Globally Optimal TIN Strategies with Time-Sharing in the MISO Interference Channel

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***Abstract*—The capacity region of the two-user multiple-input single-output (MISO) interference channel is an open problem, and various achievable rate regions have been discussed in the literature. In this paper, we assume that the transmit signals are Gaussian and that the receivers treat interference as noise (TIN), i.e., we focus on the TIN rate region with Gaussian inputs. Our aim is to compute the rate region boundary for the case of proper Gaussian signaling with time-sharing, i.e., the data rates and required transmit powers may be averaged over several transmit strategies. To this end, we apply methods from convex optimization (in particular Lagrange duality and the cutting plane algorithm), and propose the novel mixed monotonic programming (MMP) framework to treat the arising nonconvex subproblems. The obtained TIN rate region with proper Gaussian signals and time-sharing is significantly larger than previously computed TIN rate regions with proper Gaussian signals, and can even outperform TIN strategies with improper signaling.**

***Index Terms*—Improper signaling, interference channel, Lagrange duality, monotonic optimization, time-sharing.**

I. INTRODUCTION

The two-user multiple-input single-output (MISO) interference channel models the concurrent transmission of two multiantenna transmitters to their respective single-antenna receivers via a shared medium. While the capacity region of the MISO interference channel is in general an open problem,[1] there are various results about achievable rate regions that are obtained by restricting the considerations to a particular class of transmit strategies. Among the most prominent examples are so-called TIN strategies, where the receivers *treat interference as noise* and do not try to decode the interference.

Methods to design transmit strategies under a restriction to TIN strategies with proper Gaussian (i.e., circularly symmetric complex Gaussian) input signals were discussed, e.g., in [2]–[8]. Centralized gradient ascent algorithms [2], distributed interference pricing [3], [4], or game-theoretic methods [5] can be used as suboptimal approaches to maximizing the (weighted) sum rate or other utility functions. Such heuristics are good candidates for practical implementation due to their low computational complexity and/or the possibility of distributed implementation. However, to assess the fundamental limitations of TIN strategies and to have a benchmark for the heuristic methods, there is also an interest in globally optimal solutions.

Due to the nonconcave rate expressions in the interference channel, globally optimal algorithms usually have exponential complexity [6], [7], [9]. An exception is the so-called rate balancing optimization, where the sum rate is maximized while keeping a fixed pre-defined ratio between the per-user rates. For this problem, an efficient solution via a sequence of convex feasibility problems is available [7], [8]. However, this approach is only applicable under a restriction to pure strategies without time-sharing, i.e., if it is not allowed to average data rates and transmit powers over several transmit strategies.

In this paper, we derive a method for globally optimal rate balancing in the case with time-sharing, which can be used to compute the Pareto boundary of the time-sharing rate region. As will be explained after introducing the system model in Section II, this region is in general larger than the convex hull of the rate region with pure strategies [10], and can thus not be obtained via algorithms for pure strategies, such as the weighted sum rate maximization from [6], [7] or the rate balancing method from [7], [8].

The solution method we propose in Section IV is based on a Lagrangian dual approach, where evaluating the dual function requires solving a nonconvex inner problem. For this purpose, we propose the framework of mixed monotonic programming (see Section III), which is a generalization of previous monotonic optimization methods that rely, e.g., on differences of monotonic functions (e.g., [6], [11, Sec. 7]).

As an application example, we use the proposed algorithm in a numerical simulation to quantify the gains by time-sharing compared to the convex hull formulation. Moreover, we observe that globally optimal time-sharing under a restriction to proper Gaussian signals can even outperform previously proposed transmit strategies with improper Gaussian signals. This conclusion is different from the one in [8], where the full potential of time-sharing was not exploited in the reference scheme with proper signaling.

Notation: Inequalities for vectors are meant element-wise. We use the all-ones vector $\mathbf{1}$, the zero vector $\mathbf{0}$, and the identity matrix $\mathbf{I}_M$ of size M. We write $\bullet^{\mathrm{T}}$ and $\bullet^{\mathrm{H}}$ for transpose and conjugate-transpose, respectively.

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[1]For the more general scenario of a multiple-input multiple-output (MIMO) interference channel, results on the capacity region or the sum capacity can be found for several special cases in [1].

