MULTIPLE-CHANNEL SIGNAL DETECTION USING THE GENERALIZED COHERENCE SPECTRUM

David Ramírez, Javier Vía and Ignacio Santamaría

Communications Engineering Dept., University of Cantabria, Santander, 39005, Spain.
e-mail: {ramirezgd, jvia, nacho}@gtas.dicom.unican.es

ABSTRACT

Recently, a generalization of the magnitude squared coherence (MSC) spectrum for more than two random processes has been proposed. The generalized MSC (GMSC) spectrum definition, which is based on the largest eigenvalue of a matrix containing all the pairwise complex coherence spectra, provides a frequency-dependent measure of the linear relationship among several stationary random processes. Moreover, it can be easily estimated by solving a generalized eigenvalue problem. In this paper we apply the GMSC spectrum for detecting the presence of a common signal from a set of linearly distorted and noisy observations. Specifically, the new statistic for the multiple-channel detection problem is the integral of the square root of the GMSC, which can be estimated as the sum of the P largest generalized canonical correlations (typically $P = 1$ is enough in practice). Unlike previous approaches, the new statistic implicitly takes into account the spectral characteristics of the signal to be detected (e.g., its bandwidth). Finally, the performance of the proposed detector is compared in terms of its receiver operating characteristic (ROC) curve with the generalized coherence (GC) showing a clear improvement in most scenarios.

***Index Terms*—** Generalized magnitude squared coherence (GMSC) spectrum, canonical correlation analysis (CCA), multiple-channel signal detection.

1. INTRODUCTION

The magnitude squared coherence spectrum (MSC) provides a frequency-dependent measure of the statistical linear relationship between two stationary random processes, which can also be interpreted as a correlation coefficient in the frequency domain [1]. For Gaussian processes it also provides a measure of the mutual information [2]. Despite its usefulness, when more than two signals are involved a commonly accepted generalization of the MSC does not exist yet and measuring all the pairwise MSC spectra is not practical. For instance, given a set of eight random processes there would be twenty-eight different MSC spectra and, obviously, it would be difficult to extract useful information for detection and estimation from such a high number of MSC spectra.

In an attempt to fill this gap, we have recently presented a generalization of the MSC for several stationary processes [3]. The generalized MSC (GMSC) is defined as a function of the largest eigenvalue of a matrix containing all the pairwise complex coherence spectra and it preserves most of the MSC properties. Moreover, in [3] we have also presented two different techniques for the estimation of the GMSC spectrum; the first one, which can be viewed as an extension of the techniques proposed in [4, 5], is based on a filterbank interpretation of the cross-spectrum. The second one is based on a generalization of canonical correlation analysis (CCA) to several data sets [6], and also extends a recently proposed MSC estimator [7].

In this paper, we derive a new statistic from the GMSC to detect the presence of a common signal from a set of observations distorted by unknown frequency-selective channels and corrupted by noise. This problem appears in many applications, such as sensor networks [8], cooperative networks with multiple relays using the amplify-and-forward (AF) scheme [9, 10], or radar detection with multiple antennas. In addition to solving the detection problem, the proposed statistic provides valuable information for cognitive processing. For instance, the frequencies with higher GMSC (corresponding to those with higher signal-to-noise ratio) would allow us to adapt the spectrum of the transmitted signal in cognitive radar [11].

The proposed detection statistic to solve this multi-channel detection problem is the integral of the square root of the GMSC spectrum. We show in the paper that it can be estimated as the sum of the P largest eigenvalues of a generalized canonical correlation analysis (CCA) problem. In practice, using only the largest generalized correlation (i.e., $P = 1$) provides good results and simplifies the detector. The proposed statistic is compared by means of simulations with the generalized coherence (GC) detector proposed by Cochran [12], which is a frequency-independent measure. It is shown that the new detector outperforms the GC in all the considered scenarios.

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2. THE GMSC SPECTRUM

In this section we present a recently proposed generalization of the magnitude squared coherence spectrum (MSC) for $M \geq 2$ signals [3]. Let us consider M zero-mean stationary complex time series $x_1[n], \ldots, x_M[n]$; and define the complex coherence spectrum [1] between the *i*-th and *j*-th signals as

$$C_{x_i x_j}(\omega) = \frac{S_{x_i x_j}(\omega)}{\sqrt{S_{x_i x_i}(\omega) S_{x_j x_j}(\omega)}}, \quad \forall i, j = 1, \ldots, M, \tag{1}$$

where $S_{x_i x_j}(\omega)$ is the cross-spectrum and $S_{x_i x_i}(\omega)$ is the power spectral density of the *i*-th signal.

