Switch-Based Hybrid Precoding in mmWave Massive MIMO Systems

Hamed Nosrati*†, Elias Aboutanios*, David Smith†, Xiangrong Wang‡
*School of Electrical and Telecommunications Engineering, University of New South Wales Australia
h.nosrati@unsw.edu.au, elias@unsw.edu.au
†Data61 CSIRO, Australia, david.smith@data61.csiro.au
‡School of Electronic and Information Engineering, Beihang University, Beijing, China , xrwang@buaa.edu.cn

***Abstract*—In mmWave communications, large-scale arrays can be very advantageous. In such arrays, switch-based hybrid precoding (beamforming) is very promising in terms of energy efficiency and reduced complexity, as opposed to phase-shifter structures for beamforming. However, switch-based structures are binary, which means that the design of an optimum beamformer, at large-scale in the analog domain, is a difficult task. We address this problem and propose a new method for the design of a switch-based hybrid precoder for massive MIMO communications in mmWave bands. We first cast the relevant maximization of mutual information as a binary, rank-constrained quadratic maximization, and solve it iteratively for each column of the analog precoder. The solution is then effectively approximated via a set of relaxations and sequential convex programming (SCP). Finally, we show the feasibility, and effectiveness of our method via numerical results.**

***Index Terms*—Hybrid beamforming, Precoding, Millimeter wave communications, Massive MIMO.**

I. Introduction

The smaller wavelengths in mmWave communication systems allow the use of large-scale antenna arrays at the transceivers. In this context, massive multiple-input multiple-output (MIMO) systems are showing great promise for alleviating spectrum congestion in future generation wireless cellular communication systems [1]. However, capitalizing on their potential through fully-digital beamforming faces many challenges as every antenna element must have its own dedicated radio frequency (RF) and baseband chain [2], [3]. The cost and complexity of the hardware chains at mmWave frequencies, along with the associated power consumption, makes fully digital processing undesirable, or even impractical [4]. Therefore, there is a need for efficient implementations of beamforming for massive MIMO systems in mmWave bands that mitigate these problems.

Hybrid beamforming is a well-established approach in massive MIMO communication systems that has received significant attention [5]. It employs a two-stage analog and digital processing configuration where the analog precoding stage, which is simpler and less power-hungry, presents a reduced-dimensional signal to the RF and baseband stages [6]. Although the use of phase shifters in the analog beamforming stage provides substantial simplification [7], their practical realization for mmWave frequencies is not a simple task [8]. Digitally controlled phase shifters can suffer from precision [8], latency [5] and power consumption problems [9]. Passive phase shifters are known to incur higher losses thus necessitating additional amplification to maintain an acceptable output signal-to-noise (SNR) [8].

Switch-based networks are a viable alternative to phase shifters as they are simple, fast and enjoy low-power consumption [5], [7]. They effectively combine subsets of the available antennas such that they leverage the sparse nature of mmWave massive MIMO channels to realize the performance gains. Such a strategy has, in fact, been successfully applied in various contexts to deliver a large aperture and satisfactory performance at reduced hardware cost and complexity [10]–[14].

The advantages of switch-based networks are realised at the cost of increased difficulty at the design stage as the optimization is over a set of binary variables. A dictionary-based solution is presented in [15] but it is scenario-specific and the size of the dictionary grows with the number of antennas. In [6], on the other hand, a unified greedy algorithm is developed, but this solution is limited to the case where the digital beamforming matrix is square with dimension equal to the number of data streams to be transmitted.

In this work, we propose a comprehensive approach for the design of the optimum switch-based hybrid beamformer in massive MIMO communications in mmWave bands. The proposed method is not restricted to any special case and the introduced convex optimization-based technique enables imposing partially connected structures, which is highly desirable in massive MIMO systems. We first decouple the joint optimization of analog and digital precoding matrices by utilizing a rank constrained subspace. Then by taking advantage of a lower bound given by QR factorization, we iteratively optimize the columns of the analog precoder such that in each iteration we maximize a quadratic form via a sequential convex programming (SCP) procedure. Finally, as well as the proposed digital precoder update method, we study the effect of using a least squares method.

