TENSOR-TRAIN MODELING FOR MIMO-OFDM TENSOR CODING-AND-FORWARDING RELAY SYSTEMS

Yassine Zniyed[⋆] *Rémy Boyer*[*] *André L. F. de Almeida*[†] *Gérard Favier*[‡]

[⋆]L2S, Univ. Paris-Sud, CNRS, CentraleSupélec, Gif-sur-Yvette, France
[*]CRIStAL, Université de Lille, Villeneuve d'Ascq, France
[†]Department of Teleinformatics Engineering, Federal University of Fortaleza, Brazil
[‡]Laboratoire I3S, Université Côte d'Azur, CNRS, Sophia Antiplois, France

ABSTRACT

In this paper, we consider a new one-way two-hop amplify-and-forward (AF) relaying scheme with a tensor space-time coding under frequency-selective fading channels. The signals received at the destination of the multi-input multi-output (MIMO) system define a 6-order tensor which satisfies a tensor-train decomposition (TTD). We propose a new TTD based receiver for a joint channel and symbol estimation. The proposed receiver avoids the use of long training sequences and resort to very few pilots to provide unique estimates of the individual channel matrices and the symbol matrix. Numerical simulations show the performance of the new proposed TTD-based semi-blind receiver.

***Index Terms*—** Channel estimation, MIMO relay systems, tensor coding, tensor-train decomposition, semi-blind receiver.

I. INTRODUCTION

The use of relay stations in MIMO communication systems has shown to have a great potential to enhance coverage and increase system capacity [8], [10], [3]. In the case of one-way two-hop MIMO relay systems, the communication can be divided into two phases. In the first one, the source node transmits the symbols to the relay. In the second one, the relay amplifies and forwards the signals to the destination node, meanwhile the source stays silent. To achieve the predictable gains of cooperative diversity, an accurate knowledge of channels associated with the multiple hops involved in the communication is required for the design of smart antenna schemes, such as beamforming or precoding [13].

In relay-assisted MIMO communication systems, the instantaneous channel for each hop is usually estimated by means of training sequences transmitted by the source and relay nodes during successive transmission phases. The works [9], [12] proposed training sequence based schemes to estimate the individual channel matrices for two-hop MIMO relaying systems. The first one [9] relies on an SVD-based solution, while the second [12] is based on a parallel factor (PARAFAC) modeling of the received signals at the destination. By aiming at a joint channel and symbol estimation in a semi-blind fashion, tensor-based receivers have been proposed in several works [15], [4] for two-hop MIMO relaying systems. Of particular interest to this work is the approach of [4], which is based on tensor coding-and-forwarding (TCF) scheme by means of a tensor space-time coding applied at both the source and relay nodes. More recently, generalizations to multihop systems [5] and to two-way systems [6] have also been proposed. All these works have assumed that the propagation channels are frequency-flat, which is the case of narrowband communication systems.

In this work, we assume a more general and practical scenario where the MIMO relaying system is operating in frequency-selective fading environment. As will be shown later, when generalizing the scheme of [4] to the wideband communication scenario, the tensor modeling of the received signals involves the estimation of larger quantities compared to the narrowband case. These quantities are represented by third-order channel tensors and one symbol matrix, the third dimension being associated to the frequency domain. By resorting to multicarrier modulation using orthogonal frequency division multiplexing (OFDM), we propose a new receiver design for a TCF MIMO-OFDM relaying system that is capable to solve the joint channel and symbol estimation problem. The proposed receiver fits the resulting 6-order received signal tensor to a tensor train decomposition (TTD), where the knowledge of the tensor coding structure is used to ensure identifiability of the channel tensors and the symbol matrix. It is important to note that our methodology is able to manage the case of any number of relays. To the best of our knowledge, this is the first work where the TTD approach is used to design a semi-blind receiver for a MIMO communication system.

