

# Free Registration Based Shape Prior for Active Contours

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**Abstract**—A novel method of active contours with shape prior knowledge is presented in this research in order to improve its robustness for partially occluded objects. Our prior is based on a free-registration shape template estimation by using a complete and stable set of invariant descriptors. The prior template is then incorporated into the level set model, using a stopping function that updates the evolving curve only in the region of variability between the active contour and the researched template so that the computation time is reduced considerably. The proposed framework is demonstrated using both simulated and real data involving the segmentation of occluded and noisy images. Results show better robustness and stability compared to the well-known methods using statistics.

**Index Terms**—Invariant descriptors, active contour, free registration

## I. INTRODUCTION

Shape prior play an important role for object segmentation in images with noise, distortion, shape deformation and partial occlusion. Given that references are generally objected of viewing transformations, it is essential to apply an alignment process. Such a task lead to complicated implementation and expensive computational costs, especially for image segmentation with multiple shape templates. To address these problems, in this paper we propose a novel segmentation formula based on complete and stable invariant descriptors, where a reference image may constrain active contour independently of the viewing point. In image processing there are a number of active contour that deal with shape, region and edge-based detection ([1], [2], [3]). However, in this paper, we specifically focus on the edge-based level-set active contour [4].

In literature, different attempts include shape prior information into the active contour models to confine the currently detected object shape within a cluster of similar shapes. Many works have been proposed which can be classified into statistical or geometric shape priors. The first work encountered in the literature that associate statistical geometric shape priors were done by Leventon et al. in [5]. At each iteration, the MAP (maximum a posteriori) of shape and position are estimated to move globally the evolving curve while local evolution is based on the gradient. The method presents promising results but it was too long to be used in real time applications. Chen et al., [6] proposed to use the estimated mean shape of a given training set of aligned curves as their shape prior model, while Zhang [7] defined their shape model to be the level

set function of a preselected reference shape. In [8], Fang et al., defined new energy functional to be minimized using the descent gradient method to find the best template to be used as shape prior. The method is based on shape registration between the evolving curve and the reference. As it was mentioned by the authors, if the variability between shapes is large, the alignment will be difficult and may lead to unsatisfactory segmentation results. In [9] authors propose shape convexity as a new high-order regularization constraint for binary image segmentation. In the context of discrete optimization, object convexity is represented as a sum of 3-clique potentials penalizing any 1-0-1 configuration on all straight lines. They show that by using an iterative confidence region approach, these non-submodular potentials can be efficiently optimized. indeed, at each iteration, the energy is estimated linearly and optimized globally in a small zone of confidence around the current solution.

To avoid the problem of shapes alignment, many approaches adopt the use of invariant descriptors to define prior knowledge on shape. Foulonneau et al. [10] proposed an energy functional based on Legendre descriptors of the target object and the evolving front. This shape constraint was incorporated into a region based active contour [2]. An extension of this model to the multi-reference case was performed in [11]. The model presents promising results but the best moments' order has to be set empirically. In [12], the authors defined an elastic, affine-invariant shape modeling to standardize planar curves. After that, an exponential map approach is used to construct the shape constraint and a Bayesian model is then proposed.

In Charmi et al., [14], an invariant set of complete descriptors is used to define shape constraint for the Snake model. The model presents invariance to rigid transformations but to the problem of dependence to the starting point was not resolved. That's why the authors perform matching between corresponding points of both the reference and the evolving curve to obtain good results. Motivated by the work of [15], in which authors define a new stopping function that incorporates shape prior only in the regions of variability between shapes. In this research we propose to constrain the curve evolution to a specific template generated using a free-registration shape's estimator. The proposed algorithm will be based on a complete and stable set of descriptors. The proposed solution is based on the estimation of a mixture of invariant descriptor between

the current front and the learning prior curves. Consequently, the template can be generated using reverse invariant. By this way, we avoid the problem of shape registration and matching since the template will be generated independently of its pose.

