Scheduling Data Embedding in Dual Function Radar Networks

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***Abstract*—In radar networks, radars may share and broadcast their respective scheduling data. In this respect, communication signals emitted from a radar platform can convey radar signal and beam characteristics. In RF restricted operation environments and towards achieving a unified aperture and bandwidth, it would be desirable to embed such information in radar pulses without having to establish any communications link. In this paper, we consider dual system functionality in radar networks in which one system function enables the other. The focus is on scheduling data that can be reasonably encoded by radar pulses within one radar coherent processing interval (CPI). In order to limit changes in the radar waveform to a minimum, we use up- and down- chirps for information embedding. We consider two different signal embedding strategies in which each radar pulse represents one bit. Information deciphering at the downlink radar receivers is delineated, along with the corresponding probability of bit error assuming a Gaussian channel. The overall channel coding paradigm using radar chirps as parity bits is discussed.**

I. Introduction

Co-existence between radar and communications can assume different schemes for the alleviation of bandwidth contention and congestion [1]–[3]. The two systems can be independently and separately deployed or integrated into one platform [4]–[8]. The latter may include a common transmitter or/and common receiver, often referred to as dual function radar communication (DFRC) system [9]–[13]. The overarching objective of a DFRC system is to enable communication systems to capitalize on the resources of radar infrastructure, while striving to be transparent to existing radar operations and mission. These resources include large bandwidth, multi-sensor beamforming, and high quality hardware, and in many cases high power, high gain antennas. Typical DFRC systems recognize radar as the primary function, and their realization is motivated by advances in radar waveform design and the ubiquitous use of multi-sensor transmit/receive radar system configurations.

We focus on embedding of relatively slow data rate communications that could be associated with sharing scheduling information among radars in the network. Possible scheduling information is depicted in Table 1. Since each scheduling data in this table may change from one coherent processing interval (CPI) to another, information embedding would occur in blocks, each uses the radar pulses within a single CPI. We remark that striving to communicate radar scheduling or target information to other network radars renders radar and communications functions of same objective. In essence, communication signal embedding is for the sole purpose of assisting and enhancing the radar network functionality. This principal feature defines a specific class of DFRC systems in which the communications function is an integral part of the radar, and is considered essential to its mission.

In this paper, we seek signal embedding schemes that lead to minimum changes to the radar function. We adopt code shift keying [14], in lieu of phase shift keying, for signal embedding. A code shift keying, implemented for a chirp radar, preserves signal bandwidth and maintains the core transmitter/receiver structure and characteristics. It can be achieved using up- and down-chirps, where an up-chirp represents bit 1 and a down-chirp is for bit 0. In this regard, the scheduled information is made up of symbols, and signal embedding amounts to encoding these symbols into bits, or chirps. Therefore, we use the words encoding and embedding interchangeably throughout this paper. Ship-to-Ship communications is taken as a scenario for cooperative radars. It is noted that binary and M-ary chirp communications have been examined and proposed for wireless communications under Gaussian and multipath channels [15]–[18].

Two embedding schemes using up- and down- chirps are presented. The first scheme builds separate dictionaries for different scheduling information parameters, with each parameter assuming a limited number of values. For example, in Table I, the carrier frequency is considered one parameter that takes on four possible values. The second scheme builds an overall single dictionary that includes all possible combinations of all parameter respective values. Whereas the former encodes, into bits, information parameters sequentially, with the combined bits spanning one CPI, the latter scheme considers the combinations of the different parameter values as a constellation and encodes, into bits, each constellation symbol over one CPI. These two schemes differ in the number of bits, or pulses, required for information channel encoding over a CPI. Since the CPI is determined by range and Doppler estimation requirements, altering the CPI to permit proper scheduling information encoding may not be a viable option. As such, the preferred encoding scheme would certainly be the one yielding fewer bits and smaller bit error rates.