In the case of $M = 2$ time series, the MSC is defined as $\gamma^2(\omega) = |C_{x_1 x_2}(\omega)|^2$ [1]. In order to extend this idea to the general case of $M \geq 2$ stationary random processes, we define the matrix $\mathbf{\Sigma}_x(\omega) \in \mathbb{C}^{M \times M}$ containing all the pairwise complex coherence spectra as

$$\mathbf{\Sigma}_x(\omega) = \begin{bmatrix} C_{x_1 x_1}(\omega) & \ldots & C_{x_1 x_M}(\omega) \\ \vdots & \ddots & \vdots \\ C_{x_M x_1}(\omega) & \ldots & C_{x_M x_M}(\omega) \end{bmatrix},$$

which can be rewritten as

$$\mathbf{\Sigma}_x(\omega) = \mathbf{D}_x^{-1/2}(\omega) \mathbf{S}_x(\omega) \mathbf{D}_x^{-1/2}(\omega), \tag{2}$$

where

$$\mathbf{S}_x(\omega) = \begin{bmatrix} S_{x_1 x_1}(\omega) & \ldots & S_{x_1 x_M}(\omega) \\ \vdots & \ddots & \vdots \\ S_{x_M x_1}(\omega) & \ldots & S_{x_M x_M}(\omega) \end{bmatrix},$$

and $\mathbf{D}_x(\omega)$ is a diagonal matrix whose entries are $[\mathbf{D}_x(\omega)]_{i,i} = S_{x_i x_i}(\omega)$.

Definition 1: *The generalized magnitude squared coherence spectrum (GMSC) is defined as* $\gamma^2(\omega)$*, where*

$$\gamma(\omega) = \frac{1}{M-1} \left(\lambda_{MAX}(\mathbf{\Sigma}_x(\omega)) - 1 \right),$$

and $\lambda_{MAX}(\mathbf{\Sigma}_x(\omega))$ *is the largest eigenvalue of the matrix* $\mathbf{\Sigma}_x(\omega)$*.*

From (2), it is easy to prove that $\lambda_{MAX}(\mathbf{\Sigma}_x(\omega))$ is also the largest eigenvalue of the following generalized eigenvalue (GEV) problem

$$\mathbf{S}_x(\omega) \tilde{\mathbf{v}}(\omega) = \lambda(\omega) \mathbf{D}_x(\omega) \tilde{\mathbf{v}}(\omega), \tag{3}$$

where $\tilde{\mathbf{v}}(\omega) = \mathbf{D}_x^{-1/2}(\omega) \mathbf{v}(\omega)$ is the generalized eigenvector and $\mathbf{v}(\omega)$ is the eigenvector of $\mathbf{\Sigma}_x(\omega)$.

2.1. GMSC spectrum properties

Property 1: *The GMSC spectrum is bounded between* 0 *and* 1*, i.e.,*

$$0 \leq \gamma^2(\omega) \leq 1.$$

Property 2: *In the case of* $M = 2$ *signals, the GMSC spectrum reduces to the standard MSC spectrum definition.*

Property 3: *The GMSC spectrum is maximum at a given frequency when the* M *time series are perfectly pairwise correlated at that frequency, and minimum when all the signals are uncorrelated.*

Proof. The proof of these properties can be found in [3]. □

Interestingly, the *i*-th coefficient $v_i(\omega)$ of the eigenvector $\mathbf{v}(\omega)$ associated to the largest eigenvalue of $\mathbf{\Sigma}_x(\omega)$ measures the contribution of the *i*-th signal to the GMSC at frequency ω. For instance, if there are M' ($M \geq M' > 1$) signals perfectly pairwise correlated at frequency ω and the remaining $M - M'$ signals are uncorrelated, then the squared modulus of the components of the eigenvector will be $|v_i(\omega)|^2 = 1/M'$ for the perfectly correlated signals and 0 for the uncorrelated ones. Let us clarify this with a toy example. Consider $M = 3$ random processes, with $M' = 2$ signals perfectly correlated at frequency ω and the third one uncorrelated with them. For this example the matrix $\mathbf{\Sigma}_x(\omega)$ is

$$\mathbf{\Sigma}_x(\omega) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

the GMSC at frequency ω becomes $\gamma^2(\omega) = 1/4$ and the corresponding eigenvector is $\mathbf{v}(\omega) = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}^T$, which clearly indicates that the third random process does not contribute to the GMSC spectrum, whereas the first two processes are perfectly correlated.