The remaining of this paper is organized as follows: In section 2, we will start by an overview of the level set based active contour. Then, a recall of the used invariant descriptor will be the purpose of section 3. In section 4, we detail the proposed framework for a multi-reference constraint. In section 5, several experimental results are presented and commented to validate the proposed approach. Finally, we conclude the work and highlight some future perspectives in section 6.

## II. OVERVIEW OF LEVEL SET BASED ACTIVE CONTOURS

In this work, we focus on the level set based active contour. The basic idea of the level set approach is to consider the initial contour as the zero level set of a higher dimension function called  $\phi$  and following the evolution of this embedding function, we deduce the contour evolution by seeking its zero level set at each iteration. Several models have been proposed in the literature that we can classify into edge-based or region-based active contours. In [4], the authors proposed the basic level set model which is based on an edge stopping function  $g$ . The evolution's equation of  $\phi$  is

$$\phi^{n+1}(x, y) = \phi^n(x, y) + \Delta t g(x, y) F(x, y) |\nabla \phi^n(x, y)|, \quad (1)$$

$\Delta t$  is the time step.  $F$  is a speed function of the form  $F = F_0 + F_1(k)$  where  $F_0$  is a constant advection term equals to  $(\pm 1)$  depends on the object inside or outside of the initial contour. The second term is of the form  $\epsilon k$  where  $k$  is the curvature at any point and  $\epsilon > 0$ .

$$g(x, y) = \frac{1}{1 + |\nabla G_\sigma * f(x, y)|^p}, \quad (2)$$

where  $f$  is the image and  $G_\sigma$  is a Gaussian filter with a deviation equals to  $\sigma$ . This stopping function has values that are closer to zero in regions of high image gradient and values that are closer to unity in regions with relatively constant intensity. Its obvious that for this model, the evolution is based on the image gradient. Thats why, this model leads to unsatisfactory results in presence of occlusions, low contrast, and even noise.

## III. COMPLETE AND STABLE INVARIANT DESCRIPTOR

This section recalls some basic facts about shape description of closed planar curves under the action of Euclidean transformations as detailed in [16].

In this case we consider a shape representation using a normalized arc-length parametrized curve  $\gamma^*$ . Let  $\Phi(t) = 1/L \int_0^t |\gamma'(u)| du$  where  $\gamma(u) = x(u) + i \cdot y(u)$  the initiale parametrisation of the curve.  $|z|$  is the modulo of the complex number  $z$  and  $L$  is the length of the curve given by :

$$L(\gamma) = \int_{\mathbb{S}^1} |\gamma'(u)| du \quad (3)$$

Given that  $|\gamma'(u)|$  is positive so  $\Phi(t)$  is strictly growing. Therefore, the arc-length parametrisation of the curve was defined as functions from the circle  $\mathbb{S}^1 = \{e^{2i\pi t}, t \in [0, 1]\}$  into the plane of the complex numbers  $\mathbb{C}$  given by:

$$\begin{aligned} \gamma^* : \mathbb{S}^1 &\longmapsto \mathbb{C} \\ t &\longmapsto \gamma^*(t) = \gamma(\Phi^{-1}(t)) \end{aligned} \quad (4)$$

In practice, the object's curve is descretized for the purpose of computer representation. Let  $\{P_n\}_{n \in [0..N]}$  be the set of  $N$  ordered points of the curve obtained by a uniform sampling according to a normalized arc-length parametrization. Consequently, by applying the harmonic analysis and eliminating the continuous component  $a_0$  the relation between two curves under euclidean transformation is:

$$a_k(\gamma_2^*) = e^{2i\pi\theta} e^{2i\pi k t_0/N} a_k(\gamma_1^*) \quad (5)$$

with  $\theta$  the angle of rotation,  $t_0$  the difference between the starting point in the curve and  $a_k$  is the discrete Fourier transform of such curve given by

