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Compressive Nonlinear Frequency Modulated Continuous Wave LADAR

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***Abstract*—We investigate through experiments and simulations a straightforward modification of frequency modulated CW (FMCW) LADAR that permits laser ranging at drastically reduced sampling rate per equivalent total range and acquisition time via compressed sensing (CS) recovery. Short range precision is investigated via gridless CS recovery in simulation. We also demonstrate in experiment that CS can achieve comparable precision to a Nyquist system in a practical nonlinear FM LADAR by construction of a suitable sparsifying dictionary.**

***Index Terms*—LADAR, LIDAR, Laser Ranging, Compressed Sensing**

I. Introduction

Frequency Modulated Continuous Wave (FMCW) LADAR [1] is a powerful technique for laser ranging that decouples the optical bandwidth (B_o) from the electronic receiver bandwidth (B_e). Optical bandwidths up to several THz have been demonstrated [2], yielding impressive resolution and precision without the need for broadband photodetectors (PDs) and ADCs required by conventional time-of-flight systems. However, these advantages come at the cost of either total range or acquisition time. Thus, we present an approach to FMCW LADAR employing compressed sensing (CS), which can improve this tradeoff by a factor $>$200 (i.e., the compression ratio). We confirm experimentally the system bandwidth, investigate the expected precision in simulations, and finally confirm the system performance compared with a Nyquist system by generalizing the CS sparsifying basis to nonlinear FM.

II. System Concept

The system principle of operation is depicted in Fig. 1. A fiber-coupled narrow linewidth continuous wave (CW) laser is swept in frequency ν (i.e., FM) through a window centered at 1550 nm. This laser is split with a fiber coupler and one arm is directed out to the range through an optical circulator while the other arm acts as a local oscillator (LO) for the returned signal. The LO is combined with the range signal and collected onto a balanced photodetector (PD) after a 2x2 50:50 fiber coupler. The interference signal then contains a beat note at frequency f_{beat}. For the case of conventional linear FM, the sweep rate or chirping rate (C) yields directly the range estimates from the beat note: $R = cf_{\text{beat}}/(2C)$ where c is the speed of light in the medium.

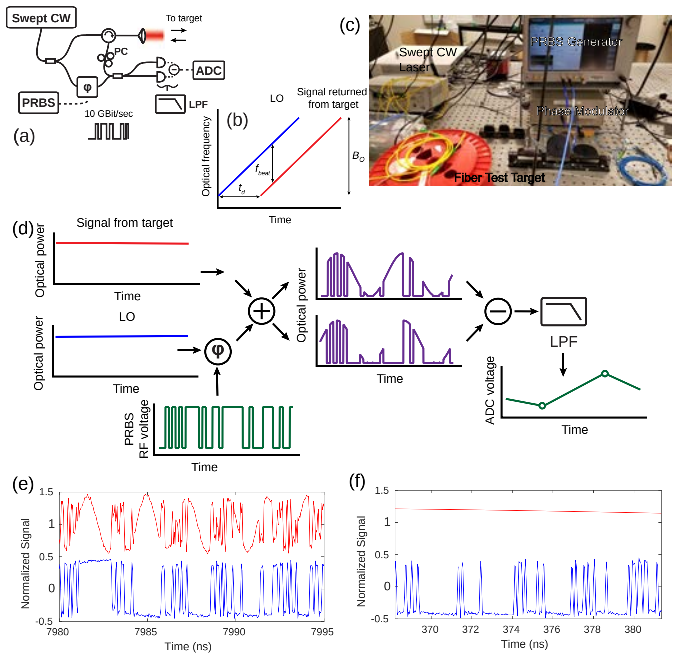

Fig. 1. (a) System diagram. (b) Linear FMCW LADAR measurement scheme depicting the relationship between the time to return from the target t_d, the optical frequency sweep of bandwidth BO, and the resulting beat note f_{beat}. (c) Photograph of experiment. (d) Conceptual operation of the Compressive FMCW technique introduced here: phase modulation of the swept LO produces complementary binary intensity modulation of the beat note, which undergoes subtraction and low-pass filtering (LPF) at the PD before digitization. (e) Raw modulation sequence (bottom) and chipped beat note detected with a fast balanced PD (top). (f) Modulation sequence and signal with slow PD and LPF.