The paper is organized as follows. In section II, we present the general problem of signal embedding in radar networks. The same section delineates the principal differences between the underlying problem and typical DFRC systems, and proposes two embedding schemes using up- and down-chirps. In Section III, we discuss cross-correlation properties and isolation

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Radar parameters	Possible values	Binary bits required to convey
Carrier frequencies	4	2 bits
CPI	2	1 bit
PRF	4	2 bits
Pulse Width	2	1 bit
Waveform Bandwidth	4	2 bits
Beam Positions	128	7 bits
Scan rates	16	4 bits
Search modes	4	2 bits
Detection modes	4	2 bits
Tracking modes	8	3 bits
Total combinations		**26 bits**

Table I: Possible scheduling information

between up- and down-chirp waveforms which is important in performance analysis. Section IV presents the probability of chirp misclassification which defines the probability of bit error. Section V discusses strategies for channel coding using the radar pulses. Section VI is the conclusions.

II. DFRC IN RADAR NETWORK

A. Operation Concept

Consider the maritime operating environment, shown by the schematic in Fig. 1, where tagging of the radar pulses signifies information embedding. The two main properties

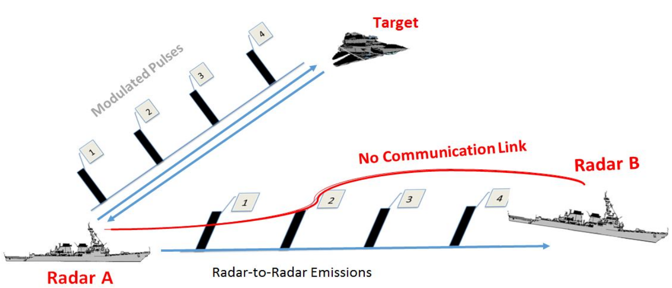

Fig. 1: Conceptual schematic of signal embedding in radar network.

that distinguish the underling DFRC system from commonly proposed dual function systems are: a) In the radar network application considered, unlike many proposed DFRC systems, the downlink communication receiver is in fact another radar; Radar-B in Fig. 1. b) In commonly proposed DFRC systems, the downlink communications receiver assumes knowledge of the radar waveform, PRF, beginning pulse time, and other key radar signal properties. However, in the radar network application considered, this information can be different for different PRFs, and in fact, it is part of the scheduling data embedded in the radar signals. Therefore, it must be deciphered by the radar receiver and it can only be gleaned through message demodulations. A list of possible scheduling information parameters, or categories, is shown in Table I. The scheduling parameters in Table I are not exhaustive and are cited just as an example. Also, the parameter values provided are nominal, and can certainly change depending on the specific radar and operating frequency band. Scheduling information can be presented using libraries and look-up tables stored at multiple radar sites. We refer to these presentations as dictionaries. In this case, communications of one radar's schedule to another needs only the transmission of a member(s) of these dictionaries over each CPI. To form a dictionary, all possible values of each scheduling data parameter, e. g., the four values of the carrier frequency, must be known to radar receivers in the network. We denote the total number of scheduling data, or parameters, as D, which is equal to 10 in Table 1. Below, we consider two possible embedding schemes.

B. Proposed Information Embedding Schemes

With the goal of causing no or limited changes to the radar signal characteristics, including bandwidth, we adopt simple, but rather effective scheduling information embedding schemes. Namely, we apply code shift keying where up-(rising) and down-(falling) chirps are used to represent bits 1 and 0, respectively. This scheme maintains the chirp signal characteristics of the radar, and most importantly avoids mixing of Doppler phase and symbol phase - a consequence of using phase shift keying. In this paper, we consider two information embedding schemes, each encodes the scheduling information every CPI. One scheme encodes the scheduling data sequentially into bits, with all bits spanning a single CPI, whereas the other scheme groups one set of the different scheduling data values into a symbol, and encodes this symbol into bits over the CPI. The two schemes permit different forms of channel coding. Error detection and corrections, in both cases, demand additional bits, and we must therefore be cognizant of the trade-off between the length of the CPI and data fidelity.

