Coverage Analysis of Relay Assisted V2I Communication in Microcellular Urban Networks

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***Abstract*—Vehicular communications have a wide range of applications and are quickly growing in last years. In our work, we investigate vehicular communications to improve the Vehicle-to-infrastructure (V2I) link and enhance the performance of the entire network. We consider a microcellular urban network and compare the coverage probability of the V2I link to the relay-assisted link. We deploy a Manhattan grid where the streets are deployed according to a Poisson line process (PLP), and micro-cell base stations (BSs) and vehicular users are placed according to a Poisson point process (PPP). We analyze the relay position that maximizes the coverage improvement depending on the density of streets, BSs and vehicular users. We derive analytical expressions for the coverage probability of both direct and relay-assisted links by exploiting tools from stochastic geometry and perform Monte Carlo system level simulations of our model.**

***Index Terms*—vehicle-to-vehicle communications, stochastic geometry, system level simulation, coverage improvement**

I. Introduction

Vehicular communications have become more and more important over the last years. From the inclusion of the Vehicle-to-everything (V2X) standard in release 14 of Long Term Evolution (LTE) [1], vehicular communications have been growing at a remarkable pace and they are envisioned to be an essential part of the 5th generation of mobile networks and beyond [2]. Among its applications, especially important is the role that vehicular communications play in the automotive sector [3]. They are used for both intelligent transport systems and road safety applications, assisting the drivers in low visibility street crossings or in a lane change.

Furthermore, vehicular communications can also be useful to enhance the coverage of mobile networks. Using dedicated relays improves the network coverage [4], but requires additional infrastructural costs. Instead, we consider idle vehicles as relay nodes to convey the information from base stations (BSs) to users with poor coverage [5]. This feature is specifically interesting in urban environments, in particular in the millimeter-wave (mm-wave) regime, where the pathloss is quite significant and the coverage deteriorates due to the increased penetration loss. In such scenarios, the deployment costs for mobile operators of a large amount of BSs would be significant, and endowing vehicles with the necessary equipment to enable vehicular communications is a cost-effective alternative. Therefore, our research is focused on the utilization of vehicular relays to improve the coverage of the network in a microcellular urban scenario. In the proposed scenario, idle vehicular users act as relays to enhance the quality of Vehicle-to-infrastructure (V2I) links. The vehicular user density in the network is a key parameter in such scenarios, since it increases the probability to have a relay close the user with poor coverage, but it also increases the interference level in the Vehicle-to-vehicle (V2V) communication. In [6], the authors group the Device-to-Device (D2D) pairs to reduce the interference among D2D transmitters. The work in [7] proposes a scheduling method with successive interference cancellation for V2V communication to improve the capacity of the network. In order to deal with the users and BSs placement and characterize the randomness of wireless networks, stochastic geometry (SG) provides tractable mathematical models to analyze random spatial patterns. In [8], the authors show the distance distribution of a homogeneous Poisson point process (PPP) in $\mathbb{R}^2$. The distance distribution among points of the process as well as the distance distribution in a finite area is investigated in [9]. In [10], the authors present a two-dimensional cellular network and compute the Signal-to-interference and noise ratio (SINR) distribution for the downlink and uplink bands as well as a multi-tier downlink case. Regarding one-dimensional point processes, in [11] the authors analyze an urban Manhattan network where the BSs are located according to a 1-D PPP in each street.

Most of the literature work done on mobile network analysis assuming random spatial patterns by leveraging tools from SG is focused on evaluating the V2I link. In our work, we introduce vehicular relaying to enhance the V2I communication and therefore we deal with an additional random process to analyze. The relaying process follows a decode-and-forward scheme, allowing the transmission in uplink of the information received in downlink. In our previous work [12], we showed the SINR and coverage improvement depending on the density of BSs, streets and users. In this paper, we derive analytical expressions for the coverage probability of the direct link and the relay assisted link and we find the optimal position for the relay that maximizes the SINR of the relay assisted link. Since the relay assisted link is composed of the BS-relay and relay-user link, several factors such as the BS density, user density and the cell edge have to be taken into consideration.

