Underwater Acoustic Channel Estimation and Equalization via Adaptive Filtering and Sparse Approximation

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***Abstract*—This paper presents a method for the identification and equalization of an underwater acoustic (UWA) channel, which is modeled as a Multi-Scale Multi-Lag (MSML) channel. The proposed approach consists of identifying the parameters of the different paths which form the UWA model using a bank of adaptive subfilters, which are applied to scaled versions of the transmitted signal and updated by considering the channel sparseness property. We first verify the accuracy of the identification procedure and then advance to a channel equalization stage using the parameters obtained during the identification process. The equalization performance is evaluated for different signal-to-noise ratios.**

***Index Terms*—Underwater acoustic channel modeling, wireless transmission, adaptive filtering, sparse systems**

I. INTRODUCTION

Underwater wireless communication is a difficult task [1]. To process high rate transmissions in an underwater channel, methods classically used in air transmissions can be employed, as for example the OFDM multicarrier modulation. This kind of modulation allows the transmission of an M-size data block on M subcarriers during a T_0 time interval, instead of sending the data sequence on one carrier during the same time interval, decreasing the transmission rate of each subcarrier but maintaining the global transmission rate [2]. Nevertheless, due to the fact that electromagnetic waves propagate badly in water, it is necessary to use another type of wave, such as acoustic waves. In fact, various copies of the transmitted signal reach the receiver, but each one following a different path, with a specific attenuation and a specific delay to suit different path lengths. It should also be observed that each path suffers from a different doppler scale due to the relative movement between transmitter and receiver. Besides, underwater channels can be considered as sparse channels, meaning that there is a relatively small number of paths through which the signal manages to reach the receiver.

This paper presents a method based on adaptive filtering techniques to take advantage of the sparseness of the underwater channel to identify and equalize the unknown channel. Most of the prior works have designed channel estimation algorithms for UWA channels by assuming a single dominant scale [7], [8], which can lead to performance degradation. In [4], [9], classical subspace methods from array signal processing were adapted to estimate the channel parameters, considering multiple scales. While offering good performance, these methods suffer from high computational complexity as they require multiple singular value decompositions or matrix inverses. In contrast, our proposed method is composed of an adaptive filter bank, such that each subfilter is associated to a scaling value on a doppler scale grid for MSML channel identification. We additionally exploit the inherent sparsity of underwater acoustic channels to enhance the quality of channel estimation. The proposed method offers performance improvements with lower complexity than what is achieved by other structured channel estimation techniques.

II. TRANSMITTED SIGNAL AND UNDERWATER CHANNEL MODEL

This section presents the properties of the transmitted signal and the underwater acoustic channel model adopted in this work.

A. Transmitted signal

The original transmitted sequence is a binary sequence $\mathbf{B}$, whose bits are grouped in q-size bit blocks to produce a 2^q-PSK signal. This sequence is denoted as $\mathbf{c}$. Symbols of the sequence $\mathbf{c}$ are grouped into M-size symbol blocks to form blocks of a zero-padding (ZP) OFDM signal, whose duration is $T = T_0 + T_g$, where T_0 is the data length of the OFDM block and T_g is the guard interval during which the signal is filled with zeros. An OFDM block is defined as

$$\tilde{s}(t) = \left[\sum_{k=0}^{M-1} c[k] e^{j2\pi \frac{k}{T_0} t} p(t)\right] e^{j2\pi f_c t} \quad (1)$$

where f_c is the carrier frequency, M is the subcarrier number, $p(t)$ is the rectangular pulse from 0 to T_0 and $c[k]$ is the k-th element of $\mathbf{c}$. It is this signal $\tilde{s}(t)$ that will be transmitted through the underwater channel.