The notations used in this paper are as follows. Scalars, vectors, matrices and tensors are represented by x, $\mathbf{x}$, $\mathbf{X}$ and $\boldsymbol{\mathcal{X}}$, respectively. The symbols $(\cdot)^T$ and $(\cdot)^{-1}$ denote, respectively, the transpose and the inverse. The Frobenius norm is defined by $||\cdot||_F$. $\mathbf{I}_N$ denotes the identity matrix of size $N \times N$. The matrix $\text{unfold}_k\boldsymbol{\mathcal{X}}$ of size $N_k \times N_1 \cdots N_{k-1}N_{k+1} \cdots N_Q$ refers to the k-mode unfolding of $\boldsymbol{\mathcal{X}}$ of size $N_1 \times \cdots \times N_Q$. The n-mode product is denoted by $\times_n$. The contraction product $\times_q^p$ [1] between $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{B}}$ of size $N_1 \times \cdots \times N_Q$ and $M_1 \times \cdots \times M_P$, with $N_q = M_p$ is a tensor of order $(Q + P - 2)$ such that

$$[\boldsymbol{\mathcal{A}} \times_q^p \boldsymbol{\mathcal{B}}]_{n_1,\ldots,n_{q-1},n_{q+1},\ldots,n_Q,m_1,\ldots,m_{p-1},m_{p+1},\ldots,m_P}$$
$$= \sum_{k=1}^{N_q} [\boldsymbol{\mathcal{A}}]_{n_1,\ldots,n_{q-1},k,n_{q+1},\ldots,n_Q} [\boldsymbol{\mathcal{B}}]_{m_1,\ldots,m_{p-1},k,m_{p+1},\ldots,m_P}.$$

The rest of the paper is organized as follows. Section II presents the new one-way two-hop TCF MIMO-OFDM system. In Section III, we recall the TTD and its ambiguity results. Section IV introduces the structure of the TT-cores when the system is decomposed in the TTD. Then, the proposed TTD based algorithm for the joint channels and symbols estimation is described. The performance of this latter is evaluated in Section V. Finally, the conclusions and some perspectives for future research are drawn in Section VI.

II. SYSTEM MODEL

In this paper, we consider a MIMO-OFDM relaying system, where the communication is divided into two hops[1] as illustrated in Fig. 1. The 6 dimensions/diversities of the system are associated with time, source code, frequency (during the first hop), relay code, frequency (during the second hop), and space. At both source and relay nodes, we consider a tensor space-time coding (TSTC) scheme, following the idea of [4]. Note, however, that the system model considered in this work is a generalization of that of [4] to a MIMO-OFDM system. Due to the added frequency dimensions at both the source $\rightarrow$ relay (SR), and relay $\rightarrow$ destination (RD) channels, the SR and RD channels are modeled as third-order tensors and denoted as $\boldsymbol{\mathcal{H}}^{(SR)}$ and $\boldsymbol{\mathcal{H}}^{(RD)}$. Let $\boldsymbol{\mathcal{X}}$ be the 6-order tensor, of dimensions

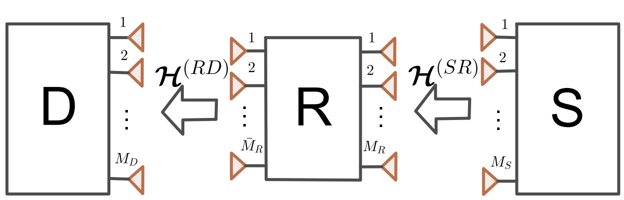

Fig. 1. One-way two-hop MIMO-OFDM relay system illustration.

$M_D \times F_1 \times K \times F_2 \times P \times N$, representing the received signals at the destination. In a free-noise scenario, this tensor $\boldsymbol{\mathcal{X}}$ can be expressed as follows:

[1]A generalization with more than two hops can be easily derived from the contribution of our work.

$$[\boldsymbol{\mathcal{X}}]_{m_D,f_1,k,f_2,p,n} = \sum_{\bar{m}_R=1}^{\bar{M}_R} \sum_{m_R=1}^{M_R} \sum_{m_s=1}^{M_S} \sum_{r=1}^{R} [\boldsymbol{\mathcal{H}}^{(RD)}]_{m_D,f_1,\bar{m}_R}$$
$$[\boldsymbol{\mathcal{C}}^{(R)}]_{\bar{m}_R,k,m_R} [\boldsymbol{\mathcal{H}}^{(SR)}]_{m_R,f_2,m_S} [\boldsymbol{\mathcal{C}}^{(S)}]_{m_S,p,r} [\mathbf{S}]_{r,n}. \quad (1)$$

We provide in Table I the description and dimensions of tensors used in eq. (1).