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} P_{n+1} e^{-2i\pi n \frac{k}{N}} \quad (6)$$

Based on the relation given by eq5, called also the shift theorem, many Fourier invariant descriptors have been proposed in literature( [17] , [18]). In this work we use the invariant descriptors introduced in( [16], [19]) given by the flowing complex number sequence  $I_n(\gamma^*)$  :

$$I_n(\gamma^*) = \begin{cases} I_{k_0} = |a_{k_0}|^{k_0 - k_1 + p}, \text{ for } k_0 \text{ and } a_{k_0} \neq 0 \\ I_{k_1} = |a_{k_1}|^{k_1 - k_0 + q}, \text{ for } k_1 \neq k_0 \text{ and } a_{k_1} \neq 0 \\ I_k = \frac{a_{k_0}^{(k)} a_{k_1}^{k_1 - k} a_{k_1}^{k - k_0}}{|a_{k_0}|^{k_1 - k - q} |a_{k_1}|^{k - k_0 - p}} \end{cases} \quad (7)$$

with  $p, q > 0$ ,  $k \in [-\frac{N}{2}, \frac{N}{2}] - \{k_0, k_1\}$ .

This set of descriptor verifies the stability propriety [20] which implies that a slight modification of invariants may not induce a noticeable shape distortion. Moreover, it is invertible. In fact, since the Fourier transform is invertible, the shape curve can be regenerated by applying the inverse Discrete Fourier Transform (inverse DFT) to the reverse obtained coefficients  $\widehat{a}_k$  given in [19] by :

$$\begin{cases} \widehat{a}_{k_0} = I_{k_0}^{\frac{1}{k_0 - k_1 + p}} e^{i\theta_0}, a_{k_0} \neq 0 \\ \widehat{a}_{k_1} = I_{k_1}^{\frac{1}{k_1 - k_0 + q}} e^{i\theta_1}, a_{k_1} \neq 0 \\ \widehat{a}_k = I_k^{\frac{1}{k_0 - k_1}} I_{k_0}^{\frac{1}{(k_0 - k_1)(k_0 - k_1 + q)}} I_{k_1}^{\frac{-q}{(k_0 - k_1)(k_1 - k_0 + p)}} \\ e^{\frac{i}{k_0 - k_1} (k(\theta_0 - \theta_1) - k_1\theta_0 + k_0\theta_1)}, \\ \forall k \in [-\frac{N}{2}, \frac{N}{2}] - \{k_0, k_1\} \end{cases} \quad (8)$$

where  $\theta_0$  and  $\theta_1$  represent respectively the complex arguments of  $a_{k_0}$  and  $a_{k_1}$ .

In addition, the resulting curve is up to an Euclidean transformation. In fact, let consider:  $\alpha = \frac{\theta_0 - \theta_1}{k_0 - k_1}$ ,  $\beta = \frac{k_0 \theta_1 - k_1 \theta_0}{k_0 - k_1}$  and

$$\begin{cases} C_{k_0} = I_{k_0}^{\frac{1}{k_0 - k_1 + p}} \\ C_{k_1} = I_{k_1}^{\frac{1}{k_1 - k_0 + q}} \\ C_k = I_k^{\frac{1}{k_0 - k_1}} I_{k_0}^{\frac{-p}{(k_0 - k_1)(k_0 - k_1 + q)}} I_{k_1}^{\frac{-q}{(k_0 - k_1)(k_1 - k_0 + p)}} \\ \forall k \in [-\frac{N}{2}, \frac{N}{2}] - \{k_0, k_1\} \end{cases} \quad (9)$$

Hence, equation 8 becomes:

$$\widehat{a}_k = e^{i\beta} e^{ik\alpha} C_k \quad (10)$$

So, the resulting curve depends on the value of  $\alpha$  and  $\beta$  which express respectively the shift on the curve and the rotation. Therefore, by choosing the adopted  $\alpha$  and  $\beta$  we generate the curve up to a specified euclidean transformation.