Finally, we prove a property of the GMSC spectrum that will be useful to apply it as a new statistic for the multiple-channel detection problem.

Property 4: *Consider the following signals*

$$y_i[n] = x_i[n] * g_i[n], \quad \forall i = 1, \ldots, M,$$

where $g_i[n]$ *denotes impulse response of some stable filters with frequency response* $G_i(\omega) = |G_i(\omega)| e^{j\phi_i(\omega)}$*, satisfying* $|G_i(\omega)| \neq 0$, $\forall \omega$*. Then, the GMSC spectra of the signals* $x_i[n]$ *and* $y_i[n]$ *are identical, i.e.*

$$\gamma_x^2(\omega) = \gamma_y^2(\omega),$$

where $\gamma_x^2(\omega)$ *and* $\gamma_y^2(\omega)$ *are respectively the GMSC spectrum of the original signals* $x_i[n]$ *and the filtered ones* $y_i[n]$*.*

Proof. The cross-spectrum between the i-th and the $j-$th filtered signals is given by

$$S_{y_i y_j}(\omega) = G_i(\omega) G_j^*(\omega) S_{x_i x_j}(\omega), \tag{4}$$

therefore, the complex coherence spectrum can be written as

$$C_{y_i y_j}(\omega) = \frac{G_i(\omega) G_j^*(\omega) S_{x_i x_j}(\omega)}{\sqrt{|G_i(\omega)|^2 S_{x_i x_i}(\omega) |G_j(\omega)|^2 S_{x_j x_j}(\omega)}} = e^{j\phi_i(\omega)} e^{-j\phi_j(\omega)} C_{x_i x_j}(\omega), \quad \forall i, j = 1, \ldots, M.$$

Hence, $\mathbf{\Sigma}_y(\omega)$ can be rewritten as follows

$$\mathbf{\Sigma}_y(\omega) = \mathbf{Q}(\omega)\mathbf{\Sigma}_x(\omega)\mathbf{Q}^H(\omega), \quad (5)$$

where $\mathbf{Q}(\omega) = \text{diag}(e^{j\phi_1(\omega)}, \ldots, e^{j\phi_M(\omega)})$ is a unitary matrix. Thus, the eigenvalue problem for the filtered signals is

$$\mathbf{\Sigma}_y(\omega)\mathbf{v}_y(\omega) = \lambda_y(\omega)\mathbf{v}_y(\omega),$$

and taking (5) into account we can rewrite it as

$$\mathbf{\Sigma}_x(\omega)\mathbf{Q}^H(\omega)\mathbf{v}_y(\omega) = \lambda_y(\omega)\mathbf{Q}^H(\omega)\mathbf{v}_y(\omega),$$

which implies that $\mathbf{v}_x(\omega) = \mathbf{Q}^H(\omega)\mathbf{v}_y(\omega)$ and $\lambda_y(\omega) = \lambda_x(\omega)$. This means that the eigenvalues are unaffected by an arbitrary linear filtering of the original signals as long as the filters are invertible. On the other hand, the eigenvectors are affected by a rotation. Since the GMSC only depends on the largest eigenvalue, and not on the eigenvectors, the GMSC spectra of the original and the filtered signals are identical, which concludes the proof. □

3. ESTIMATION OF THE GMSC SPECTRUM

In [3] we have presented and compared two different techniques to estimate the GMSC spectrum from finite registers. In particular, we have considered a filter-bank approach and a new technique based on the maximum variance canonical correlation analysis (CCA) technique [6]. Here, we summarize the CCA-based technique, which provides the best performance and thus will be applied to the multi-channel signal detection problem in the next section.