We introduce compression into the system by adding a lithium niobate electro-optic phase modulator into the LO arm of the system driven by a 10-Gbit/s pseudorandom binary sequence (PRBS). This phase modulation is converted to complementary amplitude modulation after the 2x2 coupler and the DC component is subtracted at the balanced detector, which also low-pass filters the modulated signal. In this way, true $+1/-1$ modulation can be achieved with a positive optical signal and the system can be treated as a random demodulator collecting $M \ll N$ measurements where N is the Nyquist dimension of the signal [3]–[5]. Figure 1(e) depicts the intended system operation in experiment. The beat note is

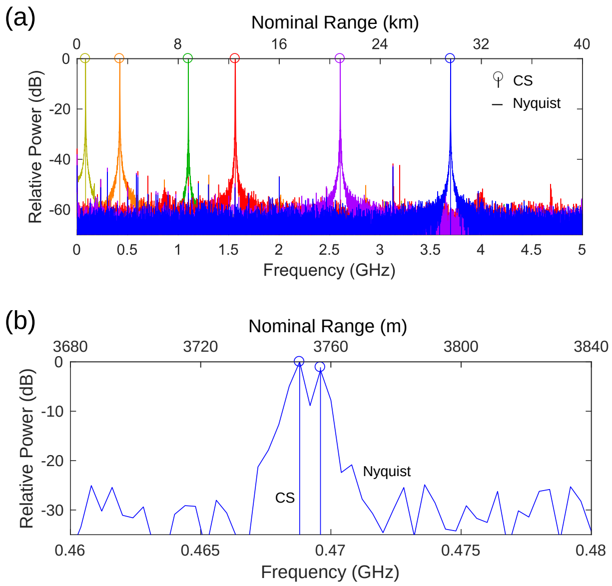

Fig. 2. (a) Nyquist measurements and CS reconstructions of several targets throughout the effective measurement bandwidth. (b) Successful reconstruction of two targets separated by one point on the CS reconstruction grid.

clearly visible on the red signal and correctly weighted $+1/-1$ by the 10-Gbit/s PRBS waveform in blue. An experimental low-pass filtered waveform input to the CS reconstruction is depicted in Fig. 1(f).

III. Linear FM

As a proof of concept experiment, we test reconstruction of signals throughout the experimental bandwidth and compare with a conventional Nyquist FMCW measurement. The swept CW laser frequency is sinusoidally FM modulated with a piezo inside the laser cavity and the interference signal is measured during only the most linear portion of the sweep. The 18.75 THz/s instantaneous C of the laser yields a maximum range of $R = cB_E/(2C) = 40$ km in air, but this will be revisited in Section V and we term this "nominal range" for now. We employ several lengths of fiber to emulate long free-space target ranges and create beat notes throughout the maximum 5 GHz electronic reconstruction bandwidth (B_e). The CS system successfully reconstructs single beat notes throughout B_e using orthogonal matching pursuit (OMP) [6] as well as closely-spaced tones separated by a single reconstruction grid point ($M = 100$ and $N = 25000$), depicted in Fig. 2.

IV. Precision

We investigate in simulations the best achievable precision per received signal power from the compressive linear FMCW system. For $B_e = 5$ GHz electronic receiver bandwidth, we include the dominant physical noise sources: thermal noise due to the random motion of electrons in the photodetector load resistance (r_L)

$$\sigma_T^2 = (4k_BT/r_L)B_e \tag{1}$$

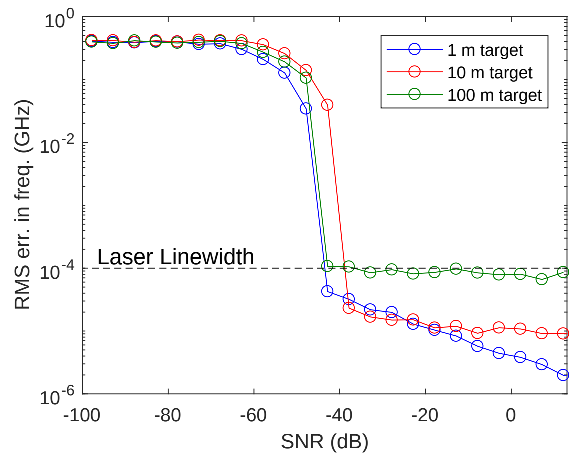

Fig. 3. Range precision convergence for a CS linear FMCW system (100× compression ratio) using gridless NOMP.

and shot noise due to the Poisson statistics of the photoelectron generation

$$\sigma_S^2 = 2qI_pB_e \tag{2}$$

where q is the electron charge and I_p is the average photocurrent (set here to $I_p = 2$ mA per detector, primarily contributed by the LO arm). We also incorporate the phase noise of the laser such that the signal and LO arms share the same Gaussian random phase fluctuations $\Delta\phi(\tau) = \phi(t+\tau) - \phi(\tau)$ [7]. We set laser linewidth due to these fluctuations to $\Delta\nu_{\text{lw}} = 100$ kHz. We then perform 100 frequency estimation tests per received power value at different random beat note frequencies (on a continuous grid) over a 2.5 µs window and measure the RMS deviation from the ground truth. Because laser frequency is tied to time in the FMCW system, the same frequency deviation (δf) corresponds to different range deviations depending on the sweep rate $\delta R = c\,\delta f/(2C)$.