II. Problem Formulation

A hybrid structure for a single-user mmWave MIMO system is depicted in Fig. 1, as proposed in [2], [7]. In this setup, the transmitter comprises N_t antennas and L_t RF transmit chains, and is required to send N_s data streams to the receiver. We

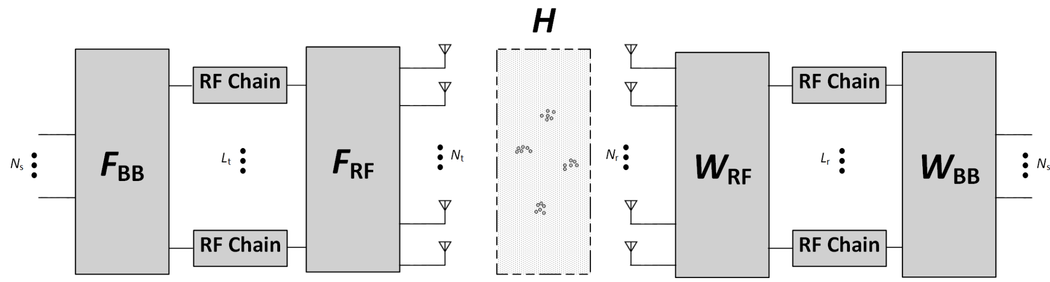

Fig. 1. Block diagram of hybrid MIMO architecture for mmWave communication with baseband and analog precoder/combiner with a clustered channel model.

assume that $N_\mathrm{s} \leq L_\mathrm{t} \leq N_\mathrm{t}$. Let the transmit digital beamforming matrix be $\mathbf{F}_\mathrm{BB}$ of size $L_\mathrm{t} \times N_\mathrm{s}$, and RF precoder matrix be $\mathbf{F}_\mathrm{RF}$ of size $N_\mathrm{t} \times L_\mathrm{t}$. $\mathbf{F}_\mathrm{RF}$ is implemented using analog phase-shifters or RF switches. The discrete-time transmit signal is then $\mathbf{x} = \mathbf{Fs}$, where $\mathbf{F} = \mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}$, and $\mathbf{s}$ is the $N_\mathrm{s} \times 1$ symbol vector such that $\mathbb{E}[\mathbf{ss}^*] = \frac{1}{N_\mathrm{s}}\mathbf{I}_{N_\mathrm{s}}$ with $\mathbb{E}$ denoting the expected value. At the receiver, N_r antennas are connected to L_r RF receive chains to recover the transmitted symbol $\mathbf{s}$. Similarly to the transmitter, the receive beamformer $\mathbf{W} = \mathbf{W}_\mathrm{RF}\mathbf{W}_\mathrm{BB}$ is composed of the $N_\mathrm{r} \times L_\mathrm{r}$ RF combining matrix $\mathbf{W}_\mathrm{RF}$ and $L_\mathrm{r} \times N_\mathrm{s}$ baseband beamforming matrix $\mathbf{W}_\mathrm{BB}$.

Given a narrowband frequency-flat channel model represented by the $N_\mathrm{r} \times N_\mathrm{t}$ channel matrix $\mathbf{H}$, with $\mathbb{E}\left[\|\mathbf{H}\|_F^2\right] = N_\mathrm{t} \times N_\mathrm{r}$, we can write the received signal as

$$\mathbf{y} = \sqrt{\rho}\mathbf{W}_\mathrm{BB}^*\mathbf{W}_\mathrm{RF}^*\mathbf{H}\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}\mathbf{s} + \mathbf{W}_\mathrm{BB}^*\mathbf{W}_\mathrm{RF}^*\mathbf{n}.$$

Here ρ is the average received power, and $\mathbf{n}$ the additive zero-mean i.i.d noise with variance σ_n^2. Also, $\mathbf{W}_\mathrm{BB}^*$ denotes the conjugate transpose of $\mathbf{W}_\mathrm{BB}$. For a clustered channel model consisting of the sum of the contributions of N_cl scattering clusters, with each cluster comprising N_ray propagation paths, the channel matrix is

$$\mathbf{H} = \gamma \sum_{i,\ell} \alpha_{i\ell}\Lambda_\mathrm{r}(\phi_{i\ell}^\mathrm{r}, \theta_{i\ell}^\mathrm{r})\Lambda_\mathrm{t}(\phi_{i\ell}^\mathrm{t}, \theta_{i\ell}^\mathrm{t})\mathbf{a}_\mathrm{r}(\phi_{i\ell}^\mathrm{r}, \theta_{i\ell}^\mathrm{r})\mathbf{a}_\mathrm{t}(\phi_{i\ell}^\mathrm{t}, \theta_{i\ell}^\mathrm{t})^*,$$

