Block Sparsity-Based DOA Estimation with Sensor Gain and Phase Uncertainties

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***Abstract*—This paper investigates the problem of direction-of-70arrival (DOA) estimation in the presence of unknown sensor gain and phase uncertainties. A novel method based on a block sparse representation is proposed to estimate the directions of sources. The data model is constructed under the framework of block sparse signal representation. Then, a convex problem is formulated to find the directions of the incident signals, and the problem can be solved using the L1-SVD algorithm. Unlike the existing eigenstructure-based methods and other sparsity-based methods which require appropriate initial values of the unknown sensor gain and phase errors for iterating between unknown sensor errors and angles of sources, the proposed block sparsity-based DOA estimation technique does not need any prior knowledge about the array errors. Numerical simulations exhibit the effectiveness and superiority of the proposed method.**

***Index Terms*—DOA estimation, sensor gain and phase error, block sparse representation**

I. Introduction

DIRECTION-OF-ARRIVAL (DOA) estimation, which is also known as direction finding, is one of the most important research topics in array signal processing, and it has found many applications in various practical fields, such as radar, sonar, wireless communication, and so on [1]. There exist many classical algorithms with high DOA estimation resolution, including multiple signal classification (MUSIC) method [2], estimation of signal parameters via rotational invariance techniques (ESPRIT) [3], minimum norm method [4], and maximum likelihood method [5], [6]. However, it is generally accepted that these high-resolution algorithms rely heavily on the exact knowledge of the array manifold, and hence their performances may suffer from great degeneration when the sensor array encounters uncertainties [7]–[9], such as unknown sensor gain and phase errors.

Some contributions have been made to circumvent the situation of sensor gain and phase uncertainties, which are classified into two types: one is non-autocalibration [10]–[14] and another is autocalibration [15]–[27]. The non-autocalibration approaches requires auxiliary sources with exactly known DOAs [10]–[12] or perfectly partly calibrated arrays [13], [14]. In general, this type of methods can calibrate the sensor array with high accuracy, but the auxiliary sources or partly calibrated sensor array may not always be available in practice.

This work of Huiping Huang is supported by the 'Excellence Initiative' of the German Federal and State Governments and the Graduate School of Computational Engineering at Technische Universität Darmstadt.

Autocalibration methods in [15]–[23] are based on eigensubspace, and therefore they may result in substantial performance degradation in low signal-to-noise ratio (SNR) or when the snapshots are limited. Recently, as the key observation that the DOAs of signals are sparse in the whole spatial domain was found [28], approaches under the framework of sparse signal representation have been proposed to tackle the direction finding problem under unknown sensor array imperfections [24]–[27]. A sparsity-based method is introduced in [24], where it is assumed that only few sensors suffer from errors. Hence, it is inapplicable to the case where all sensors are biased. Iterative approaches are proposed in [25]–[27] to jointly estimate DOAs of signals and calibrate the unknown sensor gain and phase uncertainties. However, they need appropriate initial values of the unknown sensor errors, which may be unavailable in some practical scenarios.

In this paper, we investigate the problem of direction finding with unknown sensor gain and phase uncertainties, and propose a block sparsity-based DOA estimation method. In contrast to the eigenstructure-based methods and the other existing sparse representation-based methods, such as [25]–[27], which need appropriate initial values of the sensor uncertainties for iterating between calculating the DOAs and the unknown sensor parameters, the proposed DOA estimation method does not require any prior information of the sensor gain and phase errors.

The paper is organized as follows. Section II introduces the signal model and the problem statement. The proposed block sparsity-based DOA estimation method is given in Section III. Simulation results are provided in Section IV, while Section V concludes this paper.