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II. System Model

A. *Network Deployment*

We consider a Manhattan network as shown in Figure 1, where both vertical and horizontal streets are generated according to Poisson line process (PLP) Φ_s with density λ_s. In every street, we randomly place BSs as well as users according to two independent PPPs Φ_b and Φ_u, with densities λ_b and λ_u, respectively. We classify the users into two groups: *relays* and *transmitters*. *Relays* are idle users which are available to enhance the signal coming from the BS to the users with poor coverage and on the other hand, *transmitters* have data to send to their BS. We follow the LTE standard and assume that V2V communication takes place in the uplink, with a half duplex scheme. In this way, the transmitters will interfere on the relay-user link. Also, we assume that the BSs perform orthogonal scheduling and we can neglect the interference from transmitters within the cell. We consider half of the users to be relays, i.e, $\lambda_r = 0.5\,\lambda_u$, and therefore the transmitter density is $\lambda_t = 0.5\,\lambda_u$.

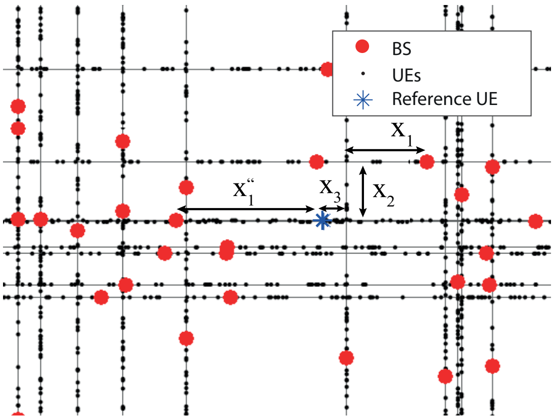

Fig. 1: System model comprising streets, BSs and users with two examples of BS-user paths, x_s and $x_s^{"}$.

B. *Pathloss model*

The pathloss model we employ identifies whether a transmitter is in the same street as the receiver, in a perpendicular street or in a parallel street to the receiver. The model is similar to the model developed in [11], based on ray tracing and measurement results. The distance between transmitter and receiver is split into several segments x_s. Each segment represents the shortest street length that the signal has to propagate through to reach the receiver. We consider that the contributions of different paths are small enough to be neglected. The distance dependent pathloss function is given by

$$L(x_s) = x_1^{\alpha_L} \prod_{s=2}^{S} x_s^{\alpha_N}. \quad (1)$$

In the Manhattan grid scenario, the maximum number of segments is $\max(S) = 3$ when the transmitter and receiver are placed in parallel streets. An example is shown in Figure 1. In a LOS link, $S = 1$ and $x_s^{"} = x_1^{"}$. In the non line-of-sight (NLOS) link depicted on the picture, $S = 3$ and $x_s = \{x_1, x_2, x_3\}$. The path from the transmitter to the receiver in a line-of-sight (LOS) scenario, or to the first corner in a NLOS scenario has a pathloss exponent α_L whereas the pathloss exponent of the remaining segments is α_N.

III. Coverage Analysis

This section is focused on finding the coverage probability expression of both direct and relay assisted links. First, some preliminary expressions are presented. In order to evaluate the SINR of the direct link we should know the distance distribution to the attached BS. With regard to the relay-user link, since we do not consider interference from transmitters within the cell, we need to find out the distance distribution to the cell edge to discard the interferers closer than such distance. The expressions are presented in Section III-A, and in Section III-B we characterize the interference.

A. *Cell edge and nearest BS distance distribution*

According to the results of [12], we can assume that the users are attached to a LOS BS with a probability close to one. To find out the distance distribution to the cell boundary, we consider the first interfering BS and assume that both boundaries are symmetric from the point of view of the user. The distance distribution to the closest BS is given by

$$f_r(r) = 2\lambda_b \exp(-2\lambda_b r). \quad (2)$$

The work in [9] presents the distance distribution to the second neighbour r', that in our case is the first interfering BS, when the distance to the first neighbour r is given for a 2-dimensional PPP. The derivation can be easily applied to the one-dimensional case. By this, the probability density function (PDF) distance to the closest interfering BS given r is expressed by

$$f_{r'}(r'|r) = 2\lambda_b \exp(-2\lambda_b(r' - r)). \quad (3)$$

To find out the distance to the cell edge d_{cell} we can perform the change of variable $d_{cell} = \frac{r'+r}{2}$. With that, the PDF of the distance to the cell edge given a distance r to the nearest BS is given by

$$f_{d_{cell}}(d_{cell}|r) = 4\lambda_b \exp(-4\lambda_b(d_{cell} - r)). \quad (4)$$