B. Modelling the underwater channel

Underwater channels allow, in general, the propagation of an acoustic signal in all directions. But the signal reaches

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the receiver only through a few specific directions. In each direction that serves as a path for the transmitted signal, the following distortion effects can be observed: attenuation and delay, which depend on the followed path, and doppler scale, which depends on the followed path and on the relative speed of the receiver with respect to the transmitter. The doppler scale acts as a resampling operation when considering a discrete-time signal. Therefore, the received signal $\tilde{r}(t)$ can be expressed as [6]

$$\tilde{r}(t) = \sum_{i=1}^{I} a_i \tilde{s}((1+\beta_i)t - \tau_i) + v(t) \tag{2}$$

where a_i is the attenuation factor, τ_i is the delay, β_i is the scaling factor, and I is the number of signal paths from the transmitter to the receiver. In the case of an underwater channel, I can be considered small (sparse channel assumption) [6] and $v(t)$ a white gaussian noise, independent from the transmitted signal.

Considering the shape of the received signal, the use of zero-padding in (1) is justified. In fact, without zero-padding, the most delayed paths would interfere with the first samples of the next OFDM block, thereby generating intersymbol interference (ISI). This interference can be avoided by adopting a guard interval $T_g > \tau_{max}$, where τ_{max} is the largest delay.

Another error source, the inter carrier interference (ICI), is caused by the loss of orthogonality among the subcarriers due to frequency scaling by doppler effect. The compensation of this type of interference will be addressed in Section III.

The appropriate identification of the underwater channel model requires accurate estimation of the parameters a_i, τ_i and β_i, as shown next.

III. UNDERWATER CHANNEL IDENTIFICATION

In this paper adaptive filtering is employed to identify the channel model parameters. Classic adaptive filtering fits the identification of a multi-lag (ML) channel, by comparing a reference signal to the output of the unknown channel to which the reference is applied. This procedure, however, is not suitable to multi-scale multi-lag (MSML) channels, because of the different scales. The proposed solution consists of using an adaptive filter bank, such that each filter is associated to an appropriate scaling value. Each filter carries out a channel identification operation by comparing the received signal (channel output) to a resampled version of the transmitted reference signal (channel input). A block diagram of this filter bank is shown in Fig.1, where a known reference OFDM block is used (called training block) by the receiver, to enable comparison with the received signal, and then process the channel identification. Each β_i block stands for a resampling operation by a factor $1+\beta_i$, and each AF$_i$ block is an L-th order finite impulse response (FIR) filter, whose coefficients are the elements of the vector $\mathbf{w}_i(k) = \left[w_0^{(i)}(k)\ w_1^{(i)}(k) \cdots w_L^{(i)}(k)\right]^T$. The optimal subfilter coefficients $w_j^{(i)}(k)$ can be obtained by means of an adaptive algorithm, such as the Affine Projection Algorithm (APA).

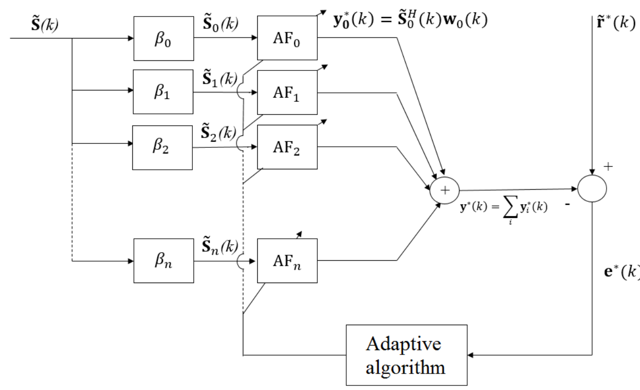

Fig. 1. Block diagram of the adaptive filter bank.

We call the resulting algorithm, derived from the APA and associated to a doppler scale grid, of MSMLAPA. The received (desired) signal is $\tilde{\mathbf{r}}^*(k) = [\tilde{r}(k)\ \tilde{r}(k-1) \cdots \tilde{r}(k-Q)]^H$ and

$$\tilde{\mathbf{S}}_i(k) = \begin{bmatrix} \tilde{s}_i(k) & \tilde{s}_i(k-1) & \cdots & \tilde{s}_i(k-Q) \\ \tilde{s}_i(k-1) & \tilde{s}_i(k-2) & \cdots & \tilde{s}_i(k-Q-1) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{s}_i(k-L) & \tilde{s}_i(k-1-L) & \cdots & \tilde{s}_i(k-Q-L) \end{bmatrix} \tag{3}$$

is the input signal matrix of AF$_i$, where Q is the reusing factor of the APA. If we suppose that each path is affected by a different doppler scale, then the expected identification result is that each filter contains at most one non-zero component, which indicates that an underwater channel is a sparse system. This sparseness property is exploited in Section III-A.