where $\gamma = \sqrt{\frac{N_\mathrm{rt}N_\mathrm{r}}{N_\mathrm{cl}N_\mathrm{ray}}}$ is a normalization factor and $\alpha_{i\ell}$ is the complex amplitude associated with the ℓ-th ray in the i-th cluster. The antenna gain at direction of departure (DoD) azimuth and elevation angles $(\phi_{i\ell}^\mathrm{t}, \theta_{i\ell}^\mathrm{t})$, and direction of arrival (DoA)$(\phi_{i\ell}^\mathrm{r}, \theta_{i\ell}^\mathrm{r})$, are denoted theby $\Lambda_\mathrm{r}(\phi_{i\ell}^\mathrm{r}, \theta_{i\ell}^\mathrm{r})$, and $\Lambda_\mathrm{t}(\phi_{i\ell}^\mathrm{r}, \theta_{i\ell}^\mathrm{r})$ respectively. The DoDs and DoAs of the scatterers are assumed randomly distributed with a Laplacian distribution [2]. The vectors, $\mathbf{a}_\mathrm{r}(\phi_{i\ell}^\mathrm{r}, \theta_{i\ell}^\mathrm{r})$ and $\mathbf{a}_\mathrm{t}(\phi_{i\ell}^\mathrm{r}, \theta_{i\ell}^\mathrm{r})$ are respectively the receive and transmit array steering vector associated with the ℓ-th ray in the i-th cluster. For an uniform planar array (UPA) located in the yz-plane, the array response is

$$\mathbf{a}(\phi, \theta) = \frac{1}{\sqrt{N}}\left[e^{jkd(m\sin(\phi)\sin(\theta)+n\cos(\theta))}\right]$$
$$0 \geq m \geq N_y, 0 \geq n \geq N_z \tag{1}$$

where N is the total number of elements, while N_y and N_z are the number of grid points in the y, and z planes respectively such that $N = N_yN_z$.

Let the transmit power be divided equally among all the data streams. Then, the mutual information is expressed as

$$\mathcal{I} = \log_2\left(\left|\mathbf{I}_{N_\mathrm{s}} + \frac{\rho}{N_\mathrm{s}}\mathbf{R}_n^{-1}\mathbf{W}_\mathrm{BB}^*\mathbf{W}_\mathrm{RF}^*\mathbf{H}\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB} \times \mathbf{F}_\mathrm{BB}^*\mathbf{F}_\mathrm{RF}^*\mathbf{H}^*\mathbf{W}_\mathrm{RF}\mathbf{W}_\mathrm{BB}\right|\right). \tag{2}$$

Here, $\mathbf{R}_n$ is the noise covariance matrix at the receiver given by $\mathbf{R}_n = \sigma^2\mathbf{W}_\mathrm{BB}^*\mathbf{W}_\mathrm{RF}^*\mathbf{W}_\mathrm{RF}\mathbf{W}_\mathrm{BB}$. The optimum beamformer is composed of the precoding and combining matrices $(\mathbf{F}_\mathrm{BB}, \mathbf{F}_\mathrm{RF}, \mathbf{W}_\mathrm{BB}, \mathbf{W}_\mathrm{RF})$ that maximise the mutual information. However, this design problem is a joint non-convex optimization that is intractable. To overcome this difficulty, we decompose it into separate transmit and receive subproblems [2], which yields the mutual information at the transmit-side

$$\mathcal{I} = \log_2\left(\left|\mathbf{I}_{N_\mathrm{s}} + \frac{\rho}{N_\mathrm{s}}\mathbf{H}\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}\mathbf{F}_\mathrm{BB}^*\mathbf{F}_\mathrm{RF}^*\mathbf{H}^*\right|\right).$$

In general, the analog precoder and combining matrices, $\mathbf{F}_\mathrm{RF}$ and $\mathbf{W}_\mathrm{RF}$, are implemented either using analog phase shifters or analog switches along with RF combiners/splitters. In this work, we focus on hybrid architectures based on switch networks and consider only the transmit-side, noting that the proposed method is equally applicable to the receive side.

III. Switch Based Hybrid Precoder Design

The general model for a hybrid precoder based on a network of analog switches, splitters, and combiners, is depicted in Fig. 2. Let us consider the singular value decomposition of the channel $\mathbf{H} = \mathbf{U\Sigma V}^*$ such that $\mathbf{U}$ is an $N_\mathrm{r} \times \mathrm{rank}(\mathbf{H})$ unitary matrix, $\mathbf{\Sigma}$ is a $\mathrm{rank}(\mathbf{H}) \times \mathrm{rank}(\mathbf{H})$ diagonal matrix of descending singular values, and $\mathbf{V}$ is a $N_\mathrm{t} \times \mathrm{rank}(\mathbf{H})$ unitary matrix. Then, the first N_s singular vectors provide the unconstrained optimum precoder $\mathbf{F}_\mathrm{opt} = \mathbf{V}_{N_\mathrm{s}}$.