B. *Interference mapping*

Since the pathloss model employed is divided in three segments, to find the coverage probability given a distance to the attached BS r we cannot follow the same derivation of previous literature for two-dimensional PPPs [10]. The authors in [11] present the coverage probability expression with a one-dimensional PPP, but the expression depends not on the distance but on the channel gain. Therefore, if we want to find the optimal position of the relay we should have an expression depending on the BS-relay or user-relay distance. We have three different interference sources: from BSs in the same street as the user, and BSs in perpendicular and parallel streets. The work in [11] concludes that the interference contribution from BSs in parallel streets can be neglected. With

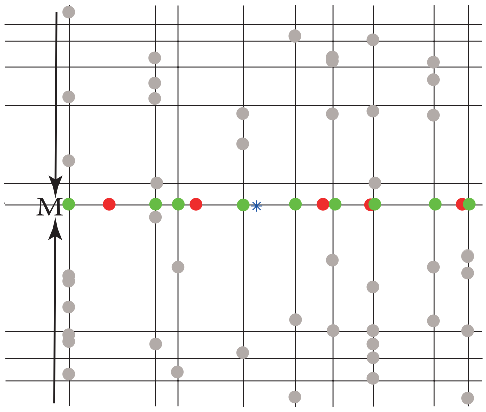

Fig. 2: System model comprising streets and BSs with the interference mapping approach. Red dots are the LOS BSs, grey dots are the original positions for the NLOS BSs and green dots represent the mapped NLOS BSs.

this assumption, we would have an interference expression depending on several PPPs: one depending on λ_b for LOS BSs, one following λ_s modeling the streets, and for each street another PPP following λ_b to represent the BSs in NLOS. In this scenario, many parameters contribute and the complexity of the interference expression is rather high. Because of that, we propose an approximation consisting of merging all interferers contributions from a perpendicular street to a single point in the street where the user is placed. We define a mapping constant M^{BS} as the BSs interference average of a single lane following [13]:

$$M^{BS} = \mathbb{E}(I) = 2\lambda_b \left(1 + \frac{1}{\alpha_L - 1}\right). \tag{5}$$

Since the current analysis is in downlink, the first segment, as explained in the pathloss model, is determined by α_L. After the mapping to the street where the user is placed, we have two sources of interference: the BSs placed in the same street, following a PPP with density λ_b, and the contribution of BSs in perpendicular streets, which are mapped onto the street following a PPP with density λ_s. In Figure 2 we plot a graphical representation of the mapping. With that, the total interference is given by:

$$I = I_L + I_N = \sum_{x_i \in \Phi_b} g_i x_i^{-\alpha_L} + M^{BS} \sum_{x_i \in \Phi_s} g_i x_i^{-\alpha_N} \tag{6}$$

We assume independent and identically distributed Rayleigh fading for both attached and interfering links, with $h, g \sim \exp(\mu, \sigma^2)$ and noise power N, the SINR expression can be written as:

$$\gamma = \frac{h r^{-\alpha}}{N + \sum\limits_{x_i \in \Phi_b} g_i x_i^{-\alpha_L} + M^{BS} \sum\limits_{x_i \in \Phi_s} g_i x_i^{-\alpha_N}} \tag{7}$$

After all users are attached to the BS that gives the smallest pathloss, we should pick the best relay among the available relays in the cell, considering that the relay boosts the signal coming from the BS. The relay-assisted link is divided into two different links: BS-relay and relay-user link. The quality of the resulting link is given by the individual link with the lower channel gain. The chosen relay should thus meet the condition:

$$j = \max_i \left(\min(\gamma_{\text{BS-relay},i}, \gamma_{\text{relay-user},i})\right) \forall i \,\epsilon \Phi_r \tag{8}$$

C. Coverage probability

We present the coverage probability expression of the direct link, as well as the individual links of the relay-assisted link.