#### IV. FREE-REGISTRATION MULTI-SHAPE CONSTRAINT FOR ACTIVE CONTOURS

We will devote this section to present our multi-reference shape prior for a level set based active contours.

In this work we use the proposed model in [15] to incorporate prior knowledge. The idea is to define a new stopping function that update the evolving level set function  $\phi$  in the region of variability between the active contour and the reference shape until convergence is obtained. The proposed stopping function is given by :

$$g_{shape}(x, y) = \begin{cases} 0, & \text{if } \phi_{prod}(x, y) \geq 0, \\ \text{sign}(\phi_{ref}(x, y)), & \text{else,} \end{cases} \quad (11)$$

where  $\phi_{prod}(x, y) = \phi(x, y) \cdot \phi_{ref}(x, y)$  with  $\phi_{ref}(x, y)$  the level set function of the reference.

The total discrete evolution's equation that is as follows

$$\frac{\phi^{n+1}(i, j) - \phi^n(i, j)}{\Delta t} = ((1 - w)g(i, j) + w g_{shape}(i, j))F(i, j)|\nabla \phi^n(i, j)|, \quad (12)$$

$w$  represents a weighting factor between the image-based and prior-based terms.

To avoid the registration problem we propose the following strategy. Given a reference shape  $C_{ref}$ , we compute its set of invariant features called  $I_{ref}$ . Then at a given number of iterations, we compute a mixture of invariant resulting from the evolving curve and the reference according to:

$$I_{int} = (I_{C_t} + I_{ref})/2 \quad (13)$$

where  $I_{C_t}$  are the invariant descriptors of the evolving curve  $C_t$  at time  $t$ , representing the zeros of the evolving level set function, and  $I_{ref}$  represent the descriptors of the considered reference. By the resulting descriptors  $I_{int}$ , we construct an intermediate front called  $C_{int}$  between the evolving curve and the reference. Since harmonics  $a_0$ ,  $a_{k_1}$  and  $a_{k_0}$  of the curve  $C_t$  are known,  $C_{int}$  is reconstructed with the same pose than

at the same pose as  $C_t$ . Using this front  $C_{int}$ , we create an intermediate level set function  $\phi_{ref}^{int}$ . This last one will be considered as prior knowledge to be used in eq.(12).

In the general case of a given several references,  $I_{ref}$  will be the mean descriptor  $I_\mu$  given by:

$$I_\mu = \frac{1}{N} \sum_1^N I^i \quad (14)$$

where  $I^i$  the invariant descriptors associated to the parametrization  $\gamma^i(t)$  of the reference curve  $C_i$ .

In some situations, using all reference may leads to unsatisfactory segmentation results [21]. For this reasons, our strategy consists in choosing the most similar references according to the target object. In fact the stability criterion of the considered invariant descriptors means that a small shape distortion doesn't induce a noticeable invariants divergence. So, the appropriate references will be selected using the following distance between theirs corresponding invariants, see [16] for more details :

$$d(\gamma, \gamma_{ref}^i) = \sum_k (|I_k(\gamma) - I_k(\gamma_{ref}^i)|^{\frac{1}{2}})^2 \quad (15)$$

The proposed algorithm is asflows:

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#### Algorithm 1 A multi-reference shape constraint

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- 1- Apply the classic level set based active contour until the evolving contour became stable eq.(1).
  - 2- Select the most similar references eq(15)
  - 3- Estimate the mean descriptor eq(14)
  - 4- Generate a mixture invariant descriptors from the selected references and the evolving front  $I_{int} = (I_{C_t} + I_\mu)/2$
  - 5- Construct an intermediate curve than an intermediate level set function to be used as prior.
  - 6- Compute the new stopping function eq.(11)
  - 7- Continue the evolution based on both the shape prior and the image data represented by the gradient with a big weight assigned to the prior shape force eq.(12).
  - 8- Go to 2 and repeats until convergence is reached.
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## V. EXPERIMENTAL RESULTS

In this section, the proposed method is tested on different datasets, containing synthetic and real images, to evaluate its detection performance.

