Time Modulated Array – A Database Approach

Jérôme Euzière*, Régis Guinvarc'h*, Israel Hinostroza Sáenz*, Raphaël Gillard† and Bernard Uguen†
*SONDRA
CentraleSupélec, Gif-sur-Yvette, France
Email: regis.guinvarch@centralesupelec.fr
†IETR-Rennes, France

***Abstract*—Time Modulated Array offers a simple way to synthesize radiation patterns with low sidelobe levels, in time-average, using an optimized strategy of switching ON and OFF the elements of the array. In this work we show that the complexity of the optimization strategy can be reduced by first shrinking the search space to a subset, or database, of useful solutions over which the optimization will be launched. This reduction is obtained by applying the constraints in a progressive and hierarchical order. In this way, this problem of exponential growth is reduced to a polynomial growth, with respect to the size of the antenna array. We present the case of designing the time modulated array for radar applications where constant directivity and ability of intereference rejection is needed.**

***Index Terms*—TMA, radar, rejection, search space, database**

I. Introduction

Time Modulated Array (TMA) [1] offers a simple way to synthesize low sidelobe radiation patterns. It relies on averaging several radiation patterns. These radiation patterns are obtained by switching (ON and OFF) the elements in the array during variable time intervals so that the average pattern has the desired characteristics. In this way, TMA offers a lot of possible degrees of freedom to which the community has been applying different optimization techniques. In order to make the optimization process acceptable, the community has focused on the time domain [2] [3] [4] [5] [6]. In this work, we reintroduce the spatial domain (weighting on the elements of the array) as a way to reduce the search space by a progressive, hierarchical application of the constraints. By discretizing the weights (amplitude and phase) on the antennas, we obtain a finite number of possible solutions (possibly large). From this, by progressively applying a sequence of filters (constraints), the search space is progressively reduced to obtain a database containing the possible solutions. In this way, this sequence of filters permits to address a large number of different problems sharing some common constraints.

In section II we recall the concepts and definitions of TMA. Next, in section III, it is shown how the filters can be applied to obtain a database of useful instantaneous patterns. Later, in section IV, we present an example of the technique considering the design of a linear antenna array for radar applications. Finally, we present some conclusions.

II. Time Modulated Array

The original TMA technique introduced by Kummer [1] uses a linear array which elements are switched ON or OFF. This switching is in fact a weigthing on the elements of the antenna array as a function of time that repeats itself after a period T. In this way, each switching combination corresponds to an instantaneous array factor, among a total of P possible ones, over a certain duration Δt_p, $p \in \{1, 2, \dots, P\}$. Next equation shows the instantaneous pattern p for a uniform linear antenna array of M elements.

$$AF_p(\theta) = \sum_{m=1}^{M} w_{mp} \cdot e^{j\cdot(m-1)k\cdot d\cdot cos(\theta)} \tag{1}$$

where k is the wave number, d is the constant distance between elements, $w_{mp} \in \mathbb{C}$ is the weight applied to the m-th element of the array ($m \in \{1, 2, \dots, M\}$) during the Δt_p time duration. Note that in [1] $w_{mp} \in \{0, 1\}$, whereas, in general, the weight w_{mp} is defined by its magnitude a_{mp} and phase φ_{mp}.

Then, the time-average pattern AF_{av} is:

$$AF_{av}(\theta) = \frac{1}{T} \sum_{p=1}^{P} AF_p(\theta) \cdot \Delta t_p \tag{2}$$

We can see that the final time-average pattern AF_{av} depends on the weighting w_{mp} of the elements and durations Δt_p. Many authors have already worked on the optimization of the latter time dimension. On the former, specially for radar, we can mention the use of sparse arrays (STMA [7]) and Phase Only Synthesis solution (POSTMA [8]). For all these cases, the number of degrees of freedom can be large (multiple amplitudes and/or phases for w_{mp}, different durations Δt_p), which gives better results, but longer optimization time. Hence, heuristic methods need to be used.