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II. System Model and Problem Formulation

For the users $k \in \{1,2\}$, we consider the data transmission

$$y_k = \boldsymbol{h}_{kk}^{\mathrm{H}} \boldsymbol{x}_k + \boldsymbol{h}_{kj}^{\mathrm{H}} \boldsymbol{x}_j + \eta_k \tag{1}$$

with $j = 3 - k$, where $\boldsymbol{h}_{kk}^{\mathrm{H}} \in \mathbb{C}^{1\times M_k}$ is the intended channel of user k, $\boldsymbol{h}_{kj}^{\mathrm{H}} \in \mathbb{C}^{1\times M_j}$ is the unintended channel from transmitter j to receiver k, and $\eta_k \sim \mathcal{CN}(0,\sigma_k^2)$ is the additive white proper Gaussian noise at receiver k. Unless otherwise stated, we assume the transmit signal $\boldsymbol{x}_k$ to be proper Gaussian. In this case, it can be generated from a scalar proper Gaussian input $s_k \sim \mathcal{CN}(0,1)$ by means of

$$\boldsymbol{x}_k = \sqrt{p_k} \boldsymbol{b}_k s_k \tag{2}$$

where $p_k \in [0; P_k]$ is the transmit power of user k and $\boldsymbol{b}_k$ is a normalized beamforming vector with $\|\boldsymbol{b}_k\| = 1$. When treating interference as noise, the achievable rate of user k is

$$r_k(\mathcal{X}) = \log_2\left(1 + \frac{p_k |\boldsymbol{h}_{kk}^{\mathrm{H}} \boldsymbol{b}_k|^2}{\sigma_k^2 + p_j |\boldsymbol{h}_{kj}^{\mathrm{H}} \boldsymbol{b}_j|^2}\right) \tag{3}$$

where $\mathcal{X} = (\boldsymbol{b}_1, \boldsymbol{b}_2, p_1, p_2)$.

Pareto-optimal rate pairs (r_1, r_2) can be computed by solving the so-called rate balancing problem

$$\max_{\mathcal{X},\, R\in\mathbb{R}} \quad R \quad \mathrm{s.t.} \quad r_k(\mathcal{X}) \geq \rho_k R, \ \ \forall k \tag{4a}$$

$$p_k \leq P_k, \ \ \forall k \tag{4b}$$

$$p_k \geq 0, \ \ \forall k \tag{4c}$$

$$\|\boldsymbol{b}_k\| = 1, \ \ \forall k \tag{4d}$$

where $\boldsymbol{\rho} = [\rho_1,\rho_2]^{\mathrm{T}} = [\rho_1, 1-\rho_1]^{\mathrm{T}}$ for various $\rho_1 \in [0;1]$ is the so-called rate profile vector. By varying the relative rate targets $\boldsymbol{\rho}$, any point on the Pareto boundary of the rate region can be computed (e.g., [12]). A solution to (4) can be found efficiently by solving a sequence of second-order cone programs (e.g., [7], [8]).

However, this problem formulation only allows for pure strategies and does not yet include the possibility of averaging data rates or transmit powers over several transmit strategies. Instead of (4), we consider the time-sharing problem

$$\max_{\substack{\mathcal{X}^{(\ell)}, L\in\mathbb{N}, R\in\mathbb{R} \\ \boldsymbol{\tau}\geq\mathbf{0}: \mathbf{1}^{\mathrm{T}}\boldsymbol{\tau}=1}} \quad R \quad \mathrm{s.t.} \quad \sum_{\ell=1}^{L} \tau_\ell r_k(\mathcal{X}^{(\ell)}) \geq \rho_k R, \ \ \forall k \tag{5a}$$

$$\sum_{\ell=1}^{L} \tau_\ell p_k^{(\ell)} \leq P_k, \ \ \forall k \tag{5b}$$

$$p_k^{(\ell)} \geq 0, \ \ \forall k, \forall \ell \tag{5c}$$

$$\|\boldsymbol{b}_k^{(\ell)}\| = 1, \ \ \forall k, \forall \ell \tag{5d}$$

where $\mathcal{X}^{(\ell)} = (\boldsymbol{b}_1^{(\ell)}, \boldsymbol{b}_2^{(\ell)}, p_1^{(\ell)}, p_2^{(\ell)})$ is the ℓth strategy, and $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_L]^{\mathrm{T}}$ is the vector of time-sharing weights, which lies inside the probability simplex. Instead of this information-theoretic notion of coded time-sharing, where both the rates and the transmit powers are averaged [10], some researchers also consider a stricter formulation, where (5b) is replaced by

$$p_k^{(\ell)} \leq P_k, \ \ \forall k, \forall \ell. \tag{6}$$

As the obtained rate region corresponds to the convex hull of the region from (4), we call this stricter version *convex hull formulation* [10]. In general, time-sharing can achieve larger rate regions than a convex hull formulation [10], [13], and this turns out to be true also in the scenario we consider.