The method presented in this paper is built upon the assumption that the mmWave system and propagation channel parameters are chosen such that a hybrid precoder $\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}$, sufficiently close to $\mathbf{F}_\mathrm{opt} = \mathbf{V}_{N_\mathrm{s}}$, is realizable. Therefore, we assume that the matrices $\mathbf{I}_{N_\mathrm{s}} - \mathbf{V}_{N_\mathrm{s}}^*\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}\mathbf{F}_\mathrm{BB}^*\mathbf{F}_\mathrm{RF}\mathbf{V}_{N_\mathrm{s}}$, and $\mathbf{V}_{\bar{N}_\mathrm{s}}^*\mathbf{F}_\mathrm{RF}\mathbf{F}_\mathrm{BB}$ have a set of sufficiently small eigenvalues [2]. Note that $\mathbf{V}_{\bar{N}_\mathrm{s}}$ denotes the eigenvectors associated with the subspace complementary to $\mathbf{V}_{N_\mathrm{s}}$. Now, by employing

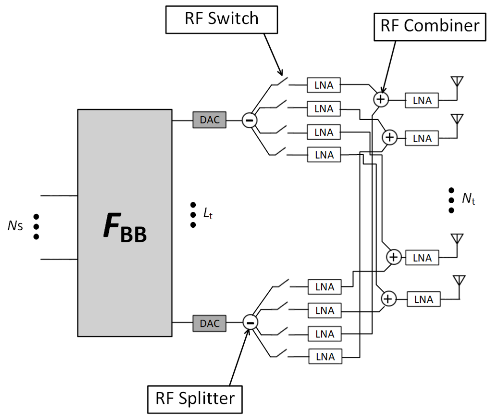

Fig. 2. Simplified analog architecture for Hybrid MIMO beamforming with analog switches, combiners, and splitters.

Sylvester's determinant theorem, and Schur's complement identity for matrix determinants, we can approximate the mutual information as

$$\begin{aligned}\mathcal{I}(\mathbf{H}) &= \log_2\left(\left|\mathbf{I}+\frac{\rho}{N_s\sigma^2}\mathbf{\Sigma}^2\mathbf{V}^*\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^*\mathbf{F}_{\mathrm{RF}}\mathbf{V}\right|\right),\\ &\approx \log_2\left(\left|\mathbf{I}+\begin{bmatrix}\frac{\rho}{N_s\sigma^2}\mathbf{\Sigma}_{N_s}^2\mathbf{V}_{N_s}^*\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^*\mathbf{F}_{\mathrm{RF}}\mathbf{V}_{N_s} & 0\\ 0 & 0\end{bmatrix}\right|\right),\\ &= \log_2\left(\left|\mathbf{I}_{N_s}+\frac{\rho}{N_s\sigma^2}\mathbf{H}_1\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^*\mathbf{F}_{\mathrm{RF}}^*\mathbf{H}_1^*\right|\right),\end{aligned} \tag{3}$$

where $\mathbf{H}_1$ is the subchannel constructed by the first N_s singular vectors and singular values of $\mathbf{H}$.

Defining a new virtual matrix $\tilde{\mathbf{H}} = \mathbf{H}_1\mathbf{F}_{\mathrm{RF}}$ of size $N_r \times L_t$ and singular value decomposition of $\tilde{\mathbf{H}} = \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^*$, we can maximize (3) by having $\mathbf{F}_{\mathrm{BB}} = \tilde{\mathbf{V}}_{N_s}$,

$$\begin{aligned}\mathcal{I}(\tilde{\mathbf{H}}) &= \log_2\left(\left|\mathbf{I}+\frac{\rho}{N_s\sigma^2}\tilde{\mathbf{H}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^*\tilde{\mathbf{H}}^*\right|\right)\\ &= \log_2\left(\left|\mathbf{I}_{N_s}+\frac{\rho}{N_s\sigma^2}\tilde{\mathbf{\Sigma}}_{N_s}^2\right|\right)\\ &= \log_2\left(\left|\mathbf{I}_{N_s}+\frac{\rho}{N_s\sigma^2}\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^*\right|\right),\end{aligned} \tag{4}$$

