Two-Way MIMO Decode-and-Forward Relaying Systems with Tensor Space-Time Coding

Walter da C. Freitas Jr., Gérard Favier, André L. F. de Almeida, and Martin Haardt

Abstract-In this paper, we present a new closed-form semiblind receiver for a two-way decode-and-forward (DF) relaying system. The proposed receiver jointly estimates the symbol and channel matrices involved in the two-way relaying system by exploiting tensor structures of the received signals at the relay and the destination, without using training sequences. The proposed receiver exploits a cross-coding approach using a third-order tensor space-time code (TSTC) at the relay, and it does not require a channel reciprocity between uplink and downlink phases, which can be of interest in frequency division duplex relaying systems. The advantages of this DF receiver compared with the amplify-and-forward (AF) receivers of [14] are three-fold: 1) use of the DF protocol which makes it possible to attenuate the propagation errors compared to the AF protocol, at the cost of an additional decoding at the relay, 2) a cross-coding approach which allows the suppression of interference between sources and therefore greatly simplifies the receivers, and 3) the closed-form aspect of the receivers based on a least squares (LSs) Kronecker product factorization algorithm. Parameter identifiability and computational complexity are analysed, and simulation results are provided to corroborate the effectiveness of the proposed semiblind receiver and coding scheme when compared with the AF receivers of [11].

Index Terms—Semi-blind receiver, block Tucker model, cooperative communications, tensor space-time coding.

I. INTRODUCTION

Cooperative wireless communications system have gained attention since signal propagation effects can be exploited, leading to increased capacity and coverage. The usefulness of tensor decompositions to derive semiblind receivers for channel and symbol estimations has been demonstrated in several works in the literature. In particular, the two-way scenario is in the spotlight nowadays due to the vehicle to everything (V2X) systems. V2X technology enables the exchange of data between vehicles and their environment using wireless communication [2].

Tensor-based receivers have been successfully used for joint symbol and channel estimation in cooperative multiple input multiple output (MIMO) communications. In this context, the usefulness of tensor decompositions to derive semiblind receivers has been demonstrated in several works (see, e.g., [3]–[5], [15] and references therein). Tensor-based receivers also have been proposed for one-way two-hop MIMO relaying, [1], [6]–[8] and for multi-hop relaying [9].

Compared with conventional LS receivers, closed-form tensor-based receivers present two main advantages: i) they avoid accumulation of channel estimation errors, and ii) they can operate under less restrictive (and more flexible) conditions on the required number of antennas at the relays and/or destination, as shown in [13].

Tensor-based receivers have also been derived for two-way cooperative MIMO systems in [10], [12], [14]. In the two-way MIMO relaying case, the communication between two sources is assisted by a multiple-antenna relay station, and the transmission consists of two phases. In the first one, the two sources transmit to the relay, while in the second phase, the relay transmits (possibly encoded) data to the sources. In [12], the authors proposed a supervised tensor-based channel estimation algorithm for a two-way AF relaying system. The authors assume channel reciprocity between uplink and downlink to achieve self-interference cancellation. In [10], a semi-blind receiver for two-way MIMO relaying systems was proposed based on a two-stage integrated alternating least squares algorithm to estimate the channels and symbols without training sequences. In [14], two different semiblind receivers were derived for jointly estimating the information symbols and channels assuming TSTC with the AF relaying protocol.

In this paper, we consider a two-way MIMO relaying wireless communication system and a new closed-form semiblind receiver assuming the DF relaying protocol is derived for jointly estimating the information symbols and channels assuming TSTC at the sources. The proposed receiver uses a cross-coding approach at the relay and does not require channel reciprocity between uplink and downlink phases. Parameter identifiability and computational complexity are analysed. As shown in our simulation results, the proposed semiblind receiver and coding strategy yield superior performances in comparison to the AF receivers of [11].