III. Database STMA

Due to the large number of degrees of freedom, the search space also becomes very large. In this section we propose a technique to reduce the size of the search space. To do so, for simplicity, we consider the case of STMA ($w_{mp} \in \{0, 1\}$ and constant time step Δt_p), hence optimization only applies to the weights of the elements.

For an antenna array of M elements having given positions and weightings, there are $N_s = 2^M - 1$ possible instantaneous array factors. Hence, one possible average pattern can be obtained by selecting P instantaneous array factors among N_s, without repetition and indifferent order of selection. Note

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that the case of selecting with repetition is a way to consider optimization along the dimension of time.

Then, for our case, the total number of possible time-average patterns $N_{av\,patt}$ is:

$$N_{av\,patt} = \binom{N_s}{P} = \frac{N_s!}{P!(N_s - P)!} \quad (3)$$

We can see that, as $M \to \infty$, $N_{av\,patt} = \mathcal{O}(2^{M \cdot P})$, which is an exponential growth with respect to the size of the antenna array. However, many of the possible instantaneous array factors may not be useful and can be discarded. The idea is then to define filters to select only useful instantaneous patterns according to the specifications. In this way we obtain a new reduced number of possible instantaneous array factors N_s^r, hence a reduction of the search space size is achieved. This database of N_s^r useful patterns is the input for the optimization process.

As an example of how this database can be built let us consider the case of radar and interference suppression using STMA.

A. Case of radar and interference suppression

1) Directivity filter: A radar antenna needs to have an average pattern with low sidelobe level (SLL) with respect to main lobe and a prescribed directivity. Additionally, during the integration time, the directivity needs to be constant when switching from one pattern to the next one. In order to assure constant directivity, the optimization needs to be restricted to a constant number of elements of the array that are turned ON. Hence, the database, under constant directivity, is composed of $N_s^{r,direct}$ solutions:

$$N_s^{r,direct} = \binom{M}{k_{OFF}} \quad (4)$$

where k_{OFF} are the number of elements of the array that are turned OFF. This means that, by considering $N_s^{r,direct}$ instead of N_s in (3), the new reduced total number of possible time-average patterns is $N_{av\,patt}^{r,direct} = \mathcal{O}(M^{k_{OFF} \cdot P})$ instead of $N_{av\,patt} = \mathcal{O}(2^{M \cdot P})$, as $M \to \infty$. Hence the optimization problem with exponential growth has been transformed into a one with polynomial growth, with respect to the size of the array.

2) Interference rejection filter: In [9], Haupt noted that, if the original TMA [1] is used for radar applications, an interference can be damaging even if the average rejection of interference is large. The problem comes from the fact that the rejection for some of the instantaneous patterns may be much higher. As an illustration, we consider the case of a linear array with N elements and $d = \lambda_0/2$ interelement spacing (λ_0 is the working wavelength). We assume the array is radiating at broadside ($\theta_{max} = 90°$) and 2 elements out of N are switched OFF. Note that this scenario will be used all along this paper.

For example, if a minimum rejection of 25 dB is needed, there is no solution for an array of 10 elements at $\theta = 50°$. In fact, the maximum rejection is 22 dB. As expected, this constraint is not a problem at other angles and for larger arrays.