1) Direct Link: First, we compute the coverage probability of the direct link relative to an SINR threshold T, assuming that the user is always attached to a BS at a distance r from the user.

$$\begin{aligned} p_c^{BS\text{-}u}(T|r) &= \mathbb{P}[\gamma > T|r] \\ &\overset{(a)}{=} \mathbb{P}[h > TR^{\alpha}(N+I)|r] \\ &\overset{(b)}{=} \mathbb{E}_I[\exp(-Tr^{\alpha}(N+I))] \\ &\overset{(c)}{=} \mathbb{E}_I[\exp(-Tr^{\alpha}(N+I_L+I_N))] \\ &\overset{(d)}{=} \exp(-Tr^{\alpha}N)\mathcal{L}_L^{BS\text{-}u}(Tr^{\alpha})\mathcal{L}_N^{BS\text{-}u}(Tr^{\alpha}) \end{aligned} \tag{9}$$

where (a) is based on the SINR definition written in (7), (b) follows the assumption of Rayleigh fading channel, (c) follows (6) and (d) is the Laplace transform of the interference following the same derivation as in [10]. With the interference mapping approach, the interference Laplace transform can be divided into two terms: $\mathcal{L}_L^{BS\text{-}u}(Tr^{\alpha})$ is the interference Laplace transform from LOS BSs, and $\mathcal{L}_N^{BS\text{-}u}(Tr^{\alpha})$ the interference Laplace transform from mapped NLOS BSs.

$$\mathcal{L}_{I_L}^{BS\text{-}u}(Tr^{\alpha}) = \exp\left(-2\lambda_b \int_r^{\infty} \frac{1}{1 + T^{-1}(\frac{x}{r})^{\alpha_L}} dx\right) \tag{10}$$

$$\mathcal{L}_{I_N}^{BS\text{-}u}(Tr^{\alpha}) = \exp\left(-2\lambda_s \int_r^{\infty} \frac{1}{1 + (M^{BS}T)^{-1}(\frac{x}{r})^{\alpha_N}} dx\right) \tag{11}$$

Note that in (11) the lower integration bound is r, since there are no BSs between the attached BS and the user, while in (11) the lower integration bound should be 0, since there might be streets between attached BS and the user. Nevertheless, our interference mapping approach results in a higher level of interference than the exact one. To compensate that effect, we assume that the BS placed in the streets between attached BS and user do not interfere. The validation of our interference mapping approximation is presented in Figure 4 of Section IV. By this we can compute the coverage probability of the direct link:

$$p_c^{BS\text{-}u}(T) = \int_0^{\infty} p_c^{BS\text{-}u}(T|r) f_r(r) dr \tag{12}$$

where $f_r(r)$ is given by (2).

2) Relay assisted Link: In this section, we compute the relay assisted link coverage probability for a given distance r to the BS. As shown in Figure 3, the BS-relay link is represented by the distance r_2, while the relay-user link by r_1.

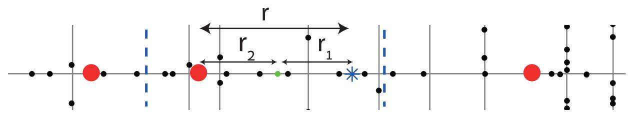

Fig. 3: Scenario comprising BSs, plotted in red, and users, represented in green. The reference user is denoted by a blue asterisk and the selected relay by a green dot.

First, we investigate the BS-relay link. The analysis is similar to the direct link. Now, the distance between transmitter and receiver is r_2, and since we have assumed that the relay is located between the BS and the user, the lower bound of the Laplace transform integral, i.e., the minimum distance to the strongest interferer, is r_2. Nevertheless, this holds only for one side, since for the side where the user is placed (right part in Figure 3), such distance would be $r + r_1$. For the sake of simplicity we will assume r_2 for both sides, which overestimates the interference slightly.