Unlike for the optimization of pure strategies in (4), there is no efficient algorithm for solving (5). In this paper, we solve this optimization problem in a globally optimal manner using Lagrange duality and mixed monotonic programming.

III. Mixed Monotonic Programming

We first introduce the mixed monotonic programming method, which we then apply to the nonconvex auxiliary problems that we are facing later on. Consider the optimization

$$\max_{\boldsymbol{x}\in\mathbb{R}^N} \ f(\boldsymbol{x}) \quad \mathrm{s.t.} \ \ \boldsymbol{x}_{\min} \leq \boldsymbol{x} \leq \boldsymbol{x}_{\max} \tag{7}$$

where $f: \mathbb{R}^N \to \mathbb{R}$ is such that there exists $F: \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}$ with the following properties:

$$F(\boldsymbol{x},\boldsymbol{y}) \leq F(\boldsymbol{x}',\boldsymbol{y}) \qquad \text{if} \ \ \boldsymbol{x} \leq \boldsymbol{x}', \tag{8}$$

$$F(\boldsymbol{x},\boldsymbol{y}) \geq F(\boldsymbol{x},\boldsymbol{y}') \qquad \text{if} \ \ \boldsymbol{y} \leq \boldsymbol{y}', \tag{9}$$

$$F(\boldsymbol{x},\boldsymbol{x}) = f(\boldsymbol{x}). \tag{10}$$

Then, we call F a *mixed monotonic (MM) function* and (7) a *mixed monotonic program (MMP)*. Introducing the placeholder $\boldsymbol{y}$ (with the same dimension as $\boldsymbol{x}$) facilitates calculating an upper bound later on. The concept could be extended to also incorporate mixed monotonic constraints, but this is not required for the optimization problems considered in this paper.

We propose to solve the MMP (7) by a branch-reduce-and-bound (BRB) approach, whose main idea is to successively split the feasible set into small boxes (branching) and to compute an upper bound to the value of the objective function inside each box (bounding). The reduction step is optional and will thus be discussed separately later on.

We can use (8)–(10) to verify that

$$U([\boldsymbol{a};\ \boldsymbol{b}]) := F(\boldsymbol{b},\boldsymbol{a}) \geq F(\boldsymbol{x},\boldsymbol{x}) = f(\boldsymbol{x}), \ \forall \boldsymbol{x} \in [\boldsymbol{a};\ \boldsymbol{b}] \tag{11}$$

is an upper bound to the value of the objective function inside a box $\mathcal{B} = [\boldsymbol{a};\ \boldsymbol{b}] = \{\boldsymbol{x} \,|\, \boldsymbol{a} \leq \boldsymbol{x} \leq \boldsymbol{b}\}$. Moreover, due to (10), this utopian upper bound converges to an achievable value as $\boldsymbol{b} - \boldsymbol{a} \to \mathbf{0}$, which is called *consistency* [14, Sec. 6.2.1].

In each iteration, the BRB algorithm chooses the box with the highest upper bound $\mathcal{B}^\star = [\boldsymbol{a}^\star;\ \boldsymbol{b}^\star]$ and subdivides it into two smaller boxes by cutting along its longest edge, i.e.,

$$\mathcal{B}_1 = \left[\boldsymbol{a}^\star;\ \boldsymbol{b}^\star - \frac{b_{n^\star}^\star - a_{n^\star}^\star}{2} \boldsymbol{e}_{n^\star}\right] \tag{12}$$

$$\mathcal{B}_2 = \left[\boldsymbol{a}^\star + \frac{b_{n^\star}^\star - a_{n^\star}^\star}{2} \boldsymbol{e}_{n^\star};\ \boldsymbol{b}^\star\right] \tag{13}$$

where $\boldsymbol{e}_n$ is the nth canonical unit vector, and

$$n^\star = \operatorname*{arg\,max}_{n\in\{1,2\}} \ b_n^\star - a_n^\star. \tag{14}$$

Whenever a new box is created, we can also check whether an arbitrarily chosen point inside this box leads to an achievable value that exceeds the best value observed so far. The procedure, which is summarized in Algorithm 1, terminates as soon as the currently highest upper bound exceeds the current best value by no more than a given error tolerance ϵ. The result is an ϵ-optimal solution, i.e., a point $\boldsymbol{x}_{\text{best}}$ such that $f(\boldsymbol{x}_{\text{best}})$ is at most ϵ worse than the global optimum.