Notation: Scalars, column vectors, matrices and tensors are denoted by lower-case, boldface lower-case, boldface upper-case, and calligraphic letters, e.g., $a, \mathbf{a}, \mathbf{A}, \mathcal{A}$, respectively. $\mathbf{A}_{i.}$ and $\mathbf{A}_{.j}$ represent the *i*-th row and the *j*-th column of $\mathbf{A} \in \mathbb{C}^{I \times J}$, respectively. The operator vec() transforms a matrix into a column vector by stacking the columns of its matrix argument. The Kronecker product is denoted by \otimes . The identity and all-zeros matrices of dimensions $N \times N$ are denoted as \mathbf{I}_N and

This work was supported by CNPq, CAPES/BRAFITEC project number 226/2017 and the CAPES/COFECUB project number Ma 830-15.

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Figure 1. Two-way model with a pair of sources i and j.

 $\mathbf{0}_N$, respectively. We use the superscripts $T, *, H, -1, \dagger$ for matrix transposition, complex conjugation, Hermitian transposition, inversion, and Moore-Penrose pseudo inversion, respectively. A Tucker decomposition of a Nth-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times \cdots \times I_N}$ is defined in terms of *n*-mode products as $\mathcal{X} = \mathcal{G} \times_1 \mathbf{A}^{(1)} \times_2 \cdots \times_N \mathbf{A}^{(N)}$, with $\mathcal{G} \in \mathbb{C}^{R_1 \times \cdots \times R_N}$ and $\mathbf{A}^{(n)} \in \mathbb{C}^{I_n \times R_n}, n = 1, \cdots, N$. A flat *n*-mode unfolding of the tensor \mathcal{X} is given by

$$\mathbf{X}_n = \mathbf{A}^{(n)} \mathbf{G}_n \left(\bigotimes_{m \neq n} \mathbf{A}^{(m)} \right) \in \mathbb{C}^{I_n \times I_1 I_2 \cdots I_{n-1} I_{n+1} \cdots I_N}.$$

II. SYSTEM MODEL

We consider a two-way MIMO relaying system composed of two sources and one relay, as illustrated in Figure 1, where the number of antennas at the sources i, j and the relay are M_{s_i} , M_{s_j} and M_r , respectively. We assume $M_{s_i} = M_{s_j} =$ M_s . The sources and relay operate in a half-duplex mode. Each source aims to estimate the information signals sent by the other source. During the uplink phase of the relaying protocol, both sources transmit their signals to the relay. In the downlink phase, the relay decodes, re-encodes and transmits the estimated signals to the sources following a DF relaying protocol. Due to the symmetry of the problem, the signal model and analysis are done for the source i, the solution for the source j being similar.

The matrix $\mathbf{H}^{(s_i r)} \in \mathbb{C}^{M_r \times M_s}$ representing the channel between the source *i* and the relay is assumed flat-fading and quasi-static during the total transmission time. The matrix $\mathbf{H}^{(rs_i)} \in \mathbb{C}^{M_s \times M_r}$ represents the channel in the opposite direction. We assume that $\mathbf{H}^{(s_i r)}$ and $\mathbf{H}^{(rs_i)}$ have complex Gaussian entries with zero-mean and variance chosen to make the received symbol energy to noise spectral density ratio (E_s/N_0) independent on the number of transmit antennas.

Define the symbol matrix transmitted by source *i* as $\mathbf{S}^{(i)} \in \mathbb{C}^{N \times R}$ containing *N* data symbols in *R* data-streams. The sources encode the signals to be transmitted using a tensor space-time code (TSTC) $\mathcal{C}^{(i)}$ and $\mathcal{C}^{(j)} \in \mathbb{C}^{R \times M_s \times P}$, respectively. The parameter *P* is the time spreading length of the codes at the sources. Assuming no channel reciprocity, $\mathbf{C}_3^{(i)}, \mathbf{C}_3^{(j)} \in \mathbb{C}^{P \times RM_s}$ are chosen as two blocks extracted from a $P \times 2RM_s$ discrete Fourier transform (DFT) matrix, such that, $\mathbf{C}_3^{(j)H}\mathbf{C}_3^{(i)} = \mathbf{0}_{RM_s}, \mathbf{C}_3^{(j)H}\mathbf{C}_3^{(j)} = \mathbf{I}_{RM_s}$. Such an orthogonal design allows to derive the closed-form semiblind receiver, the details of which will be shown later in Section III.