where $\tilde{\mathbf{H}}_1$ denotes the new virtual channel representation achieved by the first N_s eigenvalues, e.g., $\tilde{\mathbf{H}}_1 = \tilde{\mathbf{U}}_{N_s}\tilde{\mathbf{\Sigma}}_{N_s}\tilde{\mathbf{V}}_{N_s}^*$. Now, if we restrict the rank of $\tilde{\mathbf{H}}$ as $\mathrm{rank}(\tilde{\mathbf{H}}) = N_s$, then $\mathbf{F}_{\mathrm{BB}} = \tilde{\mathbf{V}}_{N_s} = \tilde{\mathbf{V}}$ becomes a unitary matrix, and therefore we can specify (4) as

$$\mathcal{I}(\tilde{\mathbf{H}}) = \log_2\left(\left|\mathbf{I}_{N_s}+\frac{\rho}{N_s\sigma^2}\tilde{\mathbf{\Sigma}}^2\right|\right) \tag{5}$$

Hence, the problem becomes a single variable maximization, that is finding the optimum binary matrix $\mathbf{F}_{\mathrm{RF}}$ satisfying the rank constraint. However, the log-determinant maximization is still a computationally expensive optimization. To address this, we propose a lower bound on (5) in the following Theorem.

Theorem 1: Assuming that $\tilde{\mathbf{H}}$ as a rank deficient matrix is factorizable by generalized QR decomposition as $\tilde{\mathbf{H}}\mathbf{P} = \mathbf{Q}\mathbf{R}$, with $\mathbf{Q}, \mathbf{R}, \mathbf{P}$ being a unitary matrix of size $N_r \times L_t$, an upper triangular matrix of size $L_t \times L_t$, and a permutation matrix of size $L_t \times L_t$, then

$$\mathcal{I}(\tilde{\mathbf{\Sigma}}^2) \geq \mathcal{I}\left(|[\mathbf{R}]_{ii}|^2\right), \tag{6}$$

where $|[\mathbf{R}]_{ii}|$ denotes the absolute value of th i-th diagonal element of $\mathbf{R}$.

Proof: We know from majorization theory that (see Lemma 4.9 in [16] or [6])

$$\prod_{i=1}^{Ns}\Sigma_i^2 \geq \prod_{i=1}^{Ns}|[\mathbf{R}]_{ii}|^2. \tag{7}$$

Therefore, we can extend this as

$$\begin{gathered}\log_2\left(\prod_{i=1}^{Ns}(1+\frac{\rho}{N_s\sigma^2}\Sigma_i^2)\right) \geq \log_2\left(\prod_{i=1}^{Ns}(1+\frac{\rho}{N_s\sigma^2}|[\mathbf{R}]_{ii}|^2)\right)\\ \mathcal{I}(\tilde{\mathbf{\Sigma}}^2) \geq \mathcal{I}\left(|[\mathbf{R}]_{ii}|^2\right)\end{gathered}$$

■

Using the properties of QR decomposition, we can write

$$|[\mathbf{R}]_{ii}|^2 = \mathbf{f}_{\mathrm{RF},i}^*\mathbf{A}_i\mathbf{f}_{\mathrm{RF},i} \tag{8}$$

where

$$\mathbf{A}_i = \mathbf{H}_1^*\Pi_{\mathbf{H}_1\mathbf{F}_{\mathrm{RF}}^i}\mathbf{H}_1,\;\; \Pi_{\mathbf{X}} = \mathbf{I}-\mathbf{X}(\mathbf{X}^*\mathbf{X})^{-1}\mathbf{X}^*.$$

The vector $\mathbf{f}_{\mathrm{RF},i}$ denotes the i-th column of $\mathbf{F}_{\mathrm{RF}}$, and matrix $\mathbf{F}_{\mathrm{RF}}^i$ represents the first i columns of $\mathbf{F}_{\mathrm{RF}}$.

In summary, the mutual information in (5) in terms of $\mathbf{H}_1\mathbf{F}_{\mathrm{RF}}$ is seen to be lower bounded by the mutual information given by the diagonal elements of $\mathbf{R}$ from a QR decomposition. Thus, we attempt to maximize the diagonal elements of $\mathbf{R}$. Now, in many practical applications, due to specific hardware limitations, it is desirable to impose some connectivity constraints on $\mathbf{F}_{\mathrm{RF}}$ in the form of a partially connected network. Considering such connectivity constraints, and by utilizing (8), we cast the maximization for each column of $\mathbf{F}_{\mathrm{RF}}$ as,