1) Embedding Scheme-1: This scheme encodes the different scheduling information parameters individually into bits, one parameter at a time, i.e., performing sequential encoding or embedding. The values assumed by each parameter form a constellation, or a dictionary on its own. The second column of Table I shows a possible constellation size of each parameter. In this case, the parameter which has a high number of symbols, i.e., large constellation size, is encoded with a high number of bits. Therefore, with each radar pulse representing one bit, different scheduling information parameters would reserve different numbers of consecutive radar pulses. For example, and according to the values in Table I, PRF takes four possible values, i.e., requires four symbols, and subsequently 2 bits, or two consecutive pulses. The number of pulses required to embed all the different scheduling data symbols should not exceed the number of pulses in CPI, N. If M_i represents the number of possible values for the i^{th} scheduling information parameter, and D is the number of parameters, then the required number of bits is,

$$N_1 = \sum_{i=1}^{D} log_2 M_i = \sum_{i=1}^{D} log_2 2^{m_i} = \sum_{i=1}^{D} m_i \qquad (1)$$

where, $m_i = log_2 M_i$ represents the number of bits. Consider the case $M_i = 4, 2, 4, 2, 4$ for $i = 1, \ldots, 5$. In this case, the required number of bits are, respectively, $m_i = 2, 1, 2, 1, 2$, yielding $N_1 = 8$ bits. That is, there must be at least 8 pulses in the CPI to match this specific case. Since the symbol constellation sizes are different for different parameters, then, if no channel coding is applied, the probability of a symbol error would vary with the type of scheduling parameter.

Denote the probability of bit error as p. With independent bit errors, the probability of the symbol error follows the binomial distribution. For example, following Table 1, the probabilities of errors in the carrier frequency information, P_{CF}, scan rates, P_{SR}, and having no errors are,

$$P_{CF} = 2p(1-p) + p^2, \quad P_1 = (1-p)^{N_1}$$
$$P_{SR} = 4p(1-p)^3 + 6p^2(1-p)^2 + 4p^3(1-p) + p^4 \quad (2)$$

In essence, this embedding scheme favors small constellations, i.e., scheduling information with limited fewer options.

2) Embedding Scheme-2: In this scheme, there is only one scheduling data constellation or dictionary whose size is determined by the product of the constellation sizes of all symbols, i.e., $M = M_1 \times M_2 \times \cdots \times M_D$. Each combination of the values of the different parameters defines one symbol of the schedule data constellation, and is represented by a binary sequence whose length is equal to the number of pulses in the CPI, N. Each pulse can be an up-or down-chirp, representing bit 1 and 0, respectively. As an example, the first parameter value of each scheduling information category may be gathered into one scheduling data constellation symbol and encoded into bits. The number of constellation symbols of covering the top five parameters in Table 1 is $4 \times 2 \times 4 \times 2 \times 4 = 256$, which requires 8 bits, or eight pulses. This answer assumes that each combination of the scheduling information parameters represents a possible overall schedule. In Scheme-2, suppose SR 1 and 2 only use carrier frequency 1 , whereas SR 3 and 4 only use carrier frequency 2, then the total number of possible combinations is 4, reducing the total number of bits required to 2, giving an advantage of embedding Scheme-2 over Scheme-1. We remark that, in terms of bit errors, assuming no channel coding is applied, a single bit error in Scheme-2 renders the entire scheduling combination incorrect, which is inferior to Scheme-1.