A. Uplink Phase

Let $\tilde{\mathcal{X}} = \mathcal{X} + \mathcal{N}$ be the noisy tensor of signals received at the relay. The entries of the noise tensor \mathcal{N} are zero-mean circularly symmetric complex-valued Gaussian random variables. During the uplink transmission phase, the signals received at the relay from the sources *i* and *j* form a tensor $\tilde{\mathcal{X}} \in \mathbb{C}^{N \times M_r \times P}$ that follows a block Tucker-2 decomposition given by

$$\tilde{\mathcal{X}} = \mathcal{C}^{(i)} \times_1 \mathbf{S}^{(i)} \times_2 \mathbf{H}^{(s_i r)} + \mathcal{C}^{(j)} \times_1 \mathbf{S}^{(j)} \times_2 \mathbf{H}^{(s_j r)} + \mathcal{N}.$$
(1)

B. Downlink Phase

Assuming a DF relaying protocol, the relay estimates and decode the symbol matrices $\hat{\mathbf{S}}^{(i)}$ and $\hat{\mathbf{S}}^{(j)}$ transmitted by the sources *i* and *j*, respectively. A cross-coding scheme for the pair of sources *i* and *j* is proposed at the relay, using $\mathcal{C}^{(j)}$ to encode $\hat{\mathbf{S}}^{(i)}$, while $\mathcal{C}^{(i)}$ is used to encode $\hat{\mathbf{S}}^{(j)}$. The signal tensor received at the source *i* from the relay are given by $\tilde{\mathcal{Y}}^{(i)} = \mathcal{Y}^{(i)} + \mathcal{V}^{(i)}$, where $\mathcal{V}^{(i)}$ is the additive noise tensor. The tensor $\tilde{\mathcal{Y}}^{(i)} \in \mathbb{C}^{N \times M_s \times P}$ $(M_r = M_s)$ follows a block Tucker-2 decomposition given by

$$\tilde{\mathcal{Y}}^{(i)} = \mathcal{C}^{(i)} \times_1 \hat{\mathbf{S}}^{(j)} \times_2 \mathbf{H}^{(rs_i)} + \mathcal{C}^{(j)} \times_1 \hat{\mathbf{S}}^{(i)} \times_2 \mathbf{H}^{(rs_i)} + \mathcal{V}.$$
(2)

Using the tensor of signals received at the relay (uplink phase) and at the source (downlink phase), in the following we describe the closed-form semiblind DF receiver for the two-way MIMO relaying system.

III. PROPOSED DF RECEIVER

In the DF receiver, the relay can estimate the Kronecker product of the symbol matrices and uplink channels for each source i and j from the flat 3-mode unfolding of $\tilde{\mathcal{X}}$, that satisfies the following equation

$$\tilde{\mathbf{X}}_{P \times NM_{r}} = \mathbf{C}_{3}^{(i)} \left(\mathbf{S}^{(i)} \otimes \mathbf{H}^{(s_{i}r)} \right)^{T} + \mathbf{C}_{3}^{(j)} \left(\mathbf{S}^{(j)} \otimes \mathbf{H}^{(s_{j}r)} \right)^{T} + \mathbf{N}_{P \times NM_{r}}.$$
(3)

Let the Kronecker products between the symbol matrices and uplink channels be defined as

$$\mathbf{Z}_{RM_s \times NM_r}^{(i)} = \left(\mathbf{S}^{(i)} \otimes \mathbf{H}^{(s_i r)}\right)^T \\
\mathbf{Z}_{RM_s \times NM_r}^{(j)} = \left(\mathbf{S}^{(j)} \otimes \mathbf{H}^{(s_j r)}\right)^T.$$
(4)