$$\max_{\mathbf{f}_{\mathrm{RF},i}} \;\; \mathbf{f}_{\mathrm{RF},i}^*\mathbf{A}_i\mathbf{f}_{\mathrm{RF},i} \tag{9a}$$
$$\text{s.t } 0 \leq \mathbf{f}_{\mathrm{RF},i} \leq 1 \tag{9b}$$
$$\mathrm{diag}\left(\mathbf{C}\mathbf{F}_{\mathrm{RF}}\right) \leq \mathbf{c} \tag{9c}$$
$$\mathrm{diag}\left(\mathbf{F}_{\mathrm{RF}}\mathbf{Q}\right) \leq \mathbf{q} \tag{9d}$$

where we relax the binary constraint. Moreover, $\mathbf{C}$ and $\mathbf{c}$ are a predefined $L_t \times N_t$ matrix, and an $L_t \times 1$ vector, respectively that impose a partially connected structure along the columns of $\mathbf{F}_{\mathrm{RF}}$. The partially connected structure along the rows of $\mathbf{F}_{\mathrm{RF}}$ is dictated by $\mathbf{Q}$ and $\mathbf{q}$ as an $N_t \times L_t$ matrix, and an $N_t \times 1$ vector, respectively.

Using (9), we find $\mathbf{f}_{\mathrm{RF},i}$ that maximizes the diagonal elements of the QR decomposition. As we are just interested in the first N_s diagonal elements, we have an upper triangular matrix $\mathbf{R}$ with N_s nonzero elements on the diagonal, and therefore a rank of N_s [17]. Consequently, we assume that the rank constraint is met and remove it from our formulation. We also temporarily remove the transmit power constraint $\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 = N_s$, and reinstate it later by scaling $\mathbf{F}_{\mathrm{BB}}$.

The maximization of the quadratic form in (9a) as a convex function is a non-convex problem. Sequential convex programming (SCP) based on iteratively linearizing the convex function is applied to reformulate the non-convex problem as a series of convex subproblems, each of which can be optimally solved using convex programming [18]. We formulate the quadratic form maximization by linearization and use a first-order Taylor expansion as a local approximation. Given $f(\mathbf{f}_{\mathrm{RF},i}) = \mathbf{f}_{\mathrm{RF},i}^{*}\mathbf{A}_i\mathbf{f}_{\mathrm{RF},i}$, we can write this approximation at point $\ell-1$ as,

$$\begin{aligned} f(\mathbf{f}_{\mathrm{RF},i},\mathbf{f}_{\mathrm{RF},i}^{(\ell-1)}) &= f(\mathbf{f}_{\mathrm{RF},i}^{(\ell-1)}) + \nabla\mathbf{f}_{\mathrm{RF},i}^{(\ell-1)}) \left(\mathbf{f}_{\mathrm{RF},i} - \mathbf{f}_{\mathrm{RF},i}^{(\ell-1)}\right) \\ &= f(\mathbf{f}_{\mathrm{RF},i}^{(\ell-1)}) + \left((\mathbf{A}_i + \mathbf{A}_i^{T})\mathbf{f}_{\mathrm{RF},i}^{(\ell-1)}\right) \left(\mathbf{f}_{\mathrm{RF},i} - \mathbf{f}_{\mathrm{RF},i}^{(\ell-1)}\right). \end{aligned}$$

The convex problem to be solved in the ℓ-th step can be expressed as:

$$\max_{\mathbf{f}_{\mathrm{RF},i}} \; f(\mathbf{f}_{\mathrm{RF},i},\mathbf{f}_{\mathrm{RF},i}^{(\ell-1)}) \tag{10a}$$

$$\text{s.t } \; 0 \leq \mathbf{f}_{\mathrm{RF},i} \leq 1 \tag{10b}$$

$$\mathrm{diag}\left(\mathbf{C}\mathbf{F}_{\mathrm{RF}}\right) \leq \mathbf{c} \tag{10c}$$

$$\mathrm{diag}\left(\mathbf{F}_{\mathrm{RF}}\mathbf{Q}\right) \leq \mathbf{q}. \tag{10d}$$

We outline the proposed method in Algorithm 1. In this algorithm, we give up the unitary structure of $\mathbf{F}_{\mathrm{BB}}$ and implement two different update mthods in steps 10, and 11. It is worth noting that the least squares update in step 11 does not preserve the structure of the precoder in terms of equal power transmission.