Table I. Description of tensors used in eq. (1).

Symbols	Description	Dimensions
$\boldsymbol{\mathcal{H}}^{(RD)}$	RD channel tensor	$M_D \times F_1 \times \bar{M}_R$
$\boldsymbol{\mathcal{C}}^{(R)}$	Coding tensor at the relay	$\bar{M}_R \times K \times M_R$
$\boldsymbol{\mathcal{H}}^{(SR)}$	SR channel tensor	$M_R \times F_2 \times M_S$
$\boldsymbol{\mathcal{C}}^{(S)}$	Coding tensor at the source	$M_S \times P \times R$
$\mathbf{S}$	Transmitted symbols matrix	$R \times N$

It is worth mentioning that the tensor $\boldsymbol{\mathcal{X}}$ results from the transmission of R data streams, each composed of N symbols, during N different time-blocks. During each block n, each antenna m_S transmits a combination of R information symbols $[\mathbf{S}]_{r,n}$ to the relay after a space-time coding by means of the coding tensor $\boldsymbol{\mathcal{C}}^{(S)}$ and through the channel $\boldsymbol{\mathcal{H}}^{(SR)}$. The signals received at the relay are encoded by means of the coding tensor $\boldsymbol{\mathcal{C}}^{(R)}$ and then transmitted to the destination through the channel $\boldsymbol{\mathcal{H}}^{(RD)}$. The received signals satisfy eq. (1). Different assumptions are considered for the model of eq. (1): (i) the coding tensors are constant during the whole transmission (ii) the channels are quasi-static, *i.e.* do not change, whitin a transmission cycle, and (iii) the coding tensors and the structural parameters (tensor dimensions) are known. Note that $M_D, \bar{M}_R, M_R,$ M_S, and R refer to the number of antennas at the destination, transmitting antennas at the relay, receiving antennas at the relay, antennas at the source, and the data streams, respectively.

III. TENSOR TRAIN DECOMPOSITION (TTD)

III-A. Definition of the TTD

Definition 1. *Let $\{R_1, \ldots, R_{Q-1}\}$ be the TT-ranks with bounding conditions $R_0 = R_Q = 1$. A Q-order tensor of size $N_1 \times \ldots \times N_Q$ admits a decomposition into a train of low-order tensors if*

$$\boldsymbol{\mathcal{X}} = \mathbf{G}_1 \times_2^1 \boldsymbol{\mathcal{G}}_2 \times_3^1 \boldsymbol{\mathcal{G}}_3 \times_4^1 \ldots \times_{Q-1}^1 \boldsymbol{\mathcal{G}}_{Q-1} \times_Q^1 \mathbf{G}_Q, \quad (2)$$

where the TT-cores $\mathbf{G}_1, \boldsymbol{\mathcal{G}}_q$, and $\mathbf{G}_Q$ are, respectively, of dimensions $N_1 \times R_1$, $R_{q-1} \times N_q \times R_q$, and $R_{Q-1} \times N_Q$, for $2 \leq q \leq Q-1$, with $\text{rank}(\mathbf{G}_1) = R_1$, $\text{rank}(\mathbf{G}_Q) = R_{Q-1}$, $\text{rank}(\text{unfold}_1\boldsymbol{\mathcal{G}}_q) = R_{q-1}$, and $\text{rank}(\text{unfold}_3\boldsymbol{\mathcal{G}}_q) = R_q$.

Using the contraction product, the model in eq. (1) can be written in a compact form such that:

$$\boldsymbol{\mathcal{X}} = \mathbf{I}_{M_D} \times_2^1 \boldsymbol{\mathcal{H}}^{(RD)} \times_3^1 \boldsymbol{\mathcal{C}}^{(R)} \times_4^1 \boldsymbol{\mathcal{H}}^{(SR)} \times_5^1 \boldsymbol{\mathcal{C}}^{(S)} \times_6^1 \mathbf{S}, \quad (3)$$

to match the definition of the TTD given in eq. (2).