As $\mathbf{C}_3^{(i)H}\mathbf{C}_3^{(i)} = \mathbf{I}_{RM_s}$ and $\mathbf{C}_3^{(i)H}\mathbf{C}_3^{(j)} = \mathbf{0}_{RM_s}$, the LS solution of the Kronecker products from Eq. (4) are given by

$$\mathbf{Z}_{RM_s \times NM_r}^{(i)} \cong \mathbf{C}_3^{(i)H} \tilde{\mathbf{X}}_{P \times NM_r}
\mathbf{Z}_{RM_s \times NM_r}^{(j)} \cong \mathbf{C}_3^{(j)H} \tilde{\mathbf{X}}_{P \times NM_r}.$$
(5)

Once $\mathbf{Z}_{RM_s \times NM_r}^{(i)}$ and $\mathbf{Z}_{RM_s \times NM_r}^{(j)}$ are estimated, the factors $(\mathbf{S}^{(i)}, \mathbf{H}^{(s_i r)})$ and $(\mathbf{S}^{(j)}, \mathbf{H}^{(s_j r)})$ of the Kronecker products can be obtained by applying the algorithm of factorization of [16], [17]. Then, the relay encodes the estimated signals using the cross-coding approach.

 Table I

 CLOSED-FORM DF SEMIBLIND RECEIVER.

Inputs: $\mathcal{X}, \mathcal{Y}^{(i)}, \mathcal{C}^{(i)}$ and $\mathcal{C}^{(j)}$.

Relay Processing

- (1.1) Compute the LS estimate of $\mathbf{Z}_{RM_s \times NM_r}^{(i)} = \left(\mathbf{S}^{(i)} \otimes \mathbf{H}^{(s_i r)}\right)^T$ using Eq. (4).
- (1.2) Use the Kronecker factorization algorithm to estimate $\mathbf{S}^{(i)}$ and $\mathbf{H}^{(s_i r)}$.
- (1.3) Remove the scaling ambiguities of $\hat{\mathbf{S}}^{(i)}$ and $\hat{\mathbf{H}}^{(s_i r)}$
- (1.4) Project the estimated symbols onto the alphabet.
- (1.5) Re-encode $\hat{\mathbf{S}}^{(i)}$ using $\mathcal{C}^{(j)}$, and $\hat{\mathbf{S}}^{(j)}$ using $\mathcal{C}^{(i)}$.
- Source *i* Processing
- (1.6) Compute the LS estimate of $\mathbf{R}_{RM_r \times NM_s}^{(i)} = (\mathbf{S}^{(j)} \otimes \mathbf{H}^{(rs_i)})^T$ using Eq. (7).
- (1.7) Use the Kronecker factorization algorithm to estimate $\mathbf{S}^{(j)}$ and $\mathbf{H}^{(rs_i)}$.
- (1.8) Remove the scaling ambiguities of $\hat{\mathbf{S}}^{(j)}$ and $\hat{\mathbf{H}}^{(rs_i)}$, and project the estimated symbols onto the alphabet.

A flat 3-mode unfolding of the tensor $\tilde{\mathcal{Y}}^{(i)}$ defined in (2) is given by

$$\mathbf{Y}_{P\times NM_s}^{(i)} = \mathbf{C}_3^{(i)} \left(\hat{\mathbf{S}}^{(j)} \otimes \mathbf{H}^{(rs_i)} \right)^T + \mathbf{C}_3^{(j)} \left(\hat{\mathbf{S}}^{(i)} \otimes \mathbf{H}^{(rs_i)} \right)^T.$$
(6)

At the source *i*, the trick is to exploit the property of the matrix codes $\mathbf{C}_{3}^{(i)H}\mathbf{C}_{3}^{(j)} = \mathbf{0}_{RM_s}$, combined with the column orthonormality of $\mathbf{C}_{3}^{(i)}$; i.e., $(\mathbf{C}_{3}^{(i)H}\mathbf{C}_{3}^{(i)} = \mathbf{I}_{RM_s})$; to deduce, from (6), the following estimate of the Kronecker product whose factors are obtained by

$$\mathbf{R}_{RM_r \times NM_s}^{(i)} \cong \mathbf{C}_3^{(i)H} \mathbf{Y}_{P \times NM_s}^{(i)} = \left(\hat{\mathbf{S}}^{(j)} \otimes \mathbf{H}^{(rs_i)}\right)^T.$$
(7)

Then, a Kronecker product factorization algorithm could be used to estimate $\mathbf{S}^{(j)}$ for the source *i* from $\mathbf{R}^{(i)}$. The same approach can be followed by source *j* to estimate $\mathbf{S}^{(i)}$ from $\mathbf{R}^{(j)}$.