A. *Adaptive algorithms for sparse system identification*

Due to the sparseness of the selected underwater channel model, the filter bank is made of filters whose coefficients are mostly equal to zero. Adaptive algorithms have been developed with the purpose of increasing the convergence rate for application in sparse systems. The sparseness index of each subfilter coefficient vector $\mathbf{w}_i(k)$ can be evaluated by using the l_0-norm $\|\mathbf{w}_i(k)\|_0$, which gives the number of non-zero components. The cost function of the l_0-norm minimization algorithm is[1]

$$\underset{\mathbf{w}_i(k+1)}{\text{argmin}}\ \frac{1}{2}\|\mathbf{w}_i(k+1) - \mathbf{w}_i(k)\|^2 + \alpha\|\mathbf{w}_i(k+1)\|_0$$

subject to

$$\tilde{\mathbf{r}}^*(k) - \sum_{i=0}^{n} \tilde{\mathbf{S}}_i^T(k)\mathbf{w}_i^*(k+1) = \mathbf{0}$$

[1]In the case of APA, $\alpha = 0$.

Using the Lagrange multipliers technique, this constrained optimization problem can be transformed into an unconstrained one, thereby yielding:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \tilde{\mathbf{S}}_i(k)\left(\tilde{\mathbf{S}}_i^H(k)\tilde{\mathbf{S}}_i(k)\right)^{-1}\mathbf{e}^*(k) + \frac{\gamma}{2}\left[\tilde{\mathbf{S}}_i(k)\left(\tilde{\mathbf{S}}_i^H(k)\tilde{\mathbf{S}}_i(k)\right)^{-1}\tilde{\mathbf{S}}_i^H(k) - \mathbf{I}\right]\nabla\|\mathbf{w}_i(k+1)\|_0 \tag{4}$$

where $\mathbf{e}^*(k) = \tilde{\mathbf{r}}^*(k) - \sum_{i=0}^{n}\tilde{\mathbf{S}}_i^T(k)\mathbf{w}_i^*(k)$. The l_0-norm of a vector $\mathbf{x}$ can be approximated as

$$\|\mathbf{x}\|_0 \approx \sum_{i=0}^{M-1}\left(1 - e^{-\rho|x_i|}\right) \tag{5}$$

where x_i, $i = 0, 1, \cdots, M-1$, are the components of $\mathbf{x}$, and hence it follows that

$$\nabla\|\mathbf{x}\|_0 = \mathbf{f}_\rho(\mathbf{x}) = [f_\rho(x_0)\ f_\rho(x_1)\cdots f_\rho(x_{M-1})]^T \tag{6}$$

where

$$f_\rho(x_i) = \frac{\partial\|\mathbf{x}\|_0}{\partial x_i} \approx \rho\text{sign}(x_i)e^{-\rho|x_i|} \tag{7}$$

In our simulations we observed that a good approximation is obtained with $\rho = 0.75$. Since the l_0-norm gradient does not vary too much from an iteration to the next one, we obtain

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu\tilde{\mathbf{S}}_i(k)\left(\tilde{\mathbf{S}}_i^H(k)\tilde{\mathbf{S}}_i(k) + \delta\mathbf{I}\right)^{-1}\mathbf{e}^*(k) + \mu\frac{\gamma}{2}\left[\tilde{\mathbf{S}}_i(k)\left(\tilde{\mathbf{S}}_i^H(k)\tilde{\mathbf{S}}_i(k) + \delta\mathbf{I}\right)^{-1}\tilde{\mathbf{S}}_i^H(k) - \mathbf{I}\right]\mathbf{f}_\rho(\mathbf{w}_i(k)) \tag{8}$$

