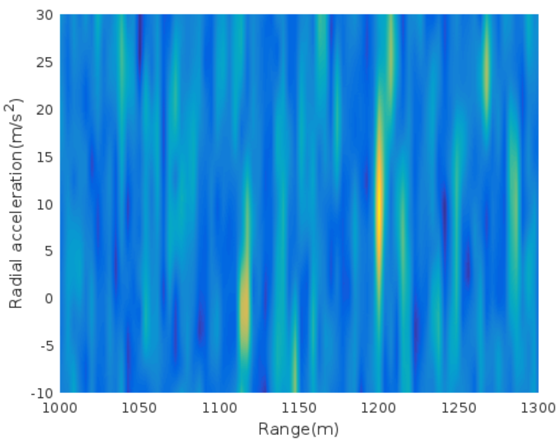
$\{0, 12, 24, 48, 72, 96, 144\}$ and the SNR of the received signal is assumed to be -18 dB. First consider an example with one moving target with initial range at 1200 m, initial velocity at 25 m/s and accelerating at 18 m/s^2. The range-radial velocity and range-radial acceleration maps are shown in Figure 1a and Figure 1b. The range-radial acceleration map clearly shows an accelerating target without range and acceleration migration while the conventional range-Doppler processing results in range and Doppler migration.

The second example consider a scenario where one target is located at 1115 m away, and is moving with constant velocity 12m/sec and another target is at the range of 1200 m, moving with velocity 25 m/s and accelerating at 10 m/s^2. Figure 2a and Figure 2b show the range-radial velocity (Doppler) map and range-radial acceleration map respectively.

The third example consider a scenario when both targets are accelerating. Both targets are located at the same ranges and moving with velocities as in the first example's scenario except the target at 1115 m is accelerating at 13 m/s^2. The range-radial velocity map and range-radial acceleration map

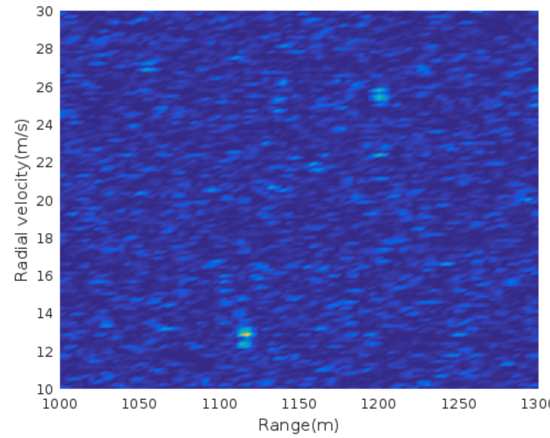


(a) Range-Radial Velocity Map

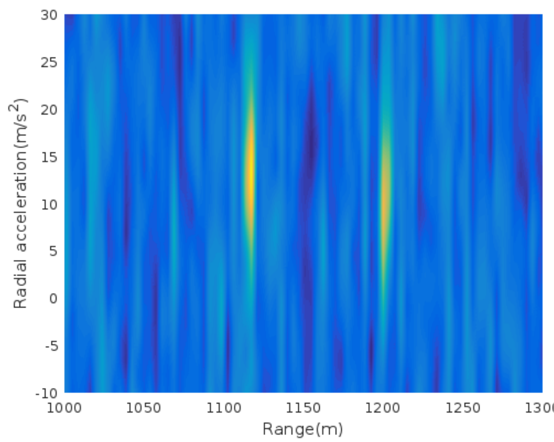


(b) Range-Radial Acceleration Map

Fig. 2: A non-accelerating target with radial velocity 12 m/s and an accelerating target at 10 m/s^2 with radial velocity 25 m/s.



(a) Range-Radial Velocity Map



(b) Range-Radial Acceleration Map

Fig. 3: Two accelerating targets, one at 13 m/s^2 with radial velocity 12 m/s and another target at 10 m/s^2 with radial velocity 25 m/s

are shown in Figure 3a and Figure 3b.

V. Discussion and Conclusions

In this paper a group theoretic technique was developed which allows the direct computation of a range-Doppler rate map that can be used to detect accelerating radar targets much more efficiently than with the optimal range-Doppler-Doppler rate map. A complete analysis of detection performance of the detector will be given in future full exposition. However, a very good indication of its performance can be obtained from by considering the results presented in [16], and upon recalling that for a given range bin the radar detection problem is that of an LFM signal.

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