Algorithm 1 BRB Method for the MMP (7)

Require: initial set $\mathbb{B} = \{[\boldsymbol{x}_{\min};\ \boldsymbol{x}_{\max}]\}$ and error tolerance ϵ

1: $\boldsymbol{x}_{\text{best}} \leftarrow \boldsymbol{x}_{\min}$
2: **while** $\mathbb{B} \neq \varnothing$ **do**
3: $\quad \mathcal{B}^\star \leftarrow \arg\max_{\mathcal{B}\in\mathbb{B}} U(\mathcal{B})$ using (11)
4: $\quad$ **if** $U(\mathcal{B}^\star) - f(\boldsymbol{x}_{\text{best}}) \leq \epsilon$, **then break**
5: $\quad$ **for** $i \in \{1,2\}$ **do**
6: $\qquad$ obtain $\mathcal{B}_i \leftarrow [\boldsymbol{a}_i;\ \boldsymbol{b}_i]$ using (12)–(13)
7: $\qquad$ **if** $f(\boldsymbol{a}_i) > f(\boldsymbol{x}_{\text{best}})$, **then** set $\boldsymbol{x}_{\text{best}} \leftarrow \boldsymbol{a}_i$
8: $\qquad \mathcal{B}_i \leftarrow \operatorname{reduce}(\mathcal{B}_i)$ using Section III-A {optional}
9: $\qquad$ **if** $\mathcal{B}_i \neq \varnothing$, **then** add it to $\mathbb{B}$
10: $\quad$ **end for**
11: $\quad$ remove $\mathcal{B}^\star$ from $\mathbb{B}$
12: **end while**
13: **return** $\boldsymbol{x}_{\text{best}}$

The BRB algorithm for difference-of-monotonic (DM) problems from [11, Sec. 7] is a special case of the proposed method since a DM problem with $f(\boldsymbol{x}) = f_1(\boldsymbol{x}) - f_2(\boldsymbol{x})$ and nondecreasing f_1 and f_2 is a special case of an MMP, where $F(\boldsymbol{x},\boldsymbol{y}) = f_1(\boldsymbol{x}) - f_2(\boldsymbol{y})$. A formal convergence proof is omitted due to space constraints (journal version in preparation), but it could be done in analogy to [11, Sec. 7.5] by exploiting that the subdivision is exhaustive (boxes converge to singletons) and that the bound (11) is consistent.

A. Reduction

The aim of reduction is to replace a box $\mathcal{B} = [\boldsymbol{a};\ \boldsymbol{b}]$ by a smaller box in a way that we drop only points which will not be required for finding an ϵ-optimal solution. If $U([\boldsymbol{a};\ \boldsymbol{b}]) < f(\boldsymbol{x}_{\text{best}}) + \epsilon$, the box is not relevant for the further execution of the BRB algorithm (see Line 4 of Algorithm 1), and we can set $\operatorname{reduce}(\mathcal{B}) = \varnothing$. Otherwise, in analogy to [11, Sec. 7.4], we set $\operatorname{reduce}(\mathcal{B}) = [\boldsymbol{a}';\ \boldsymbol{b}']$ with

$$\boldsymbol{a}' = \boldsymbol{b} - \sum\nolimits_{n=1}^{N} \varphi_n (b_n - a_n) \boldsymbol{e}_n \tag{15}$$

$$\boldsymbol{b}' = \boldsymbol{a}' + \sum\nolimits_{n=1}^{N} \psi_n (b_n - a'_n) \boldsymbol{e}_n \tag{16}$$

where we can use bisection search to find

$$\varphi_n = \sup_{\varphi \in \,]0;1]} F(\boldsymbol{b} - \varphi(b_n - a_n)\boldsymbol{e}_n,\, \boldsymbol{a}) \geq f(\boldsymbol{x}_{\text{best}}) \tag{17}$$

$$\psi_n = \sup_{\psi \in \,]0;1]} F(\boldsymbol{b},\, \boldsymbol{a}' + \psi(b_n - a'_n)\boldsymbol{e}_n) \geq f(\boldsymbol{x}_{\text{best}}). \tag{18}$$

Performing a reduction can reduce the number of iterations because the convergence of the boxes towards singletons might be accelerated, but it increases the computational complexity of each iteration. Whether or not the reduction step is beneficial in terms of the overall computational cost, thus depends on the problem under consideration.