III-B. The TTD multiplicative ambiguities

As shown in eq. (3), the considered MIMO system is modeled as a 6-order TTD with a priori known TT-ranks $\{M_D, \bar{M}_R, M_R, M_S, R\}$. The multiplicative ambiguities in the TTD correspond to post- and pre-multiplications by nonsingular matrices, *i.e.*, we can replace two successive TT-cores $\boldsymbol{\mathcal{G}}_q$ and $\boldsymbol{\mathcal{G}}_{q+1}$ in eq. (2), respectively, by $\boldsymbol{\mathcal{G}}'_q$ and $\boldsymbol{\mathcal{G}}'_{q+1}$ such that

$$\boldsymbol{\mathcal{G}}'_q = \boldsymbol{\mathcal{G}}_q \times_3^1 \mathbf{U}_q^{-1},$$
$$\boldsymbol{\mathcal{G}}'_{q+1} = \mathbf{U}_q \times_2^1 \boldsymbol{\mathcal{G}}_{q+1},$$

to recover the same tensor $\boldsymbol{\mathcal{X}}$ of eq. (2), where $\mathbf{U}_q$ is a nonsingular matrix of size $R_q \times R_q$.

Applying the state-of-art TT-SVD algorithm [11], [2] to tensor $\boldsymbol{\mathcal{X}}$ allows to recover the original TT-cores $\boldsymbol{\mathcal{G}}_q$ up to these nonsingular matrices, called $\mathbf{U}_q$ in the sequel. The TT-SVD algorithm is based on sequentially truncated SVD(s). Knowing the TT-ranks and in the noise-free case, the TT-SVD algorithm recovers exactly the TT-cores. In the context of the TT-SVD algorithm these nonsingular matrices correspond to transformation (*change-of-basis*) matrices due to the extraction of the dominant singular subspaces using the SVD.

IV. TT-SVD BASED RECEIVER

IV-A. TT-cores structure

The following theorem gives the structure of the TT-cores when the TT-SVD algorithm is applied to tensor $\boldsymbol{\mathcal{X}}$ in eq. (3).

Theorem 1. *Consider the* 6*-order tensor* $\boldsymbol{\mathcal{X}}$ *defined in eq.* (3)*. In the noise-free scenario and knowing a priori the TT-ranks, the structure of the recovered TT-cores is given by*

$$\begin{aligned}
&\mathbf{G}_1 = \mathbf{U}_1^{-1},\\
&\boldsymbol{\mathcal{G}}_2 = \boldsymbol{\mathcal{H}}^{(RD)} \times_1 \mathbf{U}_1 \times_3 \mathbf{U}_2^{-T},\\
&\boldsymbol{\mathcal{G}}_3 = \boldsymbol{\mathcal{C}}^{(R)} \times_1 \mathbf{U}_2 \times_3 \mathbf{U}_3^{-T},\\
&\boldsymbol{\mathcal{G}}_4 = \boldsymbol{\mathcal{H}}^{(SR)} \times_1 \mathbf{U}_3 \times_3 \mathbf{U}_4^{-T},\\
&\boldsymbol{\mathcal{G}}_5 = \boldsymbol{\mathcal{C}}^{(S)} \times_1 \mathbf{U}_4 \times_3 \mathbf{U}_5^{-T},\\
&\mathbf{G}_6 = \mathbf{U}_5\mathbf{S},
\end{aligned}$$

where $\mathbf{U}_1, \cdots, \mathbf{U}_5$ *are square nonsingular transformation matrices of ranks* $M_D, \bar{M}_R, M_R, M_S$ *and* R*, respectively, corresponding to the TT-ranks of the model.*

It is worth noting that the above theorem is the result of the application of the multiplicative ambiguities of the TTD given in eq. (3). This means that the cores of eq. (3) are estimated up to post- and pre-multiplications by nonsingular matrices as shown in Section III-B. In Theorem 1, the TT-cores follow a Tucker decomposition (TD) with the following equivalence:

$$\boldsymbol{\mathcal{T}}' = \mathbf{A}_1 \times_2^1 \boldsymbol{\mathcal{T}} \times_3^1 \mathbf{A}_2 = \boldsymbol{\mathcal{T}} \times_1 \mathbf{A}_1 \times_3 \mathbf{A}_2^T.$$

The TD formalism helps us to introduce our estimation scheme in the next section.