IV. Simulation Results

In this section, we evaluate the performance of the proposed method through simulations. We maximize the spectral efficiency by maximizing the mutual information at the transmit side, and assume that there is an ideal combiner at the receiver. We use a clustered channel model with $N_{\mathrm{cl}} = 8$ clusters and $N_{\mathrm{ray}} = 10$ rays in each cluster with randomly distributed AoDs, and AoAs sampled from a Laplacian distribution.

Algorithm 1: Switch Based Hybrid Design by QR Decomposition with Quadratic Update (SHD-QRQU)

Input : $\mathbf{H}$

1 Decompose $\mathbf{H} = \mathbf{U}\mathbf{S}\mathbf{V}^{*}$
2 Initialize $\Pi_{\mathbf{H}_1\mathbf{F}_{\mathrm{RF}}^{1}} = \mathbf{I}$ and $\mathbf{F}_{\mathrm{RF}} = \mathbf{0}$
3 **for** $i = 1$ *to* L_{t} **do**
4   Initialize $\mathbf{f}_{\mathrm{RF},i}^{(0)}$ at random
5   **for** L *iterations* **do**
6   Solve (10) and update $\mathbf{f}_{\mathrm{RF},i}^{(\ell)}$
7   **end**
8   Update $[\mathbf{F}_{\mathrm{RF}}]_i = \mathbf{f}_{\mathrm{RF},i}$, and $\mathbf{A}_i$
9 **end**
10 Round $\mathbf{F}_{\mathrm{RF}}$ and Construct $\tilde{\mathbf{H}} = \mathbf{F}_{\mathrm{RF}}\mathbf{H}_1$ and after decomposition update $\mathbf{F}_{\mathrm{BB}}^{\mathrm{D}} = \left(\mathbf{F}_{\mathrm{RF}}^{T}\mathbf{F}_{\mathrm{RF}}\right)^{-0.5}\tilde{\mathbf{V}}$
11 Update $\mathbf{F}_{\mathrm{BB}}^{\mathrm{LS}} = \left(\mathbf{F}_{\mathrm{RF}}^{T}\mathbf{F}_{\mathrm{RF}}\right)^{-1}\mathbf{F}_{\mathrm{RF}}^{T}\mathbf{F}_{\mathrm{opt}}$ and normalize $\mathbf{F}_{\mathrm{BB}} = \frac{\sqrt{N_{\mathrm{s}}}\mathbf{F}_{\mathrm{BB}}}{\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F}$

Output: $\mathbf{F}_{\mathrm{RF}}, \mathbf{F}_{\mathrm{BB}}^{\mathrm{D}}, \mathbf{F}_{\mathrm{BB}}^{\mathrm{LS}}$

We also assume that the complex amplitude of the rays are sampled from a complex normal distribution with an average power of unity in each cluster. The transmit and receive antenna arrays are uniform planar arrays (UPA) with inter-element spacing d of a half-wavelength. We assume a sector azimuth angle of $60°$, and sector elevation angle of $30°$ at the transmit side, while at the receive side we assume omni-directional antennas.

For each scenario, we calculate the optimal unconstrained precoder (OUP) achieved by the first N_{s} eigenmodes of the channel. Moreover, we compare the performance to a phase-shift network. We implement an algorithm called hybrid design by the least squares relaxation (HD-LSR) proposed in [3] as a fast method to design the hybrid network with phase shifters in analog section. In the case of $N_{\mathrm{s}} = L_{\mathrm{t}}$, we implement the greedy algorithm in [6] in switch network mode. We call this algorithm switch based hybrid design by a unified greedy algorithm (SHD-UG).

We then design the switch network with the proposed algorithm, switch-based hybrid design by QR decomposition with quadratic update (SHD-QRQU). For this method we update $\mathbf{F}_{\mathrm{BB}}$ with both decomposition (SHD-QRQU-D), and least squares (SHD-QRQU-LS). We also implement SHD-QRQU for designing the switch based precoder in a partially connected network (SHD-QRQU-PC). We only impose the constraints along the columns and use the parameters $\mathbf{P} = \mathbf{1}_{N_{\mathrm{t}}\times L_{\mathrm{t}}}, \mathbf{p} = 64\times\mathbf{1}_{L_{\mathrm{t}}}$. The spectral efficiency for each value of SNR is the averaged value of 100 random channel realizations. Furthermore, we use the CVX package to solve the convex optimizations in Algorithm 1 [19].