This algorithm is called Affine Projection for Sparse System Identification (AP-SSI) [3]. Morover, by ignoring the constraint of the *a posteriori* error as being equal to zero, it leads to the Quasi AP-SSI (QAP-SSI) algorithm, with simplified update equation

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu\tilde{\mathbf{S}}_i(k)\left(\tilde{\mathbf{S}}_i^H(k)\tilde{\mathbf{S}}_i(k) + \delta\mathbf{I}\right)^{-1}\mathbf{e}^*(k) + \mu\frac{\gamma}{2}\mathbf{f}_\rho(\mathbf{w}_i(k)) \tag{9}$$

Associated to a doppler scale grid for MSML channel identification, we refer to this algorithm as MSMLQAP-SSI.

B. Performance of the adaptive algorithms for underwater channel identification

In this section, we present simulation results illustrating the performance of the MSMLQAP-SSI. These results are compared with the ones of MSMLAPA, which does not take into account the system sparseness.

To begin, we consider an underwater channel called CHAN01 with two paths, defined by the parameters (a_0, τ_0, β_0)=(1, 1, 0) and (a_1, τ_1, β_1)=(23, −0.78, 10^{-4}), and by an ambient noise with SNR= 20 dB in the input of the receiver. Assuming that the doppler scales β_i are known *a priori*, we build a grid $G = \{\beta_0, \beta_1\}$, in which β_0 and β_1 are

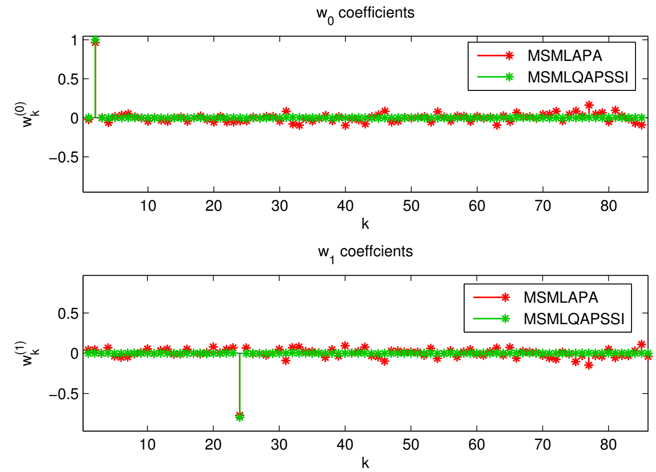

Fig. 2. Subfilters obtained after convergence of channel identification algorithms for CHAN01.

associated, respectively, to the FIR filters $\mathbf{w}_0$ and $\mathbf{w}_1$, both of order $N = \lceil T_g/T_e \rceil = 85$, where T_e is the signal sampling period. Accordingly, it is expected that each subfilter identifies the delay and the attenuation of the path that is affected by the doppler scale associated to the subfilter. The filters obtained by MSMLAPA and MSMLQAP-SSI algorithms are shown in Fig. 2. In this figure, we observe that each subfilter reached the correct identification of the component affected by a doppler scale equal to that associated to the considered subfilter. In addition, the MSMLQAP-SSI, which focuses on the sparseness of the solution, sets to zero the coefficients of the filters that do not correspond to a path of the underwater channel in a more efficient way than the MSMLAPA.

Let us now consider the case of an underwater channel, called CHAN02, which consists of 5 paths having SNR= 15 dB. In this case, we do not assume a priori knowledge of the doppler scales. It is thus necessary to use a doppler scale grid with enough values to improve the probability to find the scales that are acting in the signal propagation amongst the grid values. For the CHAN02 channel identification, it is known, due to the fact that it is a simulated channel, that for the five scales the grid assumes values in the interval from -5×10^{-4} to 5×10^{-4}. Accordingly, we chose the grid $G = \{-5 \times 10^{-4}, -4 \times 10^{-4}, \cdots, 5 \times 10^{-4}\}$, which contains 11 values. The channel identification is processed using MSMLAPA and MSMLQAP-SSI, whose performances are evaluated by computing the mean square error (MSE) and the coefficient misalignment over time, as shown in Figs. 3 and 4, respectively. In these figures, we can see that the algorithm convergence is not affected in case one grid contains more values than the propagation path number. This indicates that it is not necessary to know *a priori* the doppler scale values, as in the previous circustance. Moreover, we observe the main advantage of using MSMLQAP-SSI by comparing the MSE (−14 dB for MSMLQAP-SSI versus