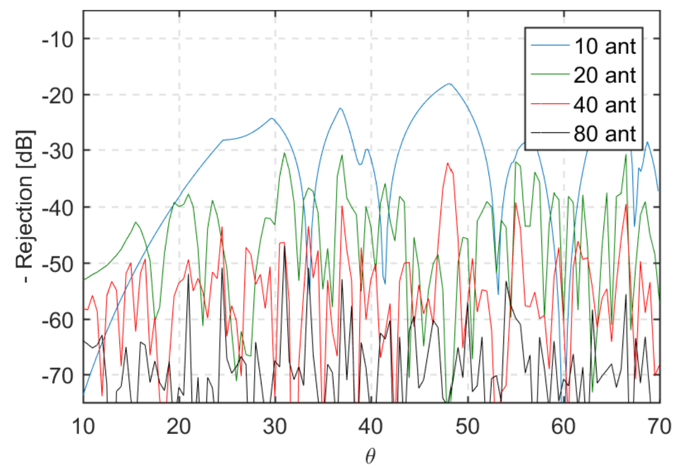

Fig. 1. Opposite of maximum rejection of interference in [dB] (relative to the main lobe level) versus angle ($\theta \in [10°,\ 70°]$, main beam at $\theta_{max} = 90°$) for different size of antenna array, blue for 8 elements out of 10, green for 18 out of 20, red for 38 out of 40 and black for 78 out of 80.

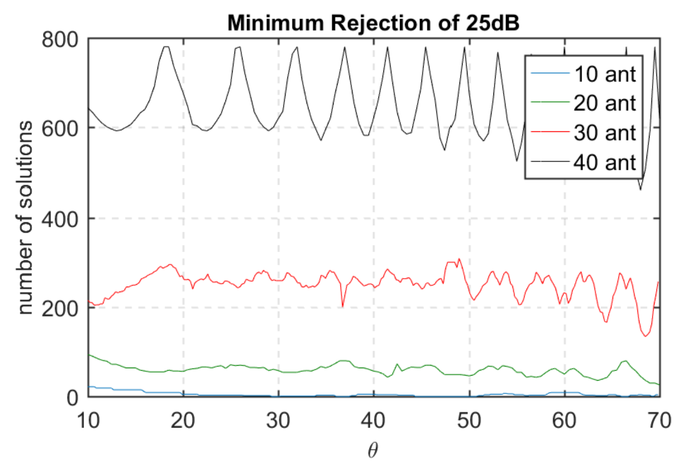

Fig. 2. Number of useful radiation patterns with minimum rejection of 25 dB versus angle ($\theta \in [10°,\ 70°]$, main beam at $\theta_{max} = 90°$) and for different size of antenna array, blue for 8 elements out of 10, green for 18 out of 20, red for 28 out of 30 and black for 38 out of 40.

For example, for 80 elements, the maximum rejection is better than 47 dB for all angles.

Hence, while building the database of useful patterns, adding an additional filter to take into account the minimum rejection will further reduce the search space. Fig. 2 shows the number of useful radiation patterns that comply with the constraint of minimum rejection of 25 dB.

From Fig. 2 we can see that the number of useful patterns is very low (no more than 22 at $\theta = 10°$) for the case of antenna arrays of 10 elements. As expected, for larger arrays this number is larger. For example, for antenna arrays of 40 elements the number of the useful patterns is at least 461 for all angles. It is interesting to note that, globally, the number of useful patterns is quite stable for all angles, once the number of elements is set.

As an example of how much reduction is obtained after

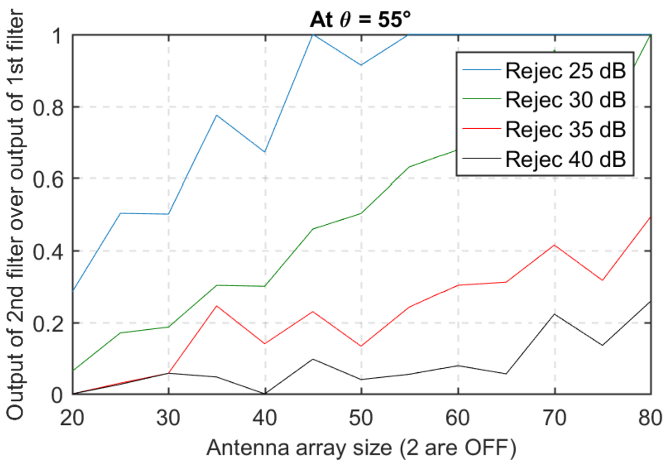

Fig. 3. Ratio of number of final useful radiation patterns (after rejection filter) over number of radiation patterns after applying directivity filter versus antenna array length, per constraint of minimum rejection at $\theta = 55°$: blue for rejection of 25 dB, green for 30 dB, red for 35 dB and black for 40 dB.