IV-B. Estimation algorithm: TT-MRS

In this section, we propose an estimation scheme of tensor channels $\boldsymbol{\mathcal{H}}^{(RD)}$, $\boldsymbol{\mathcal{H}}^{(SR)}$, and of the transmitted symbols matrix $\mathbf{S}$, assuming the knowledge of the code tensors $\boldsymbol{\mathcal{C}}^{(R)}$ and $\boldsymbol{\mathcal{C}}^{(S)}$. It is a TT-based semi-blind receiver for MIMO relay systems (TT-MRS). The idea of Algorithm 1 is to eliminate the latent ambiguity matrices $\mathbf{U}_1, \cdots, \mathbf{U}_5$ of Theorem 1 using the knowledge of tensors $\boldsymbol{\mathcal{C}}^{(R)}$ and $\boldsymbol{\mathcal{C}}^{(S)}$. The proposed algorithm is a 3 steps scheme that is based on: (i) the TT-SVD algorithm to decompose $\boldsymbol{\mathcal{X}}$ into the TTD, (ii) the estimation of the transformation matrices using $\boldsymbol{\mathcal{C}}^{(R)}$ and $\boldsymbol{\mathcal{C}}^{(S)}$, and (iii) channels and symbols estimation using the TT-cores of step 1 and the ambiguity matrices of step 2. It is worth mentioning that the processing of step 2 (same for step 3) can be done in a parallel way.

Note that tensors $\boldsymbol{\mathcal{C}}^{(R)}$ and $\boldsymbol{\mathcal{C}}^{(S)}$ represent the core tensors of the respective TDs of $\boldsymbol{\mathcal{G}}_3$ and $\boldsymbol{\mathcal{G}}_5$, as shown in Theorem 1, and we have

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Algorithm 1 TT-MRS algorithm

Input: 6-order tensor $\boldsymbol{\mathcal{X}}$, $\boldsymbol{\mathcal{C}}^{(R)}$ and $\boldsymbol{\mathcal{C}}^{(S)}$ defined in eq. (3).
Output: $\hat{\boldsymbol{\mathcal{H}}}^{(RD)}$, $\hat{\boldsymbol{\mathcal{H}}}^{(SR)}$, and $\hat{\mathbf{S}}$.

1: TTD: (using the TT-SVD algorithm)

$$\boldsymbol{\mathcal{X}} = \mathbf{G}_1 \times_2^1 \boldsymbol{\mathcal{G}}_2 \times_3^1 \boldsymbol{\mathcal{G}}_3 \times_4^1 \boldsymbol{\mathcal{G}}_4 \times_5^1 \boldsymbol{\mathcal{G}}_5 \times_6^1 \mathbf{G}_6.$$

2: Transformation matrices estimation:

$$\begin{aligned}
\hat{\mathbf{U}}_1^{-1} &= \mathbf{G}_1.\\
[\hat{\mathbf{U}}_2, \hat{\mathbf{U}}_3^{-T}] &= \text{Tucker-ALS}(\boldsymbol{\mathcal{G}}_3, \boldsymbol{\mathcal{C}}^{(R)}).\\
[\hat{\mathbf{U}}_4, \hat{\mathbf{U}}_5^{-T}] &= \text{Tucker-ALS}(\boldsymbol{\mathcal{G}}_5, \boldsymbol{\mathcal{C}}^{(S)}).
\end{aligned}$$

3: Channels and symbols estimation:

$$\begin{aligned}
\hat{\boldsymbol{\mathcal{H}}}^{(RD)} &= \boldsymbol{\mathcal{G}}_2 \times_1 \hat{\mathbf{U}}_1^{-1} \times_3 \hat{\mathbf{U}}_2^T.\\
\hat{\boldsymbol{\mathcal{H}}}^{(SR)} &= \boldsymbol{\mathcal{G}}_4 \times_1 \hat{\mathbf{U}}_3^{-1} \times_3 \hat{\mathbf{U}}_4^T.\\
\hat{\mathbf{S}} &= \hat{\mathbf{U}}_5^{-1}\mathbf{G}_6.
\end{aligned}$$

$$\text{unfold}_1\boldsymbol{\mathcal{G}}_3 = \mathbf{U}_2 \cdot \text{unfold}_1\boldsymbol{\mathcal{C}}^{(R)} \cdot \left(\mathbf{U}_3^{-T} \otimes \mathbf{I}_K\right)^T, \quad (4)$$
$$\text{unfold}_2\boldsymbol{\mathcal{G}}_3 = \mathbf{I}_K \cdot \text{unfold}_2\boldsymbol{\mathcal{C}}^{(R)} \cdot \left(\mathbf{U}_3^{-T} \otimes \mathbf{U}_2\right)^T, \quad (5)$$
$$\text{unfold}_3\boldsymbol{\mathcal{G}}_3 = \mathbf{U}_3^{-T} \cdot \text{unfold}_3\boldsymbol{\mathcal{C}}^{(R)} \cdot \left(\mathbf{I}_K \otimes \mathbf{U}_2\right)^T. \quad (6)$$