Then, we find the coverage probability of the relay-user link, assuming that they are at a distance of r_1. Since we assume that the BS performs orthogonal scheduling, and therefore the users in our cell do not interfere in the uplink, the lower bound of the interference Laplace transform changes according to the cell boundary, which depends on r as shown in (4). Therefore, the coverage probability of the relay-user link is given by the expression

$$p_c^{r\text{-}u}(T, r_1) = \int_0^\infty \int_0^{d_{cell}} \exp(-T r_1^\alpha N) \mathcal{L}_L^{r\text{-}u}(T r_1^\alpha) \mathcal{L}_N^{r\text{-}u}(T r_1^\alpha) f_{d_{cell}}(d_{cell}|r) dd_{cell} f_r(r) dr \tag{13}$$

where $f_{d_{cell}}(d_{cell}|r)$ is defined in (4), $f_r(r)$ in (2) and

$$\mathcal{L}_{I_L}^{r\text{-}u}(Tr^\alpha) = \exp\left(-2\lambda_b \int_{d_{cell}}^\infty \frac{1}{1 + T^{-1}(\frac{x}{r_1})^{\alpha_L}} dx\right), \tag{14}$$

$$\mathcal{L}_{I_N}^{r\text{-}u}(Tr^\alpha) = \exp\left(-2\lambda_s \int_{d_{cell}}^\infty \frac{1}{1 + (M^{tx}T)^{-1}(\frac{x}{r_1})^{\alpha_N}} dx\right), \tag{15}$$

where M^{tx} is the interference mean in one lane caused by the transmitters, similar to expression (5), but taking into account the transmitter density instead of the BS density.

IV. Performance Evaluation

In this section we present simulation results from 3.000 Monte Carlo system level simulation realizations with different parameter sets in a square simulation area of $4.8\,\text{km}^2$, carried out with the Vienna 5G System Level Simulator [14]. The simulation parameters are summarized in Table I.

In Figure 4, we plot the coverage probability of the direct link and compare it to the theoretical expression given by (12). As one can see, simulation results and analytical results almost overlap. We plot the results for different values of λ_b and λ_s. The highest street density and highest BS density yields the worst performance since it increases the interference level. Nevertheless, it also increases the probability to have a nearer BS and thus the difference between the results for the parameter sets are not significantly large.

TABLE I: Simulation Parameters

Parameter	Value
Pathloss exponent in LOS, α_L	2
Pathloss exponent in NLOS, α_N	4
Noise power spectral density, W_0	-174 dBm/Hz
Bandwidth, B	10 MHz
Fading mean and variance, $\{\mu, \sigma^2\}$	$\{0, 1\}$

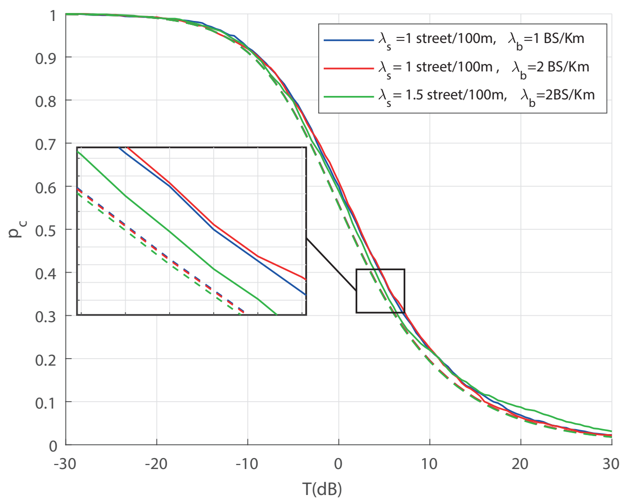

Fig. 4: Theory results (dashed line) of the coverage probability with interference mapping and simulation results (solid line) over BS and street density.

In order to evaluate the relation between the relay position and the relay density, we fix the BS-user distance r and integrate the coverage probability expressions shown in (13) and (12) to analyze the average SINR. We sweep over the distance relay-user r_1, and therefore over the BS-relay distance $r_2 = r - r_1$ as well. In Figure 5 the results for $r = 200\,m$ for several users densities are shown. Note that the user density only plays a role in the interference term of the relay-user link, since we sweep over r_1 and do not consider the user density to select the best relay in the cell. If we consider a group of potential relays, the chosen relay according to (8) would be the closest one to the optimal point. Therefore, to maximize the SINR of the relay assisted link for a given street density, the optimal relay position is located where the curves of the relay-user SINR and BS-relay intersect. When the user density increases, the optimal point gets closer to the user, i.e., r_1 becomes smaller, since the interference from the users in neighboring cells becomes higher.