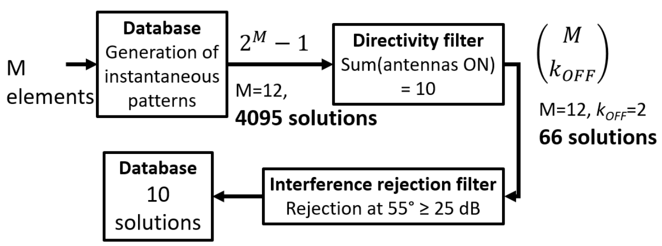

Fig. 4. Example of how a database is built by applying filters. Considering an antenna array of 12 elements: constant directivity of 10 elements ON, minimum rejection of 25 dB at $\theta = 55°$.

applying the second filter, we can consider different values of minimum rejection at $\theta = 55°$ as in Fig. 3, which shows the ratio between the output of the second filter and the output of the first filter versus array length. Fig. 3 also shows how effective is the second filter considering the constraints and array size. It can be seen that, eventually, the number of solutions after applying the second filter (rejection) approaches the number of solutions after applying only the first filter (directivity) as long as the size of the antenna array is large enough.

TABLE I
CONSTRAINTS WHEN DESIGNING TMA OF 12 ELEMENTS (CF. SECTION IV)

Constraint	Value
Const. **instant.** directivity (1st filter)	10 elements ON
Min. **instant.** Rejection at $\theta = 55°$ (2nd filter)	25 dB
Number of instant. patterns for **time-average**	4
Average Rejection at $\theta = 55°$	lowest
Average Sidelobe level	lowest

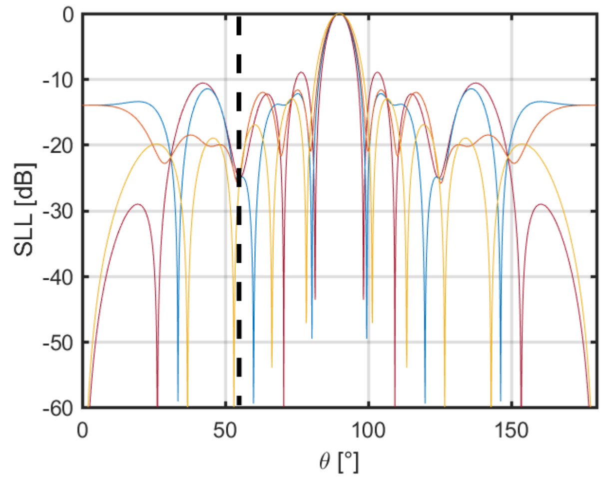

Fig. 5. The 10 instantaneous patterns (not all of them seen since they have same amplitude but different phases) of the database STMA after applying the filters (directivity, rejection) and under the constraints of Tab. I. Note that SLL at $\theta = 55°$ is barely lower than -25 dB.

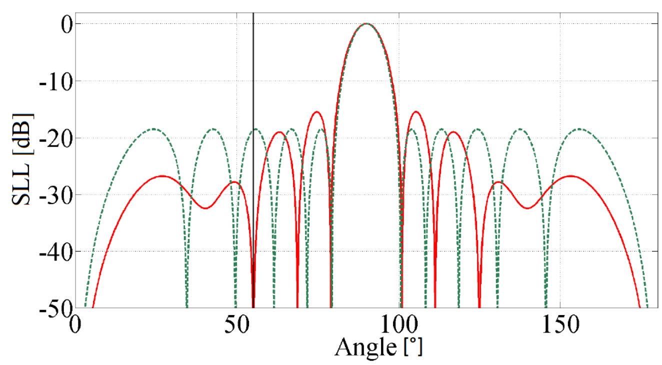

Fig. 6. Result of the optimization for low SLL, stable beamwidth and maximum rejection at $\theta = 55°$ (see constraints in Tab. I) choosing 4 instantaneous patterns from the database TMA (see Fig. 4 and Fig. 5). In red the time-average pattern of optimal solution and in green for reference a Chebyshev pattern at -18.5 dB of SLL

IV. EXAMPLE: RADAR AND INTER. REJEC. AT 55°

In the previous section it has been shown how the number of useful possibles solutions is reduced by applying different filters. Let us now consider the case of finding the best TMA configuration considering an antenna array of 12 elements under the constraints specified in Tab. I. Fig. 4 shows how a database, for later optimization, is built for this case.