Regarding eq. (4) and eq. (6), we can notice that recovering matrices $\mathbf{U}_2$ and $\mathbf{U}_3$ (the same reasoning is valid for $\mathbf{U}_4$ and $\mathbf{U}_5$), using $\boldsymbol{\mathcal{C}}^{(R)}$ and $\boldsymbol{\mathcal{G}}_3$, can be done in a general case using an iterative Tucker-ALS algorithm [7].

One may note that $\mathbf{U}_2$ and $\mathbf{U}_3$ invloved in eq. (5) are estimated up to a scalar multiplication, since

$$\mathbf{A} \otimes \mathbf{B} = \alpha \cdot \mathbf{A} \otimes (\frac{1}{\alpha}) \cdot \mathbf{B}.$$

Applying this property on the proposed scheme, and taking into account that matrices $\mathbf{U}_2, \cdots, \mathbf{U}_5$ are estimated up to scalar multiplication, the outputs of Algorithm 1 are then expressed as

$$\hat{\boldsymbol{\mathcal{H}}}^{(RD)} = \alpha_1 \cdot \boldsymbol{\mathcal{H}}^{(RD)},$$
$$\hat{\boldsymbol{\mathcal{H}}}^{(SR)} = \alpha_2 \cdot \boldsymbol{\mathcal{H}}^{(SR)},$$
$$\hat{\mathbf{S}} = (\frac{1}{\alpha_1 \alpha_2}) \cdot \mathbf{S},$$

this means that the parameters of interest are estimated up to a scalar multiplication. To resolve this ambiguity, two assumptions are possible. Firstly, the knowledge of the first-order statistic (the mean) of channels $\boldsymbol{\mathcal{H}}^{(RD)}$ and $\boldsymbol{\mathcal{H}}^{(SR)}$ can be assumed as in [14], or, secondly, one may assume the knowledge of one entry of tensors $\boldsymbol{\mathcal{H}}^{(RD)}$ and $\boldsymbol{\mathcal{H}}^{(SR)}$ as assumed in [4].

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed receiver by means of numerical computer simulations for various system configurations. The aim of the following experiments is twofold. First, we want to validate the TT modeling for the proposed MIMO relay system, and show that the proposed method allows a joint channels and symbols estimation. Second, we want to evaluate the influence of the system configurations on the quality of estimation, in particular, the choice and interest of the new introduced parameter F. The 6-order MIMO relay system is generated with random channel and coding tensors whose elements are drawn from a Gaussian distribution with zero mean and unit variance. The transmitted symbols are uniform random variables from a 4-QAM constellation. The additive noise tensors at relay and destination, noted respectively $\boldsymbol{\mathcal{N}}^{(R)}$ and $\boldsymbol{\mathcal{N}}^{(D)}$, are assumed to be composed of elements which are zero-mean Gaussian variables, with a unit variance. The final noise, noted $\boldsymbol{\mathcal{N}}^{(SRD)}$ corresponding to $\boldsymbol{\mathcal{N}}^{(D)}$ at destination, and $\boldsymbol{\mathcal{N}}^{(R)}$ at relay filtered by the relay coding tensor $\boldsymbol{\mathcal{C}}^{(R)}$ and the channel $\boldsymbol{\mathcal{H}}^{(RD)}$ is expressed as

$$\boldsymbol{\mathcal{N}}^{(SRD)} = \boldsymbol{\mathcal{H}}^{(RD)} \times_3^1 \boldsymbol{\mathcal{C}}^{(R)} \times_4^1 \boldsymbol{\mathcal{N}}^{(R)} + \boldsymbol{\mathcal{N}}^{(D)},$$