To consider the upper limits on the precision of the compressive linear FMCW system, we set the sensing matrix size to $M = 250, N = 25000$ and adopt a recent gridless CS approach known as Newtonized OMP (NOMP) [8], which iteratively selects sinusoids and then performs Newton refinements of the detected sinusoids in order to reconstruct frequency-sparse signals across a continuum of frequencies. We note that above a detection threshold the system precision improves linearly on log-log scale for small range values (e.g., 1 m) as predicted by the Cramer-Rao lower bound. However, the effect of the laser phase noise creeps in with increasing range, degrading the best achievable precision regardless of SNR. At ranges on the order of 100 m and above the precision converges above the detection threshold directly to a level approximately equal to the phase noise and stays there: this holds either for a Nyquist or compressive approach.

V. Nonlinear FM

We consider now the case of nonlinear laser FM, focusing on the practical implementation of the CS LADAR system

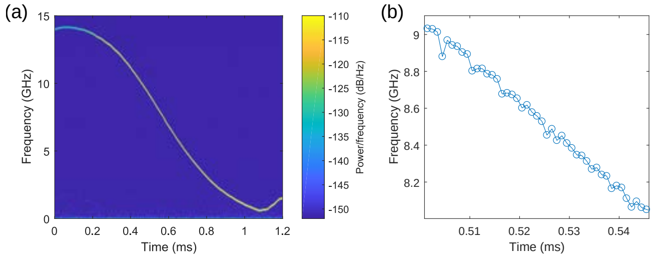

Fig. 4. (a) Spectrogram of a typical beat signal between the sweeping laser and the reference laser for calibration of the range estimates. (b) MUSIC estimation of the reference beat note for $1\mu s$ intervals, depicting nonlinear regions along the sweep.

with a free-running commercial laser. We add a 90:10 splitter to the swept CW laser and heterodyne it with a much narrower linewidth (specified < 1 kHz) distributed feedback fiber laser (at frequency ν_{ref} slightly below the swept laser's range) acting as a reference before detection with a fast (> 10 GHz) photodetector. This permits high resolution external measurement of the swept CW frequency over time. To cover larger optical bandwidths at lower cost, a fine free spectral range (FSR) filter or gas cell and slow photodetector can easily replace this reference laser.

We discover using the external frequency reference that the swept laser is very nonlinear throughout the majority of the sweep both at coarse and fine timescales. This occurs under all of the driving conditions investigated for a fast maximum sweep rate and high sweep frequency. The conventional equation for extracting a single range estimate from a swept source using an external frequency reference is [1]

$$R = \frac{\Delta\phi}{2\pi}\frac{c}{2\Delta\nu} \tag{3}$$

where

$$\Delta\phi = \int_0^{t_{\text{meas}}} 2\pi f_{\text{beat}}(t)dt \tag{4}$$

is the accumulated phase of the beat note over the duration of the measurement t_{meas} and $\Delta\nu$ is the length of the optical frequency sweep. ($\Delta\phi/2\pi$ can be considered the number of fringes in the beat note.) However, using the reference signal to determine $\Delta\nu$ over windows of time from several microseconds to nearly the full ~millisecond sweep duration produces wildly inaccurate range estimates, biased 10% or more off of the correct value with very large standard deviation (Fig. 6(a)). The nonlinearity of the laser sweep is illustrated on both coarse (ms) and fine (μs) timescales in Fig. 4. Figure 4(a) depicts a typical spectrogram and (b) depicts a MUSIC [9] estimate of the reference signal over 1-μs intervals.

To address the nonlinearity of the sweep, we modify the range estimation for the Nyquist LADAR by incorporating the measured history $x_{\text{laser}}(t)$ of the downshifted laser frequency $f(t) = \nu(t) - \nu_{\text{ref}}$:

$$x_{\text{laser}}(t) = a(t)\cos[2\pi f(t)t + \phi]. \tag{5}$$

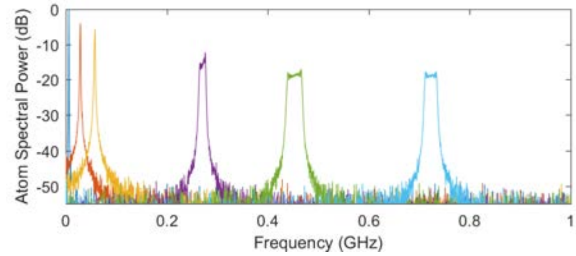

Fig. 5. Spectral power for several atoms $\mathbf{y}(\tau)$ in a typical computed sparsifying dictionary for the nonlinear FMCW CS system.