3.1. Estimation of the GMSC spectrum based on CCA

Let us start by considering the theoretical cross-spectrum between the *i*-th and *j*-th signals, which is given by

$$S_{x_ix_j}(\omega) = \mathbf{f}^H(\omega)\mathbf{R}_{x_ix_j}\mathbf{f}(\omega),$$

where $\mathbf{R}_{x_ix_j}$ is the infinite Toeplitz cross-correlation matrix and $\mathbf{f}(\omega)$ is the Fourier vector of infinite length at frequency ω. Similarly, the theoretical matrices $\mathbf{S}_x(\omega)$ and $\mathbf{D}_x(\omega)$ can be written as

$$\mathbf{S}_x(\omega) = \mathbf{F}^H(\omega)\overbrace{\begin{bmatrix}\mathbf{R}_{x_1x_1} & \cdots & \mathbf{R}_{x_1x_M}\\ \vdots & \ddots & \vdots \\ \mathbf{R}_{x_Mx_1} & \cdots & \mathbf{R}_{x_Mx_M}\end{bmatrix}}^{\mathbf{R}}\mathbf{F}(\omega), \quad (6)$$

$$\mathbf{D}_x(\omega) = \mathbf{F}^H(\omega)\underbrace{\begin{bmatrix}\mathbf{R}_{x_1x_1} & \cdots & \mathbf{0}\\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{R}_{x_Mx_M}\end{bmatrix}}_{\mathbf{D}}\mathbf{F}(\omega), \quad (7)$$

where

$$\mathbf{F}(\omega) = \begin{bmatrix}\mathbf{f}(\omega) & \cdots & \mathbf{0}\\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{f}(\omega)\end{bmatrix}.$$

Taking now into account that the matrices $\mathbf{R}_{x_ix_j}$ are diagonalized by the Fourier vectors, the eigenvalue (EV) problem $\mathbf{\Sigma}_x(\omega)\mathbf{v}(\omega) = \lambda(\omega)\mathbf{v}(\omega)$ can be rewritten as

$$\mathbf{D}^{-1/2}\mathbf{R}\mathbf{D}^{-1/2}\mathbf{w}(\omega) = \lambda(\omega)\mathbf{w}(\omega), \quad (8)$$

where $\mathbf{R}$ and $\mathbf{D}$ have been defined in (6) and (7), respectively, and $\mathbf{w}(\omega) = \mathbf{F}(\omega)\mathbf{v}(\omega)$ is the following vector

$$\mathbf{w}(\omega) = \left[\mathbf{w}_1^T(\omega), \ldots, \mathbf{w}_M^T(\omega)\right]^T,$$

with $\mathbf{w}_k(\omega) = v_k(\omega)\mathbf{f}(\omega)$.

The EV problem in (8) is the classical formulation of the maximum variance canonical correlation analysis (CCA) technique for several data sets [6]. Analogously to (3), the CCA problem can be rewritten as [13]

$$\mathbf{R}\tilde{\mathbf{w}}(\omega) = \lambda(\omega)\mathbf{D}\tilde{\mathbf{w}}(\omega),$$

where $\tilde{\mathbf{w}}(\omega) = \mathbf{D}^{-1/2}\mathbf{w}(\omega)$.

From (8) it is clear that, in the asymptotic case of infinite size cross-correlation matrices, the GMSC spectrum can be directly obtained from the eigenvalues of $\mathbf{D}^{-1/2}\mathbf{R}\mathbf{D}^{-1/2}$. However, in practice we work with finite size ($L \times L$) cross-correlation matrices that must also be estimated from a limited number of observations. This provokes a difference between the theoretical eigenvectors $\mathbf{w}(\omega) = \mathbf{F}(\omega)\mathbf{v}(\omega)$ and the actual (estimated) ones $\hat{\mathbf{w}}^{(p)} = \left[\hat{\mathbf{w}}_1^{(p)T}, \ldots, \hat{\mathbf{w}}_M^{(p)T}\right]^T$, which are obtained by solving

$$\hat{\mathbf{D}}^{-1/2}\hat{\mathbf{R}}\hat{\mathbf{D}}^{-1/2}\hat{\mathbf{w}}^{(p)} = \hat{\lambda}^{(p)}\hat{\mathbf{w}}^{(p)}, \quad (9)$$

where $\hat{\mathbf{D}} \in \mathbb{C}^{LM \times LM}$ and $\hat{\mathbf{R}} \in \mathbb{C}^{LM \times LM}$ are the estimated finite size versions of $\mathbf{D}$ and $\mathbf{R}$.