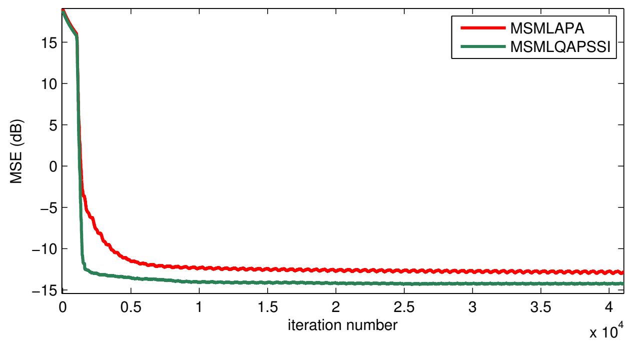

Fig. 3. MSE evolution for the identification of channel CHAN02.

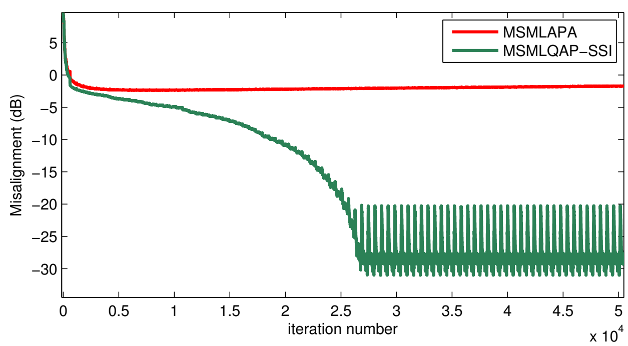

Fig. 4. Misadjustment evolution for the identification of channel CHAN02.

-12 dB for the MSMLAPA) and the misalignment (only -2 dB for MSMLAPA versus -25 dB for MSMLQAP-SSI). In conclusion, in spite of a fast convergence of MSE (around 10^4 iterations for MSMLQAP-SSI), more iterations (around 3×10^4) are necessary to obtain correct estimations of the channel parameters. In conclusion, we note that the use of a bank of adaptive sparse subfilters is able to identify the parameters (a_i, τ_i, β_i) of an underwater channel. Associated to its own specific scaling, each subfilter selects, from the received signal, attenuated and delayed copies of the reference signal scaled by the same value as that of the associated subfilter. We observed a convergence improvement when the adaptive algorithm was applied to sparse systems, in comparison with the conventional adaptive algorithm. Accurate identification is an important first step for the equalization of the received signal.

IV. Equalization of underwater channel

A. Computation of equalization

After estimating the parameters (a_i,τ_i,β_i) with the help of a reference signal known *a priori* by the receiver, we proceed to the equalization step, which consists of estimating the original emitted signal from the received signal, knowing the distortions generated by the channel. The received signal

$$\tilde{r}(t) = \sum_{i=0}^{I-1} a_i \tilde{s}((1+\beta_i)(t-\tau_i')) + v(t) \tag{10}$$

which is a reformulation of (2) with $\tau_i' = \tau_i/(1+\beta_i)$, is applied to a demodulation step so that the data sent through each subcarrier of the OFDM signal can be extracted. The symbol r_m received on the m-th subcarrier is given by

$$r_m = \frac{1}{T_0} \int_0^{T_0+T_g} \tilde{r}(t) e^{-j2\pi f_c t} e^{-j2\pi \frac{m}{T_0} t} \mathrm{d}t. \tag{11}$$