II. Signal Model and Problem Statement

Consider K far-field narrowband incoming signals from DOAs $\mathbf{\Theta} = [\theta_1, \theta_2, \cdots, \theta_K]^T$, impinging on a uniform linear array (ULA) with M omnidirectional sensors, where $(\cdot)^T$ stands for the transpose operator. The array observations $\mathbf{x}(t)$ can be modelled as follows

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $t = 1, 2, \cdots, T$ is the time index, T is the total number of snapshots, and $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T$ and $\mathbf{n}(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$ represent the signal waveform and additive Gaussian noise, respectively. $\mathbf{A}$ denotes the steering matrix, i.e., $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)]$ with the steering vectors $\mathbf{a}(\theta_k), k = 1, 2, \cdots, K$ being defined as

$$\mathbf{a}(\theta_k) \triangleq [1, \beta(\theta_k), \cdots, \beta^{M-1}(\theta_k)]^T. \tag{2}$$

Here $\beta(\theta_k) = e^{j\frac{2\pi}{\lambda}d\sin(\theta_k)}$ with λ denoting the wavelength of the incoming signals, and d is the inter-element spacing.

If the sensor array encounters unknown gain and phase errors, the observation data model (1) becomes

$$\mathbf{y}(t) = \mathbf{\Gamma}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{3}$$

where $\mathbf{\Gamma}$ is a diagonal matrix with the elements in its main diagonal standing for sensor gain and phase uncertainties as

$$\mathbf{\Gamma} = \mathrm{diag}\{[\rho_1, \rho_2, \cdots, \rho_M]^T\} \tag{4}$$

where $\mathrm{diag}\{\cdot\}$ denotes an operator returning a diagonal matrix whose main diagonal is composed of the bracketed vector, and $\rho_m \in \mathbb{C}, m = 1, 2, \cdots, M$.

The problem addressed here is as follows: Given the array observation data $\mathbf{y}(t),\ t = 1, 2, \cdots, T$, estimate the unknown incoming directions of sources $\mathbf{\Theta} = [\theta_1, \theta_2, \cdots, \theta_K]^T$ in the presence of the unknown sensor gain and phase uncertainties.

III. Proposed Method

In this section, we propose a novel estimation algorithm for direction finding, where the data model in (3) is first formulated under the framework of block sparse signal representation, then a convex problem is presented and solved via the L1-SVD algorithm.

We start with $\mathbf{\Gamma}\mathbf{A}$ in (3) as follows

$$\begin{aligned}\mathbf{\Gamma}\mathbf{A} &= \mathbf{\Gamma}[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)] \\ &= [\mathbf{\Gamma}\mathbf{a}(\theta_1), \mathbf{\Gamma}\mathbf{a}(\theta_2), \cdots, \mathbf{\Gamma}\mathbf{a}(\theta_K)].\end{aligned} \tag{5}$$

Recalling that for $k = 1, 2, \cdots, K$,

$$\begin{aligned}\mathbf{\Gamma}\mathbf{a}(\theta_k) &= \begin{bmatrix} \rho_1 & & & \\ & \rho_2 & & \\ & & \ddots & \\ & & & \rho_M \end{bmatrix} \begin{bmatrix} 1 \\ \beta(\theta_k) \\ \vdots \\ \beta^{M-1}(\theta_k) \end{bmatrix} \\ &= \begin{bmatrix} 1 & & & \\ & \beta(\theta_k) & & \\ & & \ddots & \\ & & & \beta^{M-1}(\theta_k) \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_M \end{bmatrix} \\ &= \mathrm{diag}\{\mathbf{a}(\theta_k)\} \cdot \boldsymbol{\gamma}\end{aligned} \tag{6}$$

where $\boldsymbol{\gamma} = [\rho_1, \rho_2, \cdots, \rho_M]^T$, we can rewrite $\mathbf{\Gamma}\mathbf{A}$ in (5) as

$$\begin{aligned}\mathbf{\Gamma}\mathbf{A} &= [\mathrm{diag}\{\mathbf{a}(\theta_1)\} \cdot \boldsymbol{\gamma}, \cdots, \mathrm{diag}\{\mathbf{a}(\theta_K)\} \cdot \boldsymbol{\gamma}] \\ &= [\mathrm{diag}\{\mathbf{a}(\theta_1)\}, \cdots, \mathrm{diag}\{\mathbf{a}(\theta_K)\}] \begin{bmatrix} \boldsymbol{\gamma} & & \\ & \ddots & \\ & & \boldsymbol{\gamma} \end{bmatrix}.\end{aligned} \tag{7}$$