IV. Rate-Region with Time-Sharing

In general, nonconvex optimization problems have a nonzero duality gap, meaning that the optimal value of the dual minimization (see Section IV-A) is not equal to the optimal value of the primal maximization, but can only serve as an upper bound. However, it was shown in [9], [15] that the duality gap of nonconvex rate maximization problems vanishes if time-sharing is allowed. In Section IV-B, we solve the Lagrangian dual problem of (5) with the help of the cutting plane method, we recover a primal solution, and we give an intuitive justification for the vanishing duality gap. The arising nonconvex subproblems are solved using mixed monotonic programming in Section IV-C.

A. Dual Approach

The Lagrangian dual problem of (5) reads as

$$\min_{\substack{\boldsymbol{\mu}\geq\boldsymbol{0}\\ \boldsymbol{\lambda}\geq\boldsymbol{0}}} \max_{\substack{L\in\mathbb{N}, R\in\mathbb{R}\\ \boldsymbol{\tau}\geq\boldsymbol{0}: \boldsymbol{1}^{\mathrm{T}}\boldsymbol{\tau}=1}} \left(1 - \boldsymbol{\mu}^{\mathrm{T}}\boldsymbol{\rho}\right) R + \sum_{k=1}^{2} \lambda_k P_k + \sum_{\ell=1}^{L} \tau_\ell \max_{\mathcal{X}^{(\ell)}} \sum_{k=1}^{2} \left(\mu_k r_k(\mathcal{X}^{(\ell)}) - \lambda_k p_k^{(\ell)}\right) \tag{19}$$

with dual variables $\boldsymbol{\mu} = [\mu_1,\mu_2]^{\mathrm{T}}$ and $\boldsymbol{\lambda} = [\lambda_1,\lambda_2]^{\mathrm{T}}$. We have exploited that $\mathcal{X}^{(\ell)}$ can be optimized separately for each ℓ, and we note that these inner problems are all equivalent. Using this observation and the fact that $\boldsymbol{1}^{\mathrm{T}}\boldsymbol{\tau} = 1$, it follows that the choice of L and $\boldsymbol{\tau}$ is arbitrary. Moreover, $\boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\mu} = 1$ must hold in the optimum of the outer minimization in (19) since the maximization over R would be unbounded otherwise.

We thus obtain the simplified formulation

$$\min_{\substack{\boldsymbol{\mu}\geq\boldsymbol{0},\boldsymbol{\lambda}\geq\boldsymbol{0}\\ \boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\mu}=1}} \quad \sum_{k=1}^{2} \lambda_k P_k + f_{\boldsymbol{\mu},\boldsymbol{\lambda}}(\mathcal{X}^\star(\boldsymbol{\mu},\boldsymbol{\lambda})) \tag{20}$$

where

$$\mathcal{X}^\star(\boldsymbol{\mu},\boldsymbol{\lambda}) = \left(\arg\max_{\mathcal{X}}\ f_{\boldsymbol{\mu},\boldsymbol{\lambda}}(\mathcal{X}) \ \text{ s.t. } \ \text{(4c)–(4d)}\right) \tag{21}$$

with

$$f_{\boldsymbol{\mu},\boldsymbol{\lambda}}(\mathcal{X}) = \sum_{k=1}^{2} (\mu_k r_k(\mathcal{X}) - \lambda_k p_k). \tag{22}$$

B. Outer Problem and Primal Recovery

To solve the dual problem (20), we can apply the cutting plane method [16], [17], which successively refines outer approximations

$$\min_{\substack{\boldsymbol{\mu}\geq\boldsymbol{0},\boldsymbol{\lambda}\geq\boldsymbol{0},z\in\mathbb{R}\\ \boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\mu}=1}} \quad \sum_{k=1}^{2} \lambda_k P_k + z \tag{23a}$$

$$\text{s.t. } \ z \geq f_{\boldsymbol{\mu},\boldsymbol{\lambda}}(\mathcal{X}^{(\ell)}) \quad \forall \ell \in \{1,\dots,L\}. \tag{23b}$$

For given constant strategies $\mathcal{X}^{(\ell)}$, this is a linear program in the variables $\boldsymbol{\mu}$, $\boldsymbol{\lambda}$, and z. By solving for the optimal $\boldsymbol{\mu}^\star$ and $\boldsymbol{\lambda}^\star$, setting $\mathcal{X}^{(L+1)} = \mathcal{X}^\star(\boldsymbol{\mu}^\star,\boldsymbol{\lambda}^\star)$, and incrementing L, a refined approximation is obtained. Evaluating $\mathcal{X}^\star$ requires solving the inner problem (21), which is discussed in Section IV-C.