Note, in Fig. 4, how the original number of possible instantaneous patterns is reduced to less than a hundredth thanks to the application of the filters. Fig. 5 shows the 10 instantaneous patterns (due to 10 different weightings) of the database. Note that the superimposition of amplitude levels of the patterns shows only 4 different curves, but their phases are different.

We then launch an optimization to choose 4 different instantaneous patterns (out of the 10 patterns from the database) in order to obtain, in time-average, minimum SLL and maximum interference rejection at $\theta = 55°$. Fig. 6 shows the result of the optimization. The time-average SLL of the optimized solution is -15.44 dB and the interference rejection at $\theta = 55°$ is 40 dB. Note that if we allow repetition in the choice of instantaneous patterns, the optimization then includes the time dimension.

V. Conclusions

It has been shown that the search space for the optimization can be strongly reduced (from exponential to polynomial growth relative to the size of the antenna array) when a progressive and hierarchical introduction of filters (constraints on the instantanenous patterns) is carried out. A database of useful instantaneous patterns can be built from which an optimal average pattern can be obtained through optimization techniques. An example has been shown on how a database is built for the case of radar and interference rejection, followed by the final choice (through optimization) of optimum instantaneous patterns which also gives the desired average pattern.

References

[1] W. Kummer, A. Villeneuve, T. Fong, and F. Terrio, *"Ultra-Low Sidelobes from Time-Modulated Arrays,"* IEEE Trans. Antennas Prop., vol. 11, no. 6, pp. 633–639, Nov. 1963.

[2] L. Manica, P. Rocca, L. Poli, and A. Massa, *"Almost Time-Independent Performance in Time-Modulated Linear Arrays,"* in IEEE Antennas and Propagation Letters, vol. 8, pp. 843–846, 2009.

[3] L. Poli, P. Rocca, A. Massa and M. D'Urso, *"Optimized Design of Sparse Time Modulated Linear Arrays,"* 7th European Conference on Antennas and Propagation, Sweden, 2013.

[4] P. Rocca, Q. Zhu, E. T. Bekele, S. Yang, and A. Massa, *"4D arrays as enabling technology for cognitive radio systems,"* IEEE Trans. on Antennas and Propagation – Special Issue on Antenna Systems and Propagation for Cognitive Radio, vol. 62, no. 3, pp. 1102–1116, 2014.

[5] S. Yang, Y. Beng Gan, A. Qing, and P. K. Tan, *"Design of a Uniform Amplitude Time Modulated Linear Array With Optimized Time Sequences,"* IEEE Trans. Antennas Propag., vol. 53, no. 7, pp. 2337–2339, Jul. 2005.

[6] F. Yang, S. Yang, J. Guo and Y. Chen, *"An effective hybrid optimization algorithm for the synthesis of 4-D linear antenna arrays,"* Progress in Electromagnetic Research Symposium (PIERS), 2016.

[7] J. Euzière, R. Guinvarc'h, R. Gillard, B. Uguen, *"Optimization of Sparse Time-Modulated Array by Genetic Algorithm for Radar Applications,"* in IEEE Antennas and Propagation Letters, Vol. 13, pp. 161–164, 2014.

[8] J. Euzière, R. Guinvarc'h, R. Gillard, B. Uguen, *"Phase Only Synthesis in Time Modulated Array,"* IEEE Phased Array Symposium, Boston, October 2013.

[9] R. L. Haupt, *"Time Modulated Receive Arrays,"* IEEE Antennas and Propagation Symposium, pp 968–971, July 2011.