Fig. 3 shows the spectral efficiency achieved in a 256×64 UPA for different values of SNR. The transmitter is assumed to have access to 4 RF chains ($L_{\mathrm{t}} = 4$). Also it is assumed that $N_{\mathrm{s}} = 2$ data streams are transmitted. Fig. 3 illustrates that the proposed methods, SHD-QRQU-D, and SHD-QRQU-LS achieve spectral efficiencies with only a small gap to those achieved by the OUP, and HD-LSR. Considering the significantly lower cost, power, and hardware complexity required by such switch-based hybrid methods, this small gap demonstrates a very good trade-off. Furthermore, the spectral efficiency achieved in a partially connected network, sits closely below that of the fully connected network and provides yet lower cost, power and complexity.

Finally, we examine the performance of the proposed method when $N_{\mathrm{s}} = 4$ in Fig. 4. This scenario is a special case as it is categorized as a hybrid network with $L_{\mathrm{t}} = N_{\mathrm{s}}$. We compare the performance of unconstrained switch-based hybrid design (SHD-QRQU) as a comprehensive solution with that SHD-UG. By increasing N_{s}, the capability of the hybrid network to approximate the optimal unconstrained precoder slightly deteriorates. This can be observed by the increased gap between HD-LSR, and OUP in Fig. 4. Compared to the previous case shown in Fig. 3, the switch-based network generally also has a larger gap to the unconstrained and phase-shift based hybrid structures. As shown in this figure the proposed algorithm outperforms the greedy method (SH-

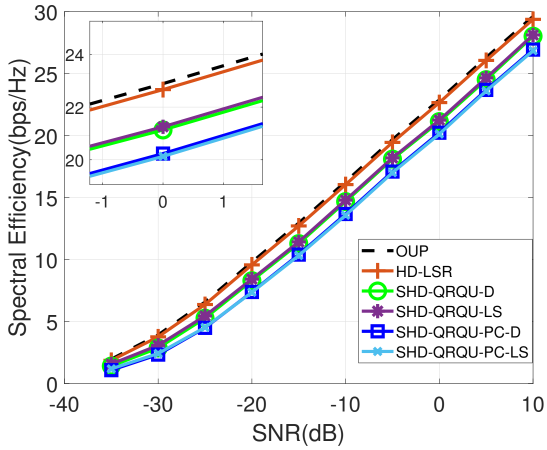

Fig. 3. Spectral Efficiency achieved by different hybrid design methods for $N_{\mathrm{t}} = 256$, $N_{\mathrm{r}} = 64$, $L_{\mathrm{t}} = 4$, and $N_{\mathrm{s}} = 2$.

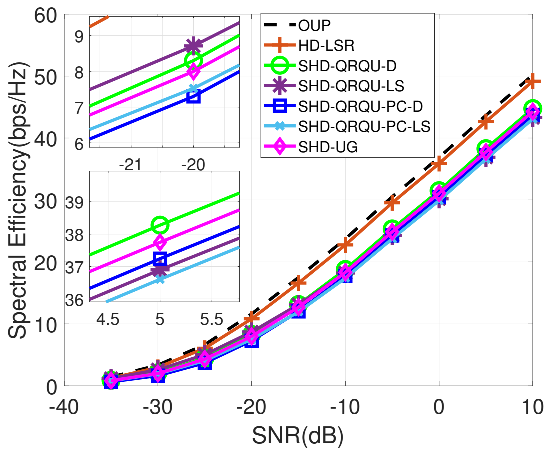

Fig. 4. Spectral Efficiency achieved by different hybrid design methods for $N_{\mathrm{t}} = 256$, $N_{\mathrm{r}} = 64$, $L_{\mathrm{t}} = 4$, and $N_{\mathrm{s}} = 4$.

UGD), with a significantly improved performance delivered by SHD-QRQU-U at low SNR. It is worth noting SHD-QRQU-LS outperforms the decomposition-based update SHD-QRQU-D at low SNRs. However as the SNR value grows SHD-QRQU-D outperforms SHD-QRQU-LS. The QRQU based method in a partially connected network, due to the relatively sparse channel with respect to the number of antennas delivers a spectral efficiency slightly lower than that of the SHD-QRQU with almost a quarter of the active switches.

V. Conclusion

We presented a new switch-based precoder for mmWave communications that decouples the problem of joint optimization of an analog and digital beamformer by confining the problem to a rank constrained subspace. We then approximated the solution through the lower bound offered by QR factorization. We also introduced linear constraints to include frequently used partially-connected structures. Finally we examined the effectiveness of the proposed method using a set of numerical examples. The results demonstrate the feasibility of the proposed method, optimizing for a variety of structures as well as providing an effective and comprehensive tool in the study of different scenarios.

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