To recover a solution of the primal problem (5), it is proposed in [17] to consider the dual linear program of the cutting plane problem (23), which is in our case given by[2]

$$\max_{\substack{\boldsymbol{\tau}\geq\mathbf{0}, R\in\mathbb{R}\\ \mathbf{1}^{\mathrm{T}}\boldsymbol{\tau}=1}} \quad R \quad \text{s.t.} \quad \sum_{\ell=1}^{L} \tau_\ell\, r_k(\mathcal{X}^{(\ell)}) \geq \rho_k R, \ \forall k \tag{24a}$$

$$\sum_{\ell=1}^{L} \tau_\ell\, p_k^{(\ell)} \leq P_k, \ \forall k. \tag{24b}$$

Using $(...)^\star$ to denote the optimal value of (...), we have

$$(5)^\star \overset{(a)}{\geq} (24)^\star \overset{(b)}{=} (23)^\star \overset{(c)}{\to} (19)^\star \overset{(d)}{\geq} (5)^\star \tag{25}$$

where (a) holds since (24) obviously delivers a feasible time-sharing strategy, (b) is the zero duality gap of linear programs [17], (c) is the convergence of the cutting plane method, and (d) is so-called weak duality, which always holds [17]. This implies that $(19)^\star = (5)^\star$, i.e., the time-sharing problem has zero duality gap (as expected due to [9], [15]), and that the method converges to a globally optimal time-sharing solution.

C. Inner Problem

Monotonic optimization methods for weighted sum rate maximization under power constraints $p_k \leq P_k$ were derived in [6], [7], but the inner problem (21) is different in two respects. On the one hand, the transmit powers p_k in (21) are unconstrained, and on the other hand, they occur in the objective function $f_{\boldsymbol{\mu},\boldsymbol{\lambda}}$, which is the difference of a weighted sum rate and a weighted sum power. While [6] could directly set $p_k = P_k$ since this was shown to be optimal [5, Proposition 1], we have to keep the powers as optimization variables.

According to [19], there exists a pair $\zeta_1,\, \zeta_2 \in [0;1]$ such that the optimal normalized beamforming vectors can be written as

$$\boldsymbol{b}_k = \tilde{\boldsymbol{b}}_k \,\|\tilde{\boldsymbol{b}}_k\|^{-1}, \qquad \tilde{\boldsymbol{b}}_k = \zeta_k \boldsymbol{b}_k^{\mathrm{MRT}} + (1-\zeta_k)\boldsymbol{b}_k^{\mathrm{ZF}}, \tag{26}$$

$$\boldsymbol{b}_k^{\mathrm{MRT}} = \boldsymbol{h}_{kk}\,\|\boldsymbol{h}_{kk}\|^{-1}, \quad \boldsymbol{b}_k^{\mathrm{ZF}} = \boldsymbol{\Pi}_{\boldsymbol{h}_{jk}}^{\perp}\boldsymbol{h}_{kk}\,\|\boldsymbol{\Pi}_{\boldsymbol{h}_{jk}}^{\perp}\boldsymbol{h}_{kk}\|^{-1} \tag{27}$$

where $\boldsymbol{\Pi}_{\boldsymbol{h}_{jk}}^{\perp} = \mathbf{I}_{M_k} - \frac{\boldsymbol{h}_{jk}\boldsymbol{h}_{jk}^{\mathrm{H}}}{\boldsymbol{h}_{jk}^{\mathrm{H}}\boldsymbol{h}_{jk}}$ is the orthogonal projection onto the orthogonal complement of the span of $\boldsymbol{h}_{jk}$.

We can rewrite the achievable rates (3) as functions[3]

$$r_k = \log_2\left(1 + \frac{p_k \alpha_k(\boldsymbol{\zeta})}{\sigma^2 + p_j\beta_j(\boldsymbol{\zeta})}\right) \tag{28}$$

of the transmit powers and the auxiliary variables ζ_1,ζ_2, where

$$\alpha_k(\boldsymbol{\zeta}) = |\boldsymbol{h}_{kk}^{\mathrm{H}}\boldsymbol{b}_k|^2 = \frac{(\zeta_k\gamma_{kk} + (1-\zeta_k)\gamma_{kj})^2}{1 - 2\zeta_k(1-\zeta_k)(1-\frac{\gamma_{kj}}{\gamma_{kk}})} \geq 0, \tag{29}$$

$$\beta_k(\boldsymbol{\zeta}) = |\boldsymbol{h}_{kj}^{\mathrm{H}}\boldsymbol{b}_j|^2 = \frac{\zeta_k^2\delta_{kj}^2\gamma_{kk}^{-2}}{1 - 2\zeta_k(1-\zeta_k)(1-\frac{\gamma_{kj}}{\gamma_{kk}})} \geq 0 \tag{30}$$

[2]The value L is obtained from the execution of the cutting plane algorithm, but usually, only few strategies obtain nonzero weights τ_ℓ. There always exists an optimal time-sharing solution with no more than 4 strategies [18].