### A. Results on simulated data

We consider here two clusters of synthetic shapes: Cross and Triangle. The objects are partially occluded, presenting missing parts and put on a cluttered background. As shown in figure 1, five references are considered to be added as prior knowledge. The used references are in different rotation, translation and scale values. The level set function of the obtained template is highlighted in the most right column. Figure 2 (A) shows the obtained results with the traditional edge-based active contour without the prior information. It

can be seen that the curves cannot detect the desired object. More, in the case of cluttered background (column (1) and (4)), the undesired gradient from the background severely interferes with the gradient of object boundary and makes it difficult to differentiate the object from the background. The results shown by figure 2 (B) prove that, by utilizing the proposed model, the interference from the background is eliminated and the original shape of objects are correctly extracted.

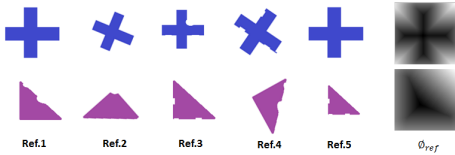


Fig. 1: The used references.

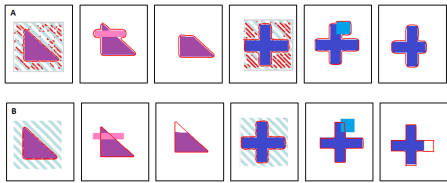


Fig. 2: Segmentation Results.

**B. Results on MPEG-7 data base**

The MPEG-7 database consists of three parts: Set A1, Set A2, and Set B. Each one contains 70 classes. By the following experimentation, we introduce some occlusion to each target image. In the first experiment, the prior information was built using the shape class (part A) in order to evaluate the proposed method under scale and rotation. By figure 3, we show the obtained segmentation results using the proposed multi-reference prior information. In (A) images, we present some of the used shapes. Images (B) show the obtained result without using the prior information. By (C) and (D) images, we illustrate the robustness of the proposed method under missing part, cluttered object, and noise. In fact, the original shape’s contours are correctly reached.

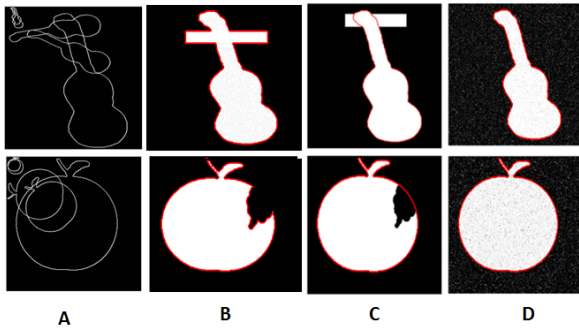


Fig. 3: The multi-reference obtained segmentation under missing part, cluttered background and noise.

TABLE I: Average segmentation result of occluded MPEG-7 shapes

Scenario	Children	Face	Bottle	Apple
without prior	0.176,	0.065	0.23	0.149
[21]	0.142	0.053	0.198	0.12
all reference	0.13	0.042	0.152	0.089
selected references	0.078	0.029	0.088	0.01

In figure 4, we show the experimental results of a comparative study between four scenarios : (a) without prior (b) using the statistical model presented in [21], (c) using all the references and (d) using a selection of references for each shape. The experimentation was performed using the MPEG-7 part B, which contains arbitrary distortions. The binary distance is used as an evaluation metric. It measures the area of non-overlapping regions defined as follows:  $d_{bin}(A, B) = \text{area}(A \cup B - A \cap B) / \text{area}(A \cup B)$  with A is the binary image resulting from the segmentation step and B is the ground truth binary image. Table I and Table II present respectively the values of  $d_{bin}$  and the execution time for each of the four scenarios. The obtained results prove that the selection of adequate references yields to the best results.