In order to obtain an accurate GMSC estimate from the solutions of (9), we propose to use a reduced-rank representation of the matrix $\hat{\mathbf{D}}^{-1/2}\hat{\mathbf{R}}\hat{\mathbf{D}}^{-1/2}$, analogously to the technique presented in [7] for the estimation of the conventional MSC. Specifically, the proposed GMSC estimate is obtained as a weighted sum of the magnitude squared Fourier transform of the P principal canonical vectors as follows

$$\hat{\gamma}(\omega) = \frac{1}{M-1}\sum_{p=1}^{P}\sum_{k=1}^{M}(\hat{\lambda}^{(p)} - 1)\left|\mathbf{f}^H(\omega)\hat{\mathbf{w}}_k^{(p)}\right|^2,$$

where $P \leq L$ is the selected rank, and $\hat{\lambda}^{(p)}$, $p = 1, \ldots, P$, are the P largest eigenvalues of (9). Finally, it is easy to prove that in the asymptotic case where $L, P \to \infty$, the proposed estimator becomes

$$\hat{\gamma}(\omega) = \frac{1}{M-1} \int_{\omega'} \sum_{k=1}^{M} (\lambda(\omega') - 1) \left| \mathbf{f}^H(\omega) \mathbf{f}(\omega') v_k(\omega') \right|^2 d\omega'$$
$$= \frac{1}{M-1} \int_{\omega'} (\lambda(\omega') - 1)\, \delta(\omega - \omega') d\omega' = \gamma(\omega),$$

which coincides with the GMSC definition.

4. MULTI-CHANNEL SIGNAL DETECTION

In this section we address the problem of deciding the presence of a common signal distorted by M different unknown channels and acquired by noisy sensors (see figure 1). Mathematically, it can be formulated as the problem of deciding between the hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, which are given by

$$\mathcal{H}_1 : x_k[n] = h_k[n] * s[n] + w_k[n], \quad k = 1, \ldots, M,$$
$$\mathcal{H}_0 : x_k[n] = w_k[n], \quad k = 1, \ldots, M,$$

where $s[n]$ is an unknown signal, $h_k[n]$ is the unknown channel impulse response between the source and the *k*-th sensor and $w_k[n]$ is the spatially uncorrelated noise at the *k*-th sensor. To highlight the generality of the above model, notice that we make no assumptions about the signal, the channels, nor the noise spectra.

The model considered in this section can be found in a large variety of applications. For instance in a sensor network [8] where the sensors measure a common signal generated by a physical phenomenon, but each sensor is distorted by a different frequency-selective and noisy channel. These signals are acquired and transmitted to a fusion center which has to decide whether the observations are generated by a common physical phenomenon or not. Other possible scenarios are cooperative networks with multiple relays using the amplify-and-forward (AF) scheme [9, 10], or radar detection with more than one receiving antenna. Let us remark finally that the frequency-dependent information provided by the GMSC could be used by a transmitter (for instance in radar applications) to adapt the spectrum of the transmitted signal in a cognitive manner, following the ideas proposed in [11].

4.1. Generalized coherence

In this subsection we present a brief review of the generalized coherence (GC). The GC is a related measure of linear dependency among several time series proposed in [12] which will be used for comparison purposes.

Definition 2: *The generalized coherence (GC) is defined as*

$$\zeta^2 = 1 - \frac{\det(\mathbf{\Psi}_x)}{r_{x_1x_1} \cdots r_{x_Mx_M}},$$

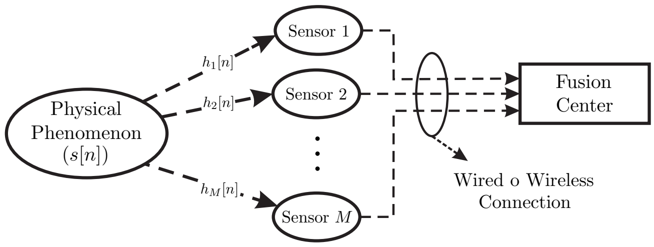

Fig. 1. Schematic diagram of a sensor network.

where

$$\mathbf{\Psi}_x = \begin{bmatrix} r_{x_1x_1} & \cdots & r_{x_1x_M} \\ \vdots & \ddots & \vdots \\ r_{x_Mx_1} & \cdots & r_{x_Mx_M} \end{bmatrix},$$

and the cross-correlation coefficients are defined as

$$r_{x_ix_j} = E[x_i[n]x_j^*[n]], \quad \forall i, j = 1, \ldots, M,$$

where $E[\cdot]$ denotes the mathematical expectation.