[3]We could also write $r_k = \log_2(\sigma^2 + p_j\beta_j(\boldsymbol{\zeta}) + p_k\alpha_k(\boldsymbol{\zeta})) - \log_2(\sigma^2 + p_j\beta_j(\boldsymbol{\zeta}))$ and apply the BRB algorithm for DM problems [11, Sec. 7], but it is easy to verify that bounding via the MMP formulation is always tighter. As good bounds are crucial for the efficiency [11, Sec. 7.5], the MMP method leads to faster convergence of the upper bound $\max_{\mathcal{B}\in\mathbb{B}} U(\mathcal{B})$ to the optimum. The quantitative gain depends, i.a., on problem parameters and the accuracy ϵ.

with $\gamma_{kk} = \|\boldsymbol{h}_{kk}\|$, $\gamma_{kj} = \|\boldsymbol{\Pi}_{\boldsymbol{h}_{jk}}^{\perp}\boldsymbol{h}_{kk}\|$, and $\delta_{kj} = |\boldsymbol{h}_{kk}^{\mathrm{H}}\boldsymbol{h}_{kj}|$ are obtained after plugging in the optimal beamforming vectors [6]. As shown in [6], $\alpha_k(\boldsymbol{\zeta})$ and $\beta_k(\boldsymbol{\zeta})$ are nondecreasing in both components of $\boldsymbol{\zeta}$.

Using this result, it is easy to verify that

$$F\left(\begin{bmatrix}\boldsymbol{\zeta}\\ \boldsymbol{p}\end{bmatrix}, \begin{bmatrix}\boldsymbol{\xi}\\ \boldsymbol{q}\end{bmatrix}\right) = \sum_{k=1}^{2}\left(\mu_k \log_2\left(1 + \frac{p_k\alpha_k(\boldsymbol{\zeta})}{\sigma^2 + q_j\beta_j(\boldsymbol{\xi})}\right) - \lambda_k q_k\right) \tag{31}$$

is an MM function for (22). The remaining task is now to identify a vector $\boldsymbol{p}_{\max}$ in a way that it is guaranteed that the optimal $\boldsymbol{p}$ lies within $[\mathbf{0};\ \boldsymbol{p}_{\max}]$. Then, (21) can be solved using the BRB method for MMPs (Algorithm 1) with $\boldsymbol{x}_{\min} = \mathbf{0}$ and $\boldsymbol{x}_{\max}^{\mathrm{T}} = [\boldsymbol{p}_{\max}^{\mathrm{T}}, 1, 1]^{\mathrm{T}}$.

To find an initialization $\boldsymbol{p}_{\max}$, consider the function

$$\hat{f}_k(p_k) = \mu_k \log_2\left(1 + \frac{p_k\|\boldsymbol{h}_{kk}\|^2}{\sigma^2}\right) - \lambda_k p_k \tag{32}$$

where the rate has been replaced by the optimal rate in an interference-free scenario. This concave function is maximized by $p_k^\star = \frac{\mu_k}{\lambda_k \ln(2)} - \frac{\sigma^2}{\|\boldsymbol{h}_{kk}\|^2}$, and we have

$$f_{\boldsymbol{\mu},\boldsymbol{\lambda}}(\mathcal{X}) \leq \sum_{i=1}^{2}\hat{f}_i(p_i) \leq \hat{f}_j(p_j^\star) + \hat{f}_k(p_k) \overset{(*)}{\leq} 0 \tag{33}$$

with $(*)$ for all $p_k \geq p_{k,0}$, where $p_{k,0}$ is the largest root of the concave function $p_k \mapsto \hat{f}_j(p_j^\star) + \hat{f}_k(p_k)$, which can be easily found, e.g., by Newton's method. By choosing $\boldsymbol{p}_{\max} = [p_{1,0}, p_{2,0}]^{\mathrm{T}}$, we can thus be sure that the optimal value of (21) is contained in the initial box of the BRB method.