The number of bits required for Scheme-2 is always less than or at most equal to the number of bits required for Scheme-1. To prove this property, we consider the case where the constellation size assumes a number different than a power of two. In general, the number of required bits for Scheme-1 is given by,

$$N_1 = \sum_{i=1}^{D} \lceil log_2 M_i \rceil \quad (3)$$

where $\lceil . \rceil$ denotes the round-off upper integer bound operation. On the other hand, for Scheme-2, the corresponding number of bits is given by,

$$N_2 = \left\lceil log_2 \prod_{i=1}^{D} M_i \right\rceil . \quad (4)$$

When all parameter constellation dimensions are power of two, (4) and (5) give the same answer. In the general case, the subset $\zeta = \{\alpha_j M_j, j = 1, 2, \ldots, S\}$, where S is the set cardinality, includes the scheduling parameter constellation dimensions that are not equal to the power of 2. We consider α_j to be less than 1 and greater than 0.5. Therefore, we deal with the case where there are fewer options for some scheduling information categories than originally considered, M_j. For Scheme-1, with the constraints put on α_j, the number of new bits, N_1 stays the same as N_1. However for Scheme-2, the new number of bits is equal to

$$N_2 = \left\lceil log_2 \prod_{i=1, i \neq j}^{D} M_i \prod_{j \in \zeta} \alpha_j M_j \right\rceil$$
$$= \left\lceil \sum_{i=1}^{D} m_i + \sum_{j \in \zeta} log_2 \alpha_j \right\rceil . \quad (5)$$

The second summation in the above expression is a sum of negative terms, and can exceed -1. This reduces the number of required bits by 1 and presents an advantage of Scheme-2 over Scheme-1, since the probability of no bit error, $P_1 = (1-p)^{N_1}$ increases. Requiring fewer bits, i.e., pulses, allows transmitting more information over the CPI.

III. RADAR RECEIVER FOR UP- AND DOWN-CHIRPS

Two LFM waveforms with the same carrier frequency and the same bandwidth, but one with a positive chirp slope (up-chirp) and the other a negative chirp slope (down-chirp), can be considered as quasi-orthogonal waveforms. These two frequency sweeping chirp waveforms are given by,

$$f_\alpha(t) = \frac{1}{\sqrt{T}} rect\left(\frac{t}{T}\right) e^{(j2\pi f_0 t + j\pi\alpha t^2)}$$
$$f_{-\alpha}(t) = \frac{1}{\sqrt{T}} rect\left(\frac{t}{T}\right) e^{(j2\pi f_0 t - j\pi\alpha t^2)}, \quad (6)$$

where f_0 is the carrier frequency, $\alpha = B/T$ is the chirp slope, T is the pulse width, B is the waveform bandwidth, and $rect(\cdot)$ is a rectangular function, defined as,

$$rect(t) \triangleq \begin{cases} 1, & |t| < 1/2, \\ \frac{1}{2}, & |t| = 1/2, \\ 0, & |t| > 1/2. \end{cases} \quad (7)$$

The application of matched filtering (MF) to the received pulse yields the auto-correlation, or the inner product, of $f_\alpha(t)$ (or $f_{-\alpha}(t)$) with itself,

$$f_\alpha(t) \cdot f_\alpha(t) = \int_{-\infty}^{\infty} f_\alpha(t) f_\alpha^*(t) dt = 1. \quad (8)$$

On the other hand, the cross-correlation, or the inner product, of $f_\alpha(t)$ and $f_{-\alpha}(t)$ equals,

$$f_\alpha(t) \cdot f_{-\alpha}(t) = \int_{-\infty}^{\infty} f_{-\alpha}(t) f_\alpha^*(t) dt$$
$$= \frac{1}{T\alpha^{1/2}} \left[\text{FresnelC}(T\alpha^{1/2}) + j\text{FresnelS}(T\alpha^{1/2}) \right], \quad (9)$$

where FresnelC and FresnelS are, respectively, cosine and sine $Fresnel$ functions [19], Both functions converge to approximately 0.5. With this approximation, we define the isolation coefficient between the two chirps as a square ratio of their inner product,

$$\rho = \frac{f_{\alpha,-\alpha}}{f_{\alpha,\alpha}} = \frac{|f_\alpha(t) \cdot f_{-\alpha}(t)|^2}{|f_\alpha(t) \cdot f_\alpha(t)|^2} \approx \frac{2 \times 0.5^2}{TB}. \quad (10)$$