Combining (1), (10) and (11), we obtain

$$\mathbf{r} = \mathbf{H}\mathbf{c} + \mathbf{v} \tag{12}$$

where

$$[\mathbf{H}]_{m,k} = \sum_{i=0}^{I-1} \frac{a_i}{1+\beta_i} e^{-j2\pi f_m \tau_i'} \mathrm{sinc}\left(\pi \phi_{m,k}^{(i)} T_0\right) e^{j\pi \phi_{m,k}^{(i)} T_0} \tag{13}$$

$$\phi_{m,k}^{(i)} = \frac{k-m}{T_0} + \frac{f_m \beta_i}{1+\beta_i} \tag{14}$$

with $f_m = f_c + \frac{m}{T_0}$, $\mathbf{c} = [c[0]\ c[1] \cdots c[M-1]]^T$ is the originally sent PSK data vector, $\mathbf{r} = [r_0\ r_1 \cdots r_{M-1}]^T$ is the received data vector after OFDM demodulation and $\mathbf{v}$ is the ambient noise vector. Therefore, $\mathbf{c}$ can be estimated by $\hat{\mathbf{c}} = \mathbf{H}^{-1}\mathbf{r}$. Singular Value Decomposition (SVD) algorithm [5] is used to invert the matrix $\mathbf{H}$. Applying the Euclidian distance criterion to find $\hat{c}[k]$, which is the closest symbol of the PSK constellation, we obtain $\hat{\mathbf{c}}'$, which is the vector comprising the PSK elements. From $\hat{\mathbf{c}}'$, we find B', which is the estimate of the original binary sequence B. Comparing B to B' we compute the bit error rate (BER).

B. Performance of equalization

In this section, various MSML channels are simulated using different SNR values. For each channel the identification procedure is carried out with one reference OFDM block, and the parameters obtained are used to process the equalization of other nine blocks which are applied to the same channel. We therefore compute various BERs which are classified according to their SNRs (150 channels for each SNR level). The equalization step does not depend only on the accurate identification of the parameters of the channel, but also on the conditioning of the matrix. In these simulations, we used 4-PSK modulation assuming $f_c = 16,384$ kHz, sampling frequency $f_s = 40,960$ kHz, $M = 512$ subcarriers, $\lfloor M/6 \rfloor = 85$ samples to fill the interval guard with zero padding and an additive white gaussian noise as ambient noise.

Analysing the BER distribution for channels that have the same SNR level, we choose to extract three measures of interest: the mean, the median and the most likely value. These measures are shown in Fig. 5. For a fixed SNR value, it can be observed that the BER distribution is far from a gaussian distribution. Figure 5 confirms this observation by showing

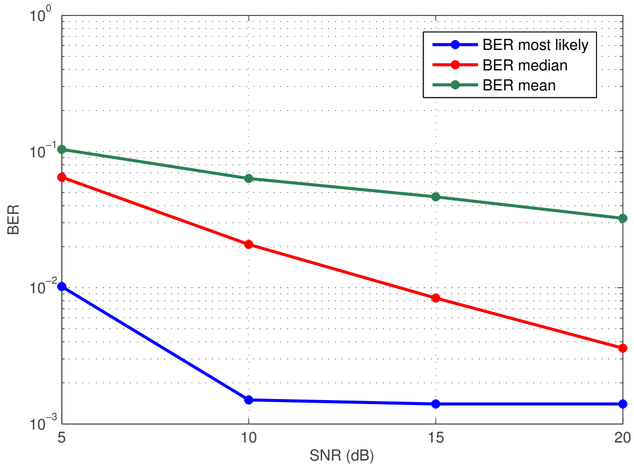

Fig. 5. BER (mean, median, most likely value) as a function of SNR regarding the equalization of various channels composed of 3 paths.

that the mean, the median and the most likely values are different. These results also point out that the mean value of BER is pushed up by some blocks that are affected by a bad performance of the algorithm, but most of the blocks were equalized with a BER close to 0, as indicated by the most likely value curve.

V. Conclusions

In this paper an efficient method for underwater acoustic channel identification was advanced. This method uses an adaptive filter bank, in which each subfilter identifies the components that are scaled by a specific value. The adaptive algorithm MSMLQAP-SSI exploited the sparseness property of the channel model by minimizing an l_0-norm approximation of each subfilter coefficient vector. Simulation results showed the advantage of using an appropriate algorithm for sparse system identification and the effectiveness of the proposed model in channel equalization applications, as verified by the BER performance.

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