where $\boldsymbol{\mathcal{N}}^{(R)}$ and $\boldsymbol{\mathcal{N}}^{(D)}$ are respectively of size $M_R \times F_1 \times P \times N$ and $M_D \times F_1 \times K \times F_2 \times P \times N$. The depicted NMSE are obtained by averaging the NSE over 10^4 independent Monte Carlo runs, with

$$\text{NSE} = \frac{\left\| \hat{\boldsymbol{\mathcal{X}}} - \boldsymbol{\mathcal{X}} \right\|_F^2}{\left\| \boldsymbol{\mathcal{X}} \right\|_F^2},$$

where $\boldsymbol{\mathcal{X}}$ and $\hat{\boldsymbol{\mathcal{X}}}$ denote, respectively, the received and reconstructed signals tensors.

Fig. 2 shows the NMSE of $\boldsymbol{\mathcal{X}}$ after the TTD using the TT-SVD algorithm (1st step of Algo. 1), *i.e.*, the reconstruction error for the TTD of the received signals at the destination, when $F_1 = F_2 = K = P = N = D_{TT}$ and $M_D = \bar{M}_R = M_R = M_S = R = R_{TT}$. This shows how the TT modeling fits successfully the considered MIMO relay system. Moreover, it can be concluded that the bigger is the dimension D_{TT} for a fixed TT-rank R_{TT}, the better is the estimation. In the opposite, when R_{TT} grows for a fixed D_{TT}, the estimation is degrading. Indeed, increasing D_{TT} implies higher diversity gains due to spreading across a higher number of subcarriers and time slots. On the other hand, increasing R_{TT} corresponds to a higher number of parameters (channel and symbols) to be estimated at the receiver. In Fig. 3, we plot the NMSE of the estimation of

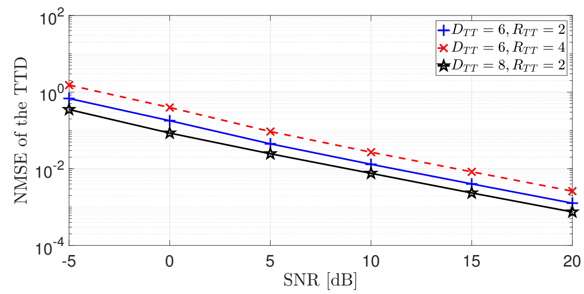

Fig. 2. NMSE vs SNR in dB with the TTD.

$\boldsymbol{\mathcal{H}}^{(RD)}$ by fixing $F_2 = K = P = R = 4$, $N = 10$ and $M_D = \bar{M}_R = M_R = M_S = 2$, and varying the parameter F_1. This result shows the correct estimation of the channel tensors using Algo. 1. Furthermore, it shows that the canal estimation becomes more difficult when several frequencies F_1 are considered.

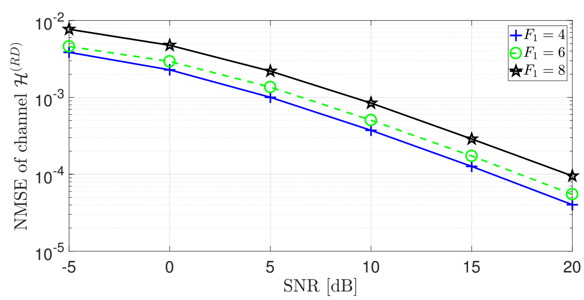

Fig. 3. NMSE of $\boldsymbol{\mathcal{H}}^{(RD)}$ for the proposed algorithm.

Finally, Fig. 4 shows the symbol error rate (SER) of the estimation of the symbols $\mathbf{S}$ as a function of the SNR. The system configuration is as follows, $F_1 = F_2 = K = P = 4$, $N = 10$ and $M_D = \bar{M}_R = M_R = M_S = 2$. The conclusions of this experiment join the previous one in the sense that the symbols matrix $\mathbf{S}$ is correctly estimated using Algo. 1, which means that this latter allows a joint channels and symbols estimation. In addition, we can see the influence of the parameter R, one may note that the estimation becomes a difficult task when there is more symbols to transmit.