The above isolation coefficient arises in the receiver analysis in the next section. It is inversely proportional to the product of the pulse width, T, and the LFM bandwidth, B (the product of time and bandwidth). The wider the bandwidth and the longer the pulse, the better the isolation. For instance, for a $10\mu s$ pulse, bandwidths of 1 MHz and 100 MHz can provide -13 dB and -33 dB isolation, respectively. It is evident from the above analysis that the two up- and down- chirps move towards orthogonality with increased TB. So, there is trade-off between probability of bit error and each of the signal bandwidth and time-duration. The conditions on both B and T imposed by frequency allocation and transceiver design limit how low of a value a probability of bit error can take. This, in turn, necessitates channel coding and, as such, reduces scheduling data size, as discussed above.

IV. RECEIVER STRUCTURE AND BIT ERROR ANALYSIS

In the underlying problem, each chirped pulse conveys a single bit of information. We consider the two communicating radar antennas of Radar-A and Radar-B to be in the line of sight (LOS). We assume that the transmitted signal is received with additive Gaussian noise. The data received over a PRI contains at most one pulse with an unknown sign of the chirp slope. The employed code-shift keying modulation problem can be in general cast as binary composite hypothesis problem, where the decision is whether the transmitted signal is up- or down-chirp, with unknown phase and time-of-arrival, which is the general case discussed in [20]. In this paper, we assume that Radar-B can accurately estimate the scheduled PRF values, and as such, only renders a binary decision between the two chirps at predetermined time-instant. In this respect, the problem reverts to finding the probability of bit errors.

Since the chirp slope is unknown, both the up- and down chirp-matched filters are applied in a parallel fashion. We denote $z_1(t)$ as the intensity output of up-chirp matched filter and $z_2(t)$ as the down-chirp matched filter, respectively. Since the received pulse is corrupted by the additive Gaussian noise, the peak of the mainlobe post pulse compression with the matched filter can be considered as a constant amplitude signal (Swerling 0 target) embedded in the Gaussian noise whose intensity has a Rice distribution of [31],

$$P_{\alpha,\alpha}(z) = \frac{1}{\sigma^2} e^{-\left(\frac{z + r\sigma^2}{\sigma^2}\right)} I_0\left(2\sqrt{\frac{rz}{\sigma^2}}\right), \qquad z \geq 0, \quad (11)$$

where σ^2 is the variance of the Gaussian noise, $z = x_I^2 + x_Q^2$ is the intensity, x_I and x_Q are the amplitude measurements of I and Q channels in the baseband, after pulse compression. The variable $r = A^2/\sigma^2$ is the SNR of the signal (the SNR here refers to the SNR post pulse compression), A is the amplitude of the mainlobe of the pulse compression, and $I_0(\cdot)$ is the modified Bessel function of the first kind. For the output of the mismatched filter, i.e. an up-chirp pulse being processed by the matched filter for the down-chirped LFM or vice versa, the distribution of the corresponding peak of the mainlobe would become,

$$P_{\alpha,-\alpha}(z) = \frac{1}{\sigma^2} e^{-\left(\frac{z + \rho r\sigma^2}{\sigma^2}\right)} I_0\left(2\sqrt{\frac{\rho rz}{\sigma^2}}\right), \quad z \geq 0, \quad (12)$$

where $\rho < 1$ is the isolation coefficient between the up- and down-chirp pulses, whose expression is given by (10).

The hypothesis test for deciding up-chirp against the down-chirp is,

$$\Lambda(z) = \frac{P_{\alpha,\alpha}(z)}{P_{\alpha,-\alpha}(z)} = e^{\left(-r\sigma^2(1-\rho)\right)} \frac{I_0(2\sqrt{rz/\sigma^2})}{I_0(2\sqrt{\rho rz/\sigma^2})} > \eta \quad (13)$$

Since $\Lambda(z)$ is a monotonically increasing function, then for $\Lambda(z) > \eta = 1$, i.e. $z_1(t_1) > z_2(t_2)$, the probability of the received signal is more likely to be an up-chirp pulse than a down-chirp pulse, and vice versa. It can be seen that the better isolation between the up- and down-chirp pulses (i.e. the smaller the value of ρ), the smaller the probability of errors.