D. Remark on the Implementation

A noteworthy aspect is that the inner problem is unbounded if $\lambda_k = 0$ for some k, and no sensible new inequality for (23b) is obtained then. For an implementation, we thus have to replace the constraint $\boldsymbol{\lambda} \geq \mathbf{0}$ by $\boldsymbol{\lambda} \geq \nu\mathbf{1}$ with $\nu > 0$. To ensure that the constraint does not become too restrictive, a small value of ν is preferable, but it has to be considered whether the resulting potentially small values of λ_k lead to numerical instabilities in the solver for the inner problem. Indeed, we have observed in numerical experiments that a branch-and-bound algorithm without reduction step can need orders of magnitude more iterations in that case, even leading to memory issues due to the high number of boxes to be stored. The reason for this seems to be that the initial box constructed at the end of Section IV-C grows significantly when the dual variables λ_k get small. Fortunately, the reduction step proposed in Section III-A solves this issue by quickly reducing the size of the boxes, so that choosing a small ν is not problematic anymore.

V. Numerical Example

Even though the MMP approach allows us to solve the nonconvex subproblems within a reasonable amount of time, it is clear that the complexity of the method is too high for online application. Instead, it can be used in offline simulations to assess the ultimate limits of the considered coding scheme, to benchmark other, less complex algorithms, and to study

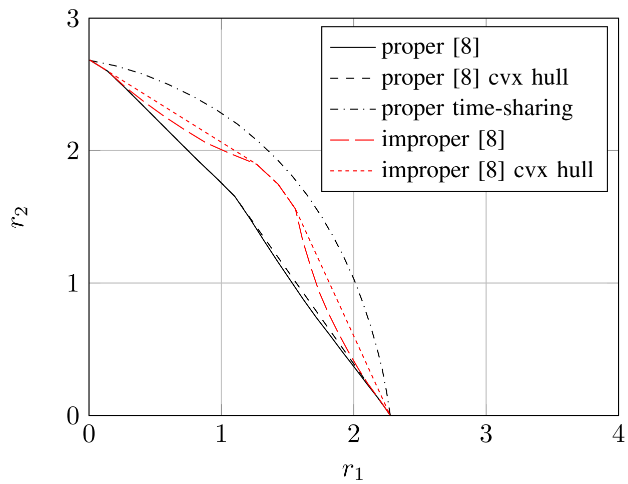

Fig. 1. Achievable rate regions at an SNR of 10dB for a particular channel realization (scenario from [8, Fig. 1], channel $\boldsymbol{H}^{(1)}$ in [8, Table II]).

fundamental aspects of the considered system model. As an example, we show how the method can be applied to numerically study the gap between the convex hull formulation and time-sharing, and to compare proper and improper Gaussian transmit signals.

In [8, Fig. 1], the globally optimal pure strategy with proper Gaussian signals was computed for a particular channel realization in the two-user MISO interference channel. This rate region is shown in Fig. 1 along with its convex hull, i.e., for the case where the rates may be averaged over time. For the case where both the rates and the transmit powers may be averaged, we have used the method from Section IV to compute the significantly larger time-sharing rate region.

As demonstrated in [8], a heuristic method with improper signals can bring a benefit over optimal proper signaling under a restriction to pure strategies. This conclusion remains valid if we extend our considerations to the convex hull formulation for both proper and improper signaling (see Fig. 1). However, the proposed global optimization method reveals that time-sharing over proper signaling strategies leads to a larger rate region than state-of-the-art methods with improper signals.

VI. Conclusion and Outlook

Using a combination of convex programming methods and a novel framework for mixed monotonic programming, we have developed an algorithm to compute the globally optimal TIN rate region for the case with time-sharing and proper Gaussian signals in the two-user MISO interference channel. Using this method, we have demonstrated that time-sharing can bring a significant gain over the convex hull formulation. Moreover, incorporating the possibility of time-sharing changes the conclusion of [8] since gains due to the improper signaling scheme from [8] can no longer be observed when compared to the time-sharing rate region.

Deriving a method to compute the time-sharing rate region with improper signaling and testing whether or not this leads to an even larger rate region is left open for future research. While the dual approach could still be used, we would need to find a method to solve the inner problem for the case of improper signaling in a globally optimal manner. However, note that it was shown analytically in [18] that improper signaling can never outperform proper signaling if time-sharing is allowed in the two-user SISO interference channel. Thus, it should also be studied whether this result can be extended to the two-user MISO interference channel, which would mean that deriving an algorithm for improper time-sharing is not even necessary.

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