By substituting (7) back into (3), and defining

$$\begin{aligned}\mathbf{b}(\theta_k) &= \mathrm{diag}\{\mathbf{a}(\theta_k)\} \\ \mathbf{B} &= [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \cdots, \mathbf{b}(\theta_K)]\end{aligned} \tag{8}$$

$$\begin{aligned}\breve{\mathbf{s}}_k(t) &= \boldsymbol{\gamma} \cdot s_k(t) \\ \bar{\mathbf{s}}(t) &= [\breve{\mathbf{s}}_1(t), \breve{\mathbf{s}}_2(t), \cdots, \breve{\mathbf{s}}_K(t)]^T\end{aligned} \tag{9}$$

we have

$$\mathbf{y}(t) = \mathbf{B}\bar{\mathbf{s}}(t) + \mathbf{n}(t) \tag{10}$$

where $\mathbf{b}(\theta_k)$ and $\mathbf{B}$ are called *block steering vector* and *block steering matrix*, respectively, and $\bar{\mathbf{s}}(t)$ is called *block signal waveform.*

If we take angle grids in the angle region of interest as $\tilde{\mathbf{\Theta}} = [\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_{\tilde{K}}]^T$, where $\tilde{K} \gg K$, (10) can be reformulated as follows

$$\mathbf{y}(t) = \tilde{\mathbf{B}}\tilde{\mathbf{s}}(t) + \mathbf{n}(t) \tag{11}$$

where $\tilde{\mathbf{B}} = [\mathbf{b}(\tilde{\theta}_1), \mathbf{b}(\tilde{\theta}_2), \cdots, \mathbf{b}(\tilde{\theta}_{\tilde{K}})]$ and $\tilde{\mathbf{s}}(t)$ is *block sparse signal waveform* with only the block-entries corresponding to the true DOAs being nonzero. Hence, the problem of direction finding using (3) is interpreted as finding the positions of nonzero block-entries in $\tilde{\mathbf{s}}(t)$ by means of (11). Considering the sampling collections, (11) can be written as

$$\mathbf{Y} = \tilde{\mathbf{B}}\tilde{\mathbf{S}} + \mathbf{N} \tag{12}$$

where $\mathbf{Y}$, $\tilde{\mathbf{S}}$, and $\mathbf{N}$ are the collection matrices of $\mathbf{y}(t)$, $\tilde{\mathbf{s}}(t)$, and $\mathbf{n}(t)$, respectively, i.e., $\mathbf{Y} = [\mathbf{y}(1), \mathbf{y}(2), \cdots, \mathbf{y}(T)]$, $\tilde{\mathbf{S}} = [\tilde{\mathbf{s}}(1), \tilde{\mathbf{s}}(2), \cdots, \tilde{\mathbf{s}}(T)]$, and $\mathbf{N} = [\mathbf{n}(1), \mathbf{n}(2), \cdots, \mathbf{n}(T)]$.

In order to solve problem (12) via the L1-SVD algorithm, we follow the procedure outlined in [28]. First, a singular value decomposition (SVD) is performed on $\mathbf{Y}$ in order to reduce both the computational complexity and the sensitivity to noise. That is,

$$\mathbf{Y} = [\mathbf{U}_s\ \mathbf{U}_n] \begin{bmatrix} \mathbf{\Sigma}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix} \tag{13}$$

where $(\cdot)^H$ denotes the Hermitian transpose operator, and one obtains $\mathbf{Y}_{SV} = \mathbf{Y}\mathbf{V}_s$, $\tilde{\mathbf{S}}_{SV} = \tilde{\mathbf{S}}\mathbf{V}_s$, and $\mathbf{N}_{SV} = \mathbf{N}\mathbf{V}_s$. Then, a modified version of the model in (12) with reduced dimensions can be stated as follows

$$\mathbf{Y}_{SV} = \tilde{\mathbf{B}}\tilde{\mathbf{S}}_{SV} + \mathbf{N}_{SV}. \tag{14}$$