The advantages of this DF approach compared with the AF with reciprocity of [14] are three-fold: 1) we do not assume reciprocity between the channel phases, 2) we are not using another tensor code at the relay, and 3) in each source we just need to know the own TSTC code.

The two-way MIMO system transmits $2NM_s$ information symbols during the uplink and downlink phases, of the same duration NP. Then, the transmission rate of the proposed DF receiver is given by $\frac{M_s}{P}\log_2\mu$, where μ is the alphabet cardinality.

IV. IDENTIFIABILITY

For the source *i*, the system parameter identifiability is linked to the uniqueness of the LS estimates of the Kronecker products $\mathbf{Z}^{(i)}$ and $\mathbf{R}^{(i)}$, i.e., the full column rank property of the matrices $\mathbf{C}_3^{(j)}$ (and $\mathbf{C}_3^{(i)}$ for source *j*), to ensure the uniqueness of their left inverse, in Eqs. (4) and (7). The codes constructions consider that a DFT matrix of dimensions $P \times 2RM_s$ is used to construct the unfoldings of the code tensors $\mathbf{C}_3^{(i)}$ and $\mathbf{C}_3^{(j)}$, implying the necessary condition $P \geq 2RM_s$.

Disregarding the noise, the matrices $\mathbf{H}^{(rs_i)}$ and $\mathbf{S}^{(i)}$ are estimated at source *i*, up to scalar scaling ambiguities

(permutation ambiguity does not exist due to the knowledge of the coding tensors). For eliminating these scalar scaling ambiguities, we assume that the elements $s_{1,1}^{(i)}$ and $s_{1,1}^{(j)}$ are known and equal to 1. Then, the final estimates of the channels and symbol matrices are given by

$$\hat{\mathbf{S}}^{(i)} \leftarrow \hat{\mathbf{S}}^{(i)} \lambda_{\mathbf{S}^{(i)}}, \ \hat{\mathbf{H}}_{M_r \times M_s}^{(s_i r)} \leftarrow \hat{\mathbf{H}}_{M_r \times M_s}^{(s_i r)} \lambda_{\mathbf{S}^{(i)}}^{-1}, \\ \hat{\mathbf{S}}^{(j)} \leftarrow \hat{\mathbf{S}}^{(j)} \lambda_{\mathbf{S}^{(j)}}, \ \hat{\mathbf{H}}_{M_s \times M_r}^{(rs_i)} \leftarrow \hat{\mathbf{H}}_{M_s \times M_r}^{(s_j r)} \lambda_{\mathbf{S}^{(j)}}^{-1},$$

where $\lambda_{\mathbf{S}^{(i)}} = 1/\hat{s}_{1,1}^{(i)}$ and $\lambda_{\mathbf{S}^{(j)}} = 1/\hat{s}_{1,1}^{(j)}$. The closed-form semiblind DF receiver is summarized in Table I.

The dominant complexity is associated with the singular value decomposition (SVD) applied to compute the factors of the Kronecker products, which are rewritten as rank-one matrices. Note that, for a matrix of dimensions $J \times K$, the complexity of its SVD computation is $\mathbb{O}(\min(J, K)JK)$. Hence, the computational complexity of the proposed receiver is basically that of the Kronecker factorization algorithm, and is dominated by steps (1.2) and (1.7). The step (1.2) has complexity $\mathbb{O}(\min(NM_r, RM_s)NM_rRM_s)$, while step (1.7) has complexity $\mathbb{O}(\min(NM_s, RM_r)NM_sRM_r)$.