V. ERROR CONTROL CODING STRATEGIES

In this section, we highlight the possible limitations that can be imposed by the radar on codewords, irrespective of the employed coding method. Key constraints on employed coding techniques are:

(a) Simplicity of the channel coding and decoding so as not to overburden the transmitter and receiver, which are fundamentally realized for radar signal emission and processing.

(b) Operating within a single CPI. The codeword, which involves many additional bits, or pulses, to the original message bits, forms a challenge to radar CPI, as the radar cannot just expand or change its CPI to achieve coding tasks. It is noted that the number of pulses in a CPI has an important role in determining the desirable Doppler resolution and also SNR.

(c) Re-transmitting the same scheduling data information if the receiver detect bit terrors is disallowed, as re-transmission would conflict with sending new scheduling data in the following CPI.

(d) If we cannot correct the errors, then we may choose to disregard the transmitted scheduling data entirely. The reason is that, when error occurs, Radar-B would receive false information about Radar-A operations. If such information is solely used by the cooperative radar or integrated together with others information, it may cause erroneous errors with undesired consequences to the entire radar network.

Stemming from the above conditions, convolution codes, which are generated by passing the information sequence through a linear finite-state shift register, may not be attractive due to the computational demand in implementing a Viterbi decoder at the receiver as well as the significant increase in the number of bits at the output of the shift register. The alternative is to use systematic block codes, where every coded data in a CPI is broken into a data part of k bits and a redundant part of $r = n - k$ bits. The code is designated as (n, k). The redundant bits are extra bits that are generated and added to the scheduling data bits to ensure that no bits were lost during the data communications. For the problem considered, two important constraints should be satisfied by any block code (n, k), beside its error detection and correction capabilities. These are:

a) The code length, n, must be an integer power of 2 to enable Doppler processing though fast Fourier transform (FFT).

b) The redundant bits $r = n - k$, which belong to scheduling data over one CPI, have to be conveyed over this same CPI.

To quantify the challenge of both detecting and correcting one bit error, consider the case of transmitting a 4-bit scheduling data. This accommodates 16 different symbols or a dictionary of size 16. To detect and correct the error, 4 parity bits are needed of a total of 8 pulses in the CPI. But an 8-pulse CPI without coding can represent a total of 256 symbols, not only 16. Therefore, there is a reduction of approximately 93 percent of symbols when attempting to transmit an error-free message. The above simple coding analysis discourages correcting the errors. A more effective course of action is to reduce probability of error by setting proper values of SNR and the isolation coefficient. The above argument also applies to other coding techniques.

VI. CONCLUSIONS

In this paper, we introduced one important and specific form of the DFRC system where the communications function is, in essence, an enabler to the radar function. In radar networks, this aid can take the form of communicating the raw or processed data gathered from one radar to other network radars. We focused on scheduling information signal embedding, and introduced two possible encoding schemes where different scheduling data are either encoded into bits, one data at a time, or first combined together into one large constellation, and then encoded into a stream of bits over the CPI. In both cases, the scheduling data are viewed as symbols and establishes dictionaries that are known to the radar receivers. We used code shift keying of rising and falling chirps as bits 1 and 0. Because of the inability to resend the information in the case of transmission errors, and due to the constraint on fitting the code length over one CPI, we considered error detection and correction coding schemes for each strategy to improve data reliability. The receiver detection problem was cast as a binary hypothesis testing with known delay time. It was shown that the probability of bit error decreases with the TB product which, in turn, reduces code length and allows flexibility in encoding scheduling data within the limits of CPI.

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