It is clear that $\mathbf{\Psi}_x$ can be seen as a frequency-independent version of $\mathbf{\Sigma}_x(\omega)$. However, the GC uses the determinant of $\mathbf{\Psi}_x$ whereas we use the largest eigenvalue of $\mathbf{\Sigma}_x(\omega)$. In practice, the GC is estimated from a limited number of observations of the signals, N, and the detection criterion is

$$\hat{\zeta}^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \nu,$$

where ν is a properly selected threshold.

4.2. The GMSC spectrum as a multi-channel signal detector

We propose to use the integral of $\gamma(\omega)$ (the squared root of the GMSC spectrum) as a detection statistic as follows

$$\Gamma = \frac{1}{2\pi} \int_0^{2\pi} \gamma(\omega) d\omega \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \nu,$$

where ν is also a threshold that must be selected according to the Bayes or the Neyman-Pearson criteria.

When $\gamma(\omega)$ is estimated using the reduced-rank CCA-based estimate described in Section 3, the proposed detection statistic is given by

$$\hat{\Gamma} = \frac{1}{2\pi} \int_0^{2\pi} \hat{\gamma}(\omega) d\omega =$$
$$= \frac{1}{2\pi} \frac{1}{M-1} \sum_{p=1}^{P} \sum_{k=1}^{M} (\hat{\lambda}^{(p)} - 1) \int_0^{2\pi} \left| \mathbf{f}^H(\omega) \hat{\mathbf{w}}_k^{(p)} \right|^2 d\omega,$$

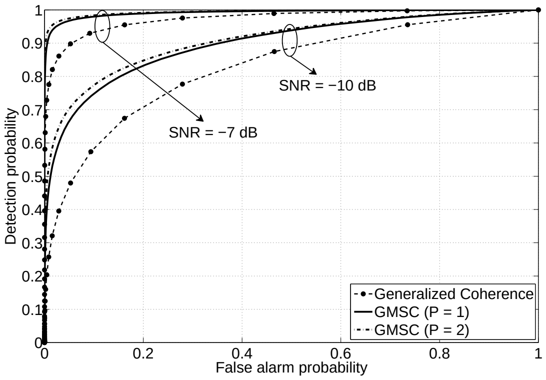

Fig. 2. ROC for the first example, $M = 3$, $N = 256$, $L = 20$.

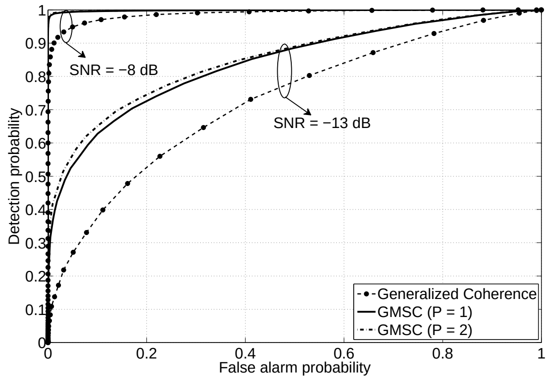

Fig. 3. ROC for the second example, $M = 5$, $N = 256$, $L = 20$.

which, by applying the Parseval's theorem, reduces to

$$\hat{\Gamma} = \frac{1}{M-1}\sum_{p=1}^{P}(\hat{\lambda}^{(p)} - 1)\sum_{k=1}^{M}\left\|\hat{\mathbf{w}}_k^{(p)}\right\|^2 = $$
$$= \frac{1}{M-1}\sum_{p=1}^{P}\left(\hat{\lambda}^{(p)} - 1\right),$$

where we have taken into account that the eigenvectors $\hat{\mathbf{w}}^{(p)} = \left[\hat{\mathbf{w}}_1^{(p)T}, \ldots, \hat{\mathbf{w}}_M^{(p)T}\right]^T$ have unit norm. Therefore, when the detection statistic is obtained from the reduced-rank GMSC estimate the integration simplifies to the sum of the P largest eigenvalues of the CCA problem (8).

An important aspect of the GMSC spectrum estimate, and consequently of the proposed detector, is how to select the model order (i.e., P). This is an old and complex problem that we will not treat here in detail. As we will show in the simulations section, for this particular application using $P = 1$ is enough to get good results. Moreover, this choice also notably reduces the computational cost of the test, since only the largest eigenvalue of the matrix $\hat{\mathbf{D}}^{-1/2}\hat{\mathbf{R}}\hat{\mathbf{D}}^{-1/2}$ must be obtained. The interpretation of this choice of the order is that from a practical standpoint the detection statistic can be formed using uniquely the most correlated frequency of the GMSC spectrum.