V. SIMULATION RESULTS

Simulation results are provided to evaluate the performance of the proposed semiblind receiver in terms of symbol error rate (SER) and normalized mean square error (NMSE) of the estimated channels, which are plotted as a function of the symbol energy to noise spectral density ratio (E_s/N_0) . Each SER and NMSE curve represents an average over at least 4×10^4 Monte Carlo runs. Each run corresponds to a different realization of the channels, transmitted symbols and noise. The symbols are randomly drawn from a unit energy quadrature amplitude modulation (QAM) alphabet. The modulation order and the spreading length P are adjusted to ensure the same transmission rate, equal to 4/5 bit per channel use, for all the configurations compared in a same figure. The number of data symbols is N = 4, data-streams is R = 2 and antennas $M_s = M_r = 2$. Recall that the code matrices are DFT matrices, as defined in Section II. Further details of the AF receivers can be found in [14].

Figure 2 compares the SER performance of the proposed DF receiver without the channel reciprocity. As a reference for comparison, we show the performance of the AF receivers with and without reciprocity from reference [14] assuming a third-order tensor coding at the sources and relay. We can see that the DF receiver performs much better than the AF one in all ranges of E_s/N_0 values. The main reason for this performance is due to the lower modulation order for the DF receiver to achieve the same transmission data rate of 4/5 bit per channel use, chosen as 16-OAM for the AF with reciprocity, 256-QAM for the case without reciprocity and 4-QAM for the DF. The two-way MIMO system in the case of the AF receiver transmits 2NR information symbols during the uplink and downlink phases, of respective duration NP and NPJ, where J is the time spreading lengths of the code at the relay. Then, the transmission rate is given by



Figure 2. SER comparison between DF and AF receivers.

 $\frac{2R}{P(J+1)}\log_2\mu$. Indeed, the DF receiver is also less sensitive to noise amplification, due to the fact that the relay forwards the estimated symbols to the destination, instead of the noisy signals received at the relay. This explains the improved SER performance of the DF receiver compared with the AF one.

Figure 3 depicts the NMSE for the estimated channels between the source i and the relay. The NMSE performance for the estimated channels for the AF receiver with assumed reciprocity between uplink and downlink phases provides a better channel estimation accuracy. This due to the reciprocity assumption, allowing to estimate the channel in the first LS estimation. For the AF case without reciprocity, the estimation suffers from error propagation, in contrast to the DF receiver, and this explains the gap between them.

In Fig. 4, we compare the DF receiver with an ideal situation where the relay re-encodes the exact symbol matrices $S^{(i)}$ and $S^{(j)}$, referred to here as "estimate-and-forward (EF) ideal". A 16-QAM modulation is assumed in this experiment. Comparing the SER results of the DF and EF ideal, we can see the impact of the estimation at the relay. As a lower bound, we also depict the performance of a "genie-aided" solution that assumes a perfect knowledge of all the channels using a Zero-Forcing (ZF) receiver. For the ZF receiver, the symbol matrix $S^{(j)}$ is estimated at the source *i* from (7) using the Kronecker product factorization algorithm presented in [18], and the solution is given by

$$\operatorname{vec}\left(\hat{\mathbf{S}}^{(j)}\right) = \left(\mathbf{I}_{R} \otimes \begin{bmatrix} \mathbf{I}_{N} \otimes \mathbf{h}_{:,1}^{(rs_{i})} \\ \mathbf{I}_{N} \otimes \mathbf{h}_{:,2}^{(rs_{i})} \\ \vdots \\ \mathbf{I}_{N} \otimes \mathbf{h}_{:,M_{r}}^{(rs_{i})} \end{bmatrix}^{\dagger} \right) \operatorname{vec}\left(\mathbf{R}_{NM_{s} \times RM_{r}}^{(i)}\right),$$
(8)

VI. CONCLUSION

We have presented a closed-form semiblind receiver for two-way MIMO DF relaying systems. The advantages of



Figure 3. NMSE of estimated channels.



Figure 4. SER comparison between semiblind DF and ZF receivers.

this DF receiver compared with the AF receivers of [11] are three-fold: 1) use of the DF protocol which makes it possible to attenuate the propagations errors compared to the AF protocol, at the cost of an additional decoding at the relay, 2) a cross-coding approach which allows the suppression of interference between sources and therefore greatly simplify the receivers, and 3) the closed-form aspect of the receivers based on an LS estimate of Kronecker products.

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