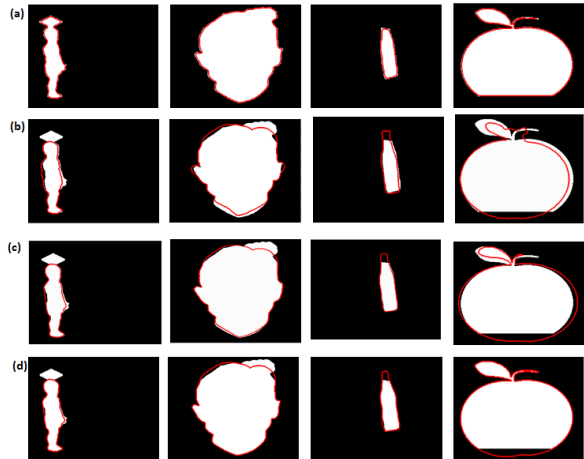


Fig. 4: Segmentation results without prior (a), using the idea proposed in [21], using all references (c) and a selection of references (d).

TABLE II: Average segmentation time of occluded MPEG-7 shape’s segmentation.

Scenario	Children	Face	Bottle	Apple
without prior	10.46	10.97	11.66	8.39
[21]	50.3	53.8	58.3	48.2
all reference	31.2	38.3	30.2	28.8
selected references	27.2	30.2	27.7	25.5

Recall that the proposed approach can be considered as a free registration tool since the prior information will be injected independently of the viewing point. More, by our approach, only adequate references participate in the process of segmentation.

### C. Tracking of human shape

Another useful application of our prior energy is real human activity recognition. The adopted strategy consists of using the previous frame's segmentation results as references to generate the prior at the frame  $t_i$ . We assume that the shape deformations between two consecutive treated frames are not important. Let  $I_j$  the invariant descriptors of the shape's curve  $C_j$  at the frame  $t_j$  with  $j \in [i - nb, i - 1]$ . The invariant descriptors of the prior information is given by:  $I_{prior} = \sum_{j=i-nb}^{i-1} (\alpha_j \cdot I_j)$  with  $\alpha_j = \frac{j-i+nb+1}{\sum_{k=i-nb}^{i-1} (k-i+nb+1)}$  and  $nb$  is the number of used frames. The estimated prior was then introduced as described above. The segmentation results with the proposed model are shown in figure 5. The number  $nb$  is fixed at 5. The used references are shown in the first column. As it can be seen (the second one), the objects to be detected are considered with background cluttered. By the third column, we prove that the shape constraint helps the model to converge to the researched curves.

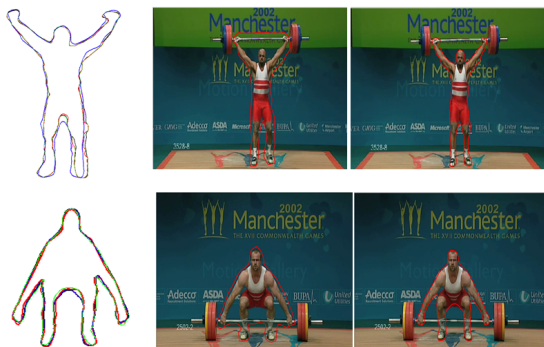


Fig. 5: Application to human activity recognition

## VI. CONCLUSION

In this work, a novel prior model is proposed for robust object detection by active contours. Based on a complete and stable set of invariants descriptors, we construct a mixture template to constrain the subsequent curve evolution. This shape constraint is then incorporated into a level set model that opts to update the evolution only in regions of variability between the current curve and the constructed shape prior. Experiments performed on synthetic and real data demonstrate that the prior model can recover the original shape of the target object in presence of occlusion, noise and missing parts in reduced execution time compared to existing works. As a perspective, we plan to extend the proposed approach to a more general class of transformations such as the affine ones [22].

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