Next, aiming at exploiting the sparsity of $\tilde{\mathbf{S}}_{SV}$, the following convex optimization problem is constructed

$$\begin{aligned}&\min_{\tilde{\mathbf{S}}_{SV}} \ \|\tilde{\mathbf{S}}_{SV}^{(l2)}\|_0 \\ &\text{s.t.} \ \ \mathbf{Y}_{SV} = \tilde{\mathbf{B}}\tilde{\mathbf{S}}_{SV} + \mathbf{N}_{SV}\end{aligned} \tag{15}$$

where $\tilde{\mathbf{S}}_{SV}^{(l2)}$ returns a vector whose elements are the L2-norm of the corresponding block-row of $\tilde{\mathbf{S}}_{SV}$, and $\|\cdot\|_0$ denotes the L0-norm of a vector. This is an NP-hard problem because of the L0-norm [29]. Replacing the L0-norm with the L1-norm, and in a least square sense, (15) can be finally reformulated as follows

$$\begin{aligned}&\min_{\tilde{\mathbf{S}}_{SV}} \ \|\tilde{\mathbf{S}}_{SV}^{(l2)}\|_1 \\ &\text{s.t.} \ \ \|\mathbf{Y}_{SV} - \tilde{\mathbf{B}}\tilde{\mathbf{S}}_{SV}\|_F \le \epsilon\end{aligned} \tag{16}$$

where $\|\cdot\|_1$ and $\|\cdot\|_F$ stand for the L1-norm of a vector and the Frobenius norm of a matrix, respectively, and ϵ is a user-defined parameter specifying how much noise we wish to allow. This is a convex problem and it can be efficiently solved by some convex optimization tools such as CVX [30]. Once a sparse solution $\tilde{\mathbf{S}}_{SV}$ is found, the DOAs of the signals are determined.

It is noteworthy that the parameter ϵ in (16) should be chosen on the basis of the noise power which may be unknown. Since it is out of the scope of this paper, we do not discuss this issue here, and more details can be found in [28] and the references therein. Besides, in order to avoid the off-grid problem, a so-called *coarse-to-fine* scheme can be adopted. That is, once we get coarse DOAs, we can take more dense angle grids around them and achieve more accurate DOAs. This scheme is carried out until satisfied DOA accuracy is obtained. However, this results in very high computational cost of the proposed method.

IV. Simulations

In this section, four numerical experiments are carried out to evaluate the performance of the proposed block sparsity-based DOA estimation method.

In the first two examples, the spatial spectrum is adopted, which is defined as follows: Denoting the sparse solution to (16) as $\tilde{\mathbf{S}}_{SV}$, and $\hat{\mathbf{s}} = \tilde{\mathbf{S}}_{SV}^{(l2)}$, the spatial spectrum is calculated by $\hat{\mathbf{s}}/\max\{\hat{\mathbf{s}}\}$, where $\max\{\cdot\}$ returns the maximal element of a vector. Note that the indicators of $\hat{\mathbf{s}}$ correspond to the angles.

Example 1: Spatial Spectrum with Multiple Sources. In this example, we use a ULA with $M = 10$ sensors to receive $K = 9$ uncorrelated far-field narrowband signals from DOAs $\boldsymbol{\Theta} = [-61°, -42°, -29°, -12°, 0°, 13°, 28°, 43°, 57°]^T$. The sensor gain and phase errors are randomly generated by drawing from uniform distributions on $[0.8, 1.2]$ and $[-30°, 30°]$, respectively. The signal-to-noise ratio is set to SNR $= 30$ dB, and the number of snapshots is $T = 60$. The density of angle grids is set to be $1°$. The spatial spectrum obtained by the proposed method is plotted in Fig. 1. It is seen from Fig. 1 that the proposed method can shape sharp peaks in the spatial spectrum, and the DOA estimates of the proposed method are quite close to the true ones.