In Figure 6 we plot the coverage probability of the direct link and the combined link, i.e, the best link of the relay-assisted link and the direct link for a fixed BS and street density. In this way, we quantify the coverage enhancement of the overall network by utilizing relay transmission. The best coverage is achieved with the highest user density, since with

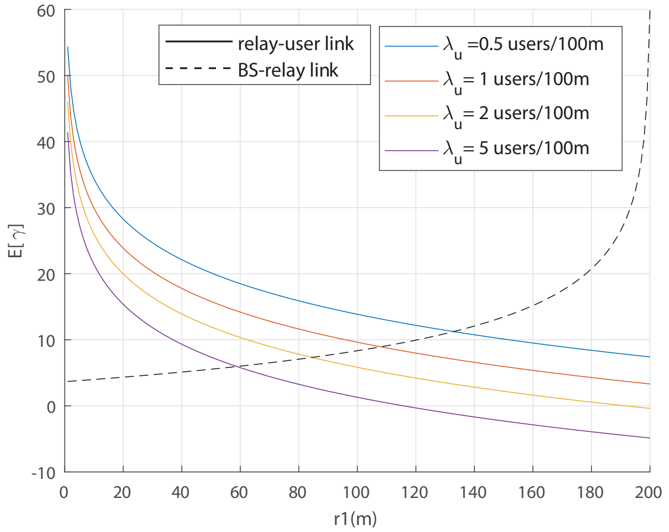

Fig. 5: Average SINR for the relay-user link (solid lines), and the BS-relay link (dashed line), for $\lambda_b = 2\,\text{BS/km}$ and $\lambda_s = 2\,\text{streets/km}$.

a higher user density, the probability to have a relay close to the optimal position is higher, even though the average SINR is smaller, as shown in Figure 5.

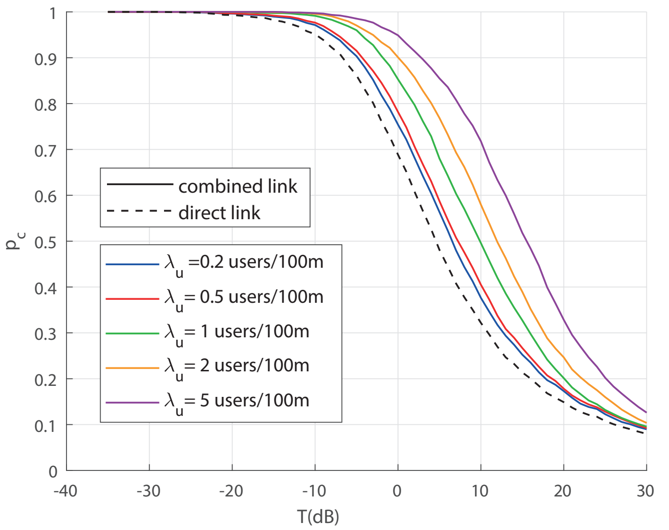

Fig. 6: Coverage probability of the direct link (dashed line) and the combined link (solid lines) for several users densities, $\lambda_b = 2\,\text{BSs/km}$ and $\lambda_s = 2\,\text{streets/km}$.

V. Conclusions

This paper presents a coverage analysis of a Manhattan grid scenario when idle users are utilized to relay the signal from the BS, and hence improve the V2I links. We provide analytical expressions of the coverage probability for the direct link and the individual parts of the relay assisted link. To elaborate this, we propose an approximation consisting of a mapping constant to deal with the interference from perpendicular streets. We validate the approach comparing analytical results with simulation results. Furthermore, we calculate the optimal position for the relay to maximize the SINR of the relay-assisted link. We also show the coverage improvement of the entire network achieved with vehicular relaying dependent on the user density.

Acknowledgements

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