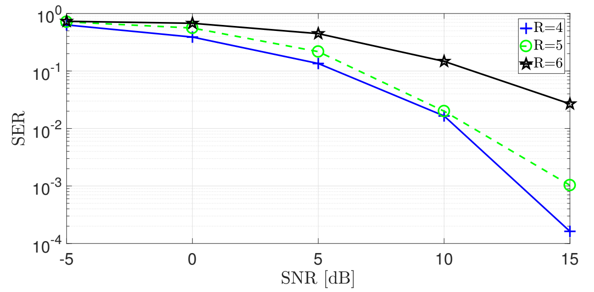

Fig. 4. NMSE of symbols $\mathbf{S}$ for the proposed algorithm.

VI. CONCLUSION

A new TTD modeling approach has been proposed for MIMO-OFDM relay systems to jointly estimate the channels and the information symbols. Our approach generalizes the NTD based system of [4] by considering the case of an OFDM relay system. A new semi-blind receiver which uses a closed-form TTD algorithm and Tucker-ALS algorithms, and which relays on some TTD ambiguity results, had also been proposed. The effectiveness of the proposed receiver is demonstrated by means of Monte Carlo simulations. Some extensions of this work include a generalization to TCF MIMO-OFDM with multiple relays.

VII. REFERENCES

[1] A. Cichocki. Era of big data processing: A new approach via tensor networks and tensor decompositions. *CoRR*, 2014.

[2] A Cichocki, D Mandic, L De Lathauwer, G Zhou, Q Zhao, C Caiafa, and Huy A Phan. Tensor decompositions for signal processing applications: From two-way to multiway component analysis. *IEEE Signal Processing Magazine*, 32(2):145–163, 2015.

[3] M. Dohler and Y. Li. *Cooperative Communications: Hardware, Channel and PHY*. John Wiley & Sons, 2010.

[4] G. Favier, C. A. R Fernandes, and A. L. F. de Almeida. Nested Tucker Tensor Decomposition with Application to MIMO Relay Systems Using Tensor Space–Time Coding (TSTC). *Signal Processing*, 128:318–331, 2016.

[5] W. C. Freitas, G. Favier, and A. L. F. de Almeida. Sequential closed-form semiblind receiver for space-time coded multihop relaying systems. *IEEE Signal Processing Letters*, 24(12):1773–1777, Dec 2017.

[6] W. C. Freitas, G. Favier, and A. L. F. de Almeida. Tensor-based joint channel and symbol estimation for two-way mimo relaying systems. *IEEE Signal Processing Letters*, 26(2):227–231, Feb 2019.

[7] Pieter M. Kroonenberg and Jan de Leeuw. Principal component analysis of three-mode data by means of alternating least squares algorithms. *Psychometrika*, 45:69–97, 1980.

[8] J. N. Laneman, D. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Transactions on Information theory*, 50(12):3062–3080, 2004.

[9] P. Lioliou, M. Viberg, and M. Coldrey. Efficient channel estimation techniques for amplify and forward relaying systems. *IEEE Transactions on Communications*, 60(11):3150–3155, Nov. 2012.

[10] K. J. R. Liu, A. K. Sadek, W. Su, and A. Kwasinski. *Cooperative Communications and Networking*. Cambridge University Press, New York, NY, USA, 2009.

[11] I V Oseledets. Tensor-train decomposition. *SIAM Journal on Scientific Computing*, 33(5):2295–2317, 2011.

[12] Y. Rong, M. Khandaker, and Y. Xiang. Channel estimation of dual-hop MIMO relay system via parallel factor analysis. *IEEE Transactions on Wireless Communications*, 11(6):2224–2233, Jun. 2012.

[13] Y. Rong, X. Tang, and Y. Hua. A Unified Framework for Optimizing Linear Nonregenerative Multicarrier MIMO Relay Communication Systems. *IEEE Trans. on Signal Process.*, 57:4837–4851, Dec. 2009.

[14] Chao-Kai Wen, Jung-Chieh Chen, and Pangan Ting. Robust Transmitter Design for Amplify-and-Forward MIMO Relay Systems Exploiting Only Channel Statistics. *IEEE Transactions on Wireless Communications*, 11:668–682, 2012.

[15] L. R Ximenes, G. Favier, and A. L. F. de Almeida. Semi-Blind Receivers for Non-Regenerative Cooperative MIMO Communications Based on Nested PARAFAC Modeling. *IEEE Transactions on Signal Processing*, 63(18):4985–4998, 2015.