Example 2: Spatial Spectrum with Close-by Sources. In this example, we consider $K = 2$ signals with close incoming angles $\boldsymbol{\Theta} = [0°, 3°]^T$ impinging on a ULA with $M = 20$ sensors. The density of angle grids is set to be $0.1°$. The other parameters are the same as those of Example 1. The spatial spectrum is plotted in Fig. 2, from which it can be seen that the proposed method can estimate two close-by sources without knowing the sensor gain and phase uncertainties.

In the following two examples, the root mean square error (RMSE) is considered as the criterion for DOA estimation. It is defined as follows

$$\text{RMSE} = \sqrt{\frac{1}{KQ}\sum_{k=1}^{K}\sum_{q=1}^{Q}(\hat{\theta}_{k,q} - \theta_k)^2} \tag{17}$$

where $\hat{\theta}_{k,q}$ represents the DOA estimate of the kth signal in the qth Monte Carlo trial and Q is the total number of trials. The eigensubspace-based DOA estimation method in [15] and the sparse representation-based method in [26] are used for comparison. The initial sensor errors of both the eigensubspace-based method and the sparse representation-based method are chosen to be an identity matrix. The MUSIC algorithm with known sensor gain and phase errors is used as a benchmark. For the sake of fairness, the density of angle grids of the proposed block sparsity-based method and the sparse representation-based method are both chosen to be $1°$.

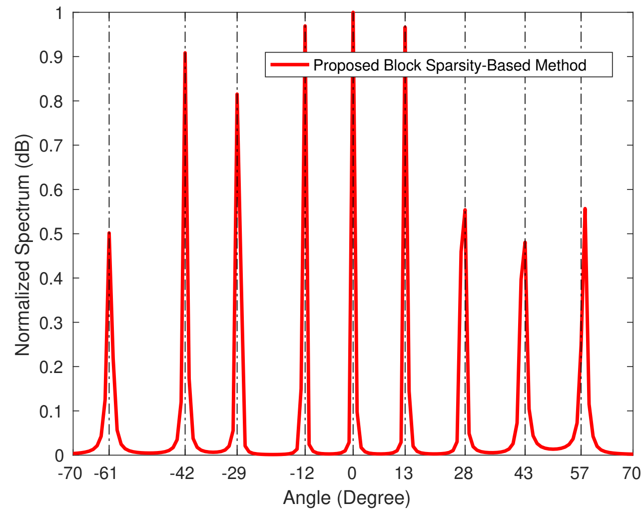

Fig. 1: Spatial spectrum with 10 sensors, 9 sources, $T = 60$ snapshots and SNR $= 30$ dB.

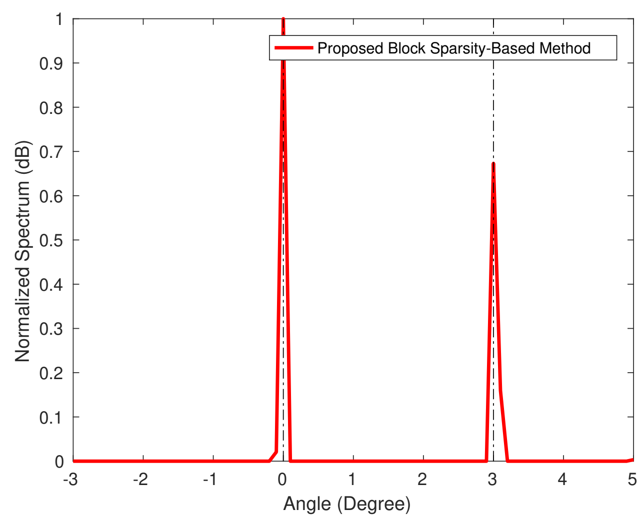

Fig. 2: Spatial spectrum with 20 sensors, 2 sources, $T = 60$ snapshots and SNR $= 30$ dB.