For any measurement window $t \in [0, t_{\text{meas}}]$, we compute predicted interference signals

$$y(t,\tau) = \text{LPF}\{[x_{\text{laser}}(t) + x_{\text{laser}}(t-\tau)]^2\} \tag{6}$$

where τ is a time shift in the range $[0, cB_E/(2C_{\text{max}})]$ (C_{max} is the maximum instantaneous chirping rate). LPF is a low-pass filter function that removes the $f(t) + f(t-\tau)$ components (for the down-chirp) from the predicted signal. The range $R = c\tau/2$ is then estimated from the measured $y_{\text{meas}}(t)$ when τ is determined by

$$\max_{\tau} |\text{Cov}[y(t,\tau), y_{\text{meas}}(t)]|. \tag{7}$$

To test the same approach with the CS system, we construct a dictionary ($\mathbf{D}$) with normalized $\mathbf{y}(\tau)$ vectors as atoms for $\tau \in [0, cB_E/C_{\text{max}}]$ and solve the so-called basis pursuit denoising problem

$$\min_{\boldsymbol{\alpha}\in\mathbb{R}^N} \frac{1}{2}||\boldsymbol{y} - \boldsymbol{\Phi}\mathbf{D}\boldsymbol{\alpha}||_2^2 + \lambda||\boldsymbol{\alpha}||_1 \tag{8}$$

using SpaRSA [10] (OMP yields nearly identical results) where $\boldsymbol{\Phi}$ is the random $+1/-1$ binary matrix with $N = 25000$. The locations of the activate elements in $\boldsymbol{\alpha}$ then correspond directly to the time delay incurred by the signal. To illustrate the difference between the dictionary atoms and the discrete Fourier basis it is designed to replace, we plot the FFT of several of the atoms in a typical $\mathbf{D}$ in Fig. 5. It is clear that the expected beat notes are much better represented by chirps of varying width and center frequency (depending on the delay and measured sweeping behavior) than by simple tones.

We solve this problem independently ($M = 100, N = 25000$) for 2.5 μs duration windows along a region of the sweep for a 770 m target range and compare in Fig. 6 with an equivalent measurement by the Nyquist system collected several minutes earlier. Both measurements identify the correct range window at each point along the sweep and have comparable standard deviations: 1.25 m for the Nyquist measurement and 1.6 m for the CS system, but the CS system collects 250× fewer measurements. Both uncertainties agree with the estimate $\delta R = c\delta f/(2C) = 2.15$ m taking into account our measured laser linewidth (over a 500-μs period) of $\delta f = 244$ kHz and the average sweep rate $C = 17$ THz/s over the interval in question.

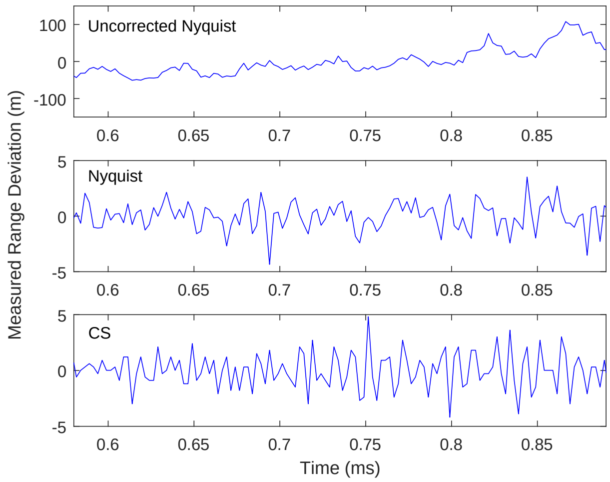

Fig. 6. Nonlinear FMCW precision measurements for a 770 m target range. Depicted are "uncorrected" Nyquist measurements using only equations (3) and (4), corrected Nyquist measurements using equation (7), and CS using the nonlinear FM approach described in Section V.

VI. Conclusion and Outlook

In conclusion, we have demonstrated the potential of compressive FMCW LADAR, considering both linear and nonlinear FM sweeps. The approach is capable of reconstructing beat notes throughout a 5-GHz equivalent electronic bandwith using up to 200× fewer measurements and digitizer bandwidth than a Nyquist system. Through continuous grid CS reconstruction, we demonstrate that the system can match the precision of Nyquist systems for single target range estimation. Finally, to consider nonlinear FM, we adapt the sparsifying basis with external knowledge about the FM sweep, constructing a dictionary of predicted beat notes. This simultaneously generalizes the sensing problem and mitigates nonidealities in the laser, affording nearly equivalent precision for large target ranges between the CS and Nyquist systems.

Future investigations will focus on improving the robustness and efficiency of the dictionary generation and experimentally verifying the precision for very small ranges (<100 m) predicted in Section IV.

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