5. SIMULATION RESULTS

In this section we evaluate the performance of the proposed detector by means of some numerical examples. Specifically, the performance of the proposed technique is compared with the generalized coherence by means of the receiver operating characteristic (ROC) curve. In all the examples the signal $s[n]$ is a narrowband zero-mean real Gaussian process with unit power and passband between 0.1 and 0.15.

In the first two examples we assume perfect channels between the source and each sensor (i.e., $h_k[n] = \delta[n]$, $\forall k$), we acquire $N = 256$ samples of each signal and use a time embedding of $L = 20$ to estimate the cross-correlation matrices. We compare the ROC of the GMSC-based detector using $P = 1$ and $P = 2$, and that of the GC detector. Figures 2 and 3 show the obtained results when there are $M = 3$ and $M = 5$ sensors, respectively: it can be seen that the proposed detector outperforms the GC-based detector in both scenarios. From these figures we also see that the performance of the proposed detector with $P = 1$ and $P = 2$ is very similar.

In the third and fourth examples we have considered normalized finite impulse response (FIR) channels with $T = 10$ taps, where each tap is an independent random variable generated according to

$$h_k[n] \sim \mathcal{N}(0, 1/T), \quad n = 0, \ldots, T-1.$$

We also use a time embedding of $L = 20$ to estimate the cross-correlation matrices in both examples. Figure 4 compares the ROCs for the different detectors when there are $M = 3$ sensors and $N = 256$ samples are acquired at each sensor output. Finally, Fig.5 shows the results for $M = 5$ sensors and $N = 128$ samples. Again, we can conclude that when frequency-selective channels are considered the proposed detector also outperforms the GC detector.

6. CONCLUSIONS

In this paper we have proposed a new statistic to detect the presence of a common signal distorted by a set of unknown frequency-selective channels and corrupted by noise. Specifically, the new detector is the integral of the square root of the generalized magnitude squared coherence (GMSC) spectrum, which has been recently proposed as a frequency-dependent measure of the statistical linear relationship between more

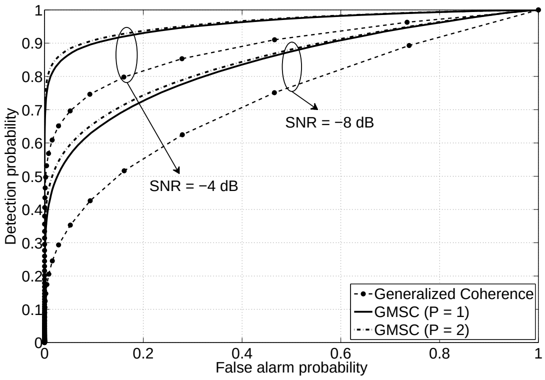

Fig. 4. ROC for the third example, $M = 3$, $N = 256$, $L = 20, T = 10$.

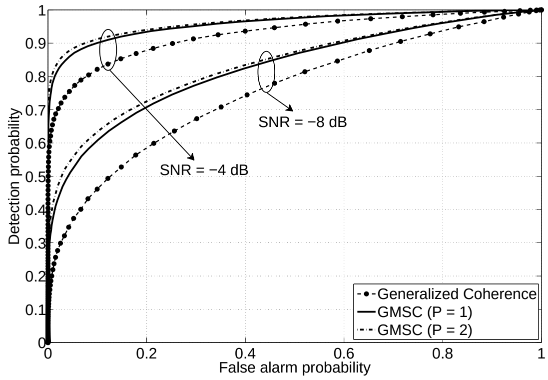

Fig. 5. ROC for the fourth example, $M = 5$, $N = 128$, $L = 20, T = 10$.

than two stationary random processes. There is a close relationship between the GMSC spectrum and canonical correlation analysis (CCA): this can be exploited to develop accurate reduced-rank estimators of the GMSC by solving a CCA problem. We have also shown that this relationship allows us to rewrite the integral in the test statistic as a summation of the P largest eigenvalues of the corresponding CCA problem. Interestingly, good results are obtained by using only the largest eigenvalue, (i.e. $P = 1$), thus avoiding the order selection problem. Finally, the performance of the GMSC-based detector is compared with the generalized coherence by means of computer simulations showing a clear improvement in most of the situations.

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