Example 3: RMSE versus SNR. We consider in this example $K = 3$ signals from DOAs $\boldsymbol{\Theta} = [-18.87°, 6.12°, 32.25°]^T$ impinging on a ULA with $M = 7$ sensors. The sensor gain and phase errors are randomly generated by drawing from uniform distributions on $[0.5, 1.5]$ and $[-10°, 10°]$, respectively. $Q =$

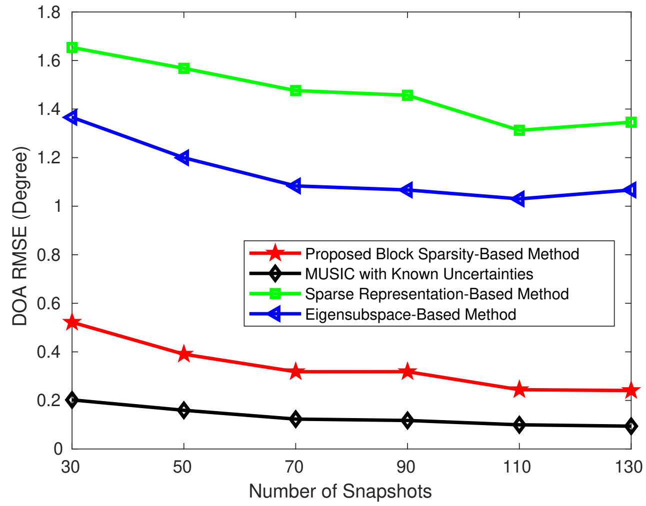

Fig. 4: RMSE versus number of snapshots with $\mathrm{SNR} = 10$ dB.

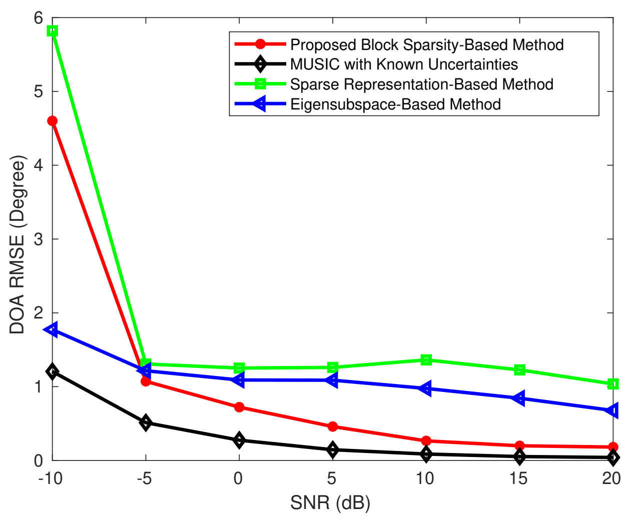

Fig. 3: RMSE versus SNR with number of snapshots $T = 60$.

100 Monte Carlo trials are performed. RMSE versus SNR is drawn in Fig. 3, with SNR varying from -10 dB to 20 dB and the number of snapshots being $T = 60$. We can see from Fig. 3 that the proposed method outperforms the other two methods when the SNR is equal to or larger than -5 dB.

Example 4: RMSE versus Number of Snapshots. In this example, the SNR is fixed to 10 dB, while the number of snapshots changes from 30 to 130. The other parameters are the same as those of Example 3. RMSE versus number of snapshots is drawn in Fig. 4, from which it can be seen again that the proposed block sparsity-based method has lowest RMSE compared to the eigensubspace-based method and the sparse representation-based method.

One of the reasons why the proposed method outperforms the eigensubspace-based method [15] and the sparse representation-based method [26] in Examples 3 and 4 may be that the latter two methods choose the identity matrix as the initial guess of the sensor gain and phase uncertainties. However, this initial guess still differs from the true sensor errors, which finally results in their worse performance in DOA estimation.

V. Conclusion

In this paper, the problem of direction finding in the presence of sensor gain and phase errors has been investigated, and a DOA estimation method based on block sparsity has been proposed. The signal model has been constructed in the framework of block sparse representation, and a convex optimization problem has been formulated and solved via L1-SVD method. It is worth mentioning that the proposed method does not need a priori knowledge about the sensor gain and phase errors. Numerical results has showed the effectiveness and superiority of the proposed block sparsity-based method.

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