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Sparsity-constrained Kalman Filter concept for damage identification in mechanical structures

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***Abstract*—This paper presents a novel framework for damage identification in mechanical structures by combining sparse solutions with an Extended Kalman Filter. Structural damage can be described by means of a sparse parameter vector, as damage leads in most cases to a very local stiffness change, while the other areas remain unchanged. In order to ensure sparsity of the estimated damage parameter vector the state space measurement equation is expanded by an additional nonlinear L_1-minimizing observation. This fictive measurement equation accomplishes stability of the Extended Kalman Filter and leads to a sparse estimation. For verification, a proof-of-concept example on a plate structure is presented.**

***Keywords*—L_1-minimization, sparse reconstruction, Extended Kalman Filter, damage identification.**

I. Introduction

Structural health monitoring (SHM) is a methodology to ensure a safe operation of mechanical structures and to reduce life cycle cost by replacing schedule-driven inspections by Condition-Based Maintenance (CBM). One major task of SHM systems is the detection and identification of damage in an early stage of structural damage evolution. An integrated sensor network is required to measure the structural vibrations excited either by an artificial or a natural source, e.g. wind and traffic loads. By means of the integrated sensor system the effect of the damage on the structural vibration response can be measured indirectly.

Now, SHM systems need smart data processing algorithms in order to draw conclusions about the exact cause for the measured effect. Thus, vibration-based damage identification can be seen as the inversion of the principle of cause and effect. This leads to a mathematical inverse problem. If there are many causes which will lead to the same measurable effect, the inverse problem is additionally ill-posed. Ill-posedness means that either the existence, the uniqueness or the stability of the solution is violated.

In the last decades many methods have been developed and a considerable amount of literature has been published on the inverse problem of structural damage identification, an overview on this topic can be found for example in [1]. Damage identification techniques can be classified as frequency or time domain or time-frequency domain methods. Classical frequency domain approaches consider the changes of the natural frequencies, modal damping, mode shapes or frequency response functions due to damage. As these quantities provide information on a global level, they are often insensitive to small local structural damage, especially if only lower structural modes are used. In the medium or higher frequency range problems may occur to identify these modes since this requires a very dense sensor network. Time domain approaches seem to be comfortable, as raw time data can be used directly.

Several time domain approaches have already been proposed, such as least-squares estimation methods [2], [3] or methods using particular filters [4], [5], [6], [7]. For the latter the Extended Kalman Filter (EKF) is the most well-known system parameter estimation method [8], [9], [10]. EKF-based system parameter identification belongs to the class of model-based approaches. Here, a reference model of the undamaged structure is tested against the actual system in each filter step. In the filter process the state vector of the Kalman Filter equations is typically augmented [10] or sometimes even replaced [11] by the system parameter to be estimated. By making use of the input-output signal an estimation of the system parameters is obtained in each filter step. Even so the original Kalman Filter is known as optimal linear filter, EKF-based damage identification is still facing some challenges, e.g. high computational effort for complex structures and intrinsic ill-posedness of the inverse problem [12]. To overcome ill-posedness usually the damage parameter space is reduced by considering only damage hot spots or by a drastic increase of the sensor number. In order to perform damage monitoring on the whole structure and to keep the required number of sensors low, a sparsity-constrained Extended Kalman Filter concept is proposed.

Therefore, a priori information about the damage properties is used to solve the inverse problem and to obtain meaningful solutions. For example, cracks can be interpreted as spatial singularities, which cause only a very local structural stiffness reduction. Thus, a system parameter vector which describes the change in structural stiffness has only a few non-zero elements corresponding to the actual damage location. Such a vector is called sparse. In many fields of applied mathematics, L_1-regularizing techniques have been proven to promote such kind of sparse solutions (e.g. Compressive Sensing [13], [14]. The proposed damage identification method links the concept of L_1-regularization with the Extended Kalman Filter by expanding the measurement equation by an additional nonlinear L_1-minimizing observation.

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The paper is structured as follows: The problem of damage parameter estimation using a non-linear state-space description is formulated in section II. In section III the concept of sparse solution is incorporated in the Extended Kalman filter concept. Various proof-of-concept simulation studies are carried out in section IV. Here the functionality of the proposed identification method is demonstrated by analyzing different damaged scenarios on a quadratic aluminum plate structure. A stochastic validation is performed by means of a Monte Carlo simulation and the capability of compensating modelling errors is also shown. Finally, concluding remarks are presented in section V.

II. Problem Statement

In general, the dynamics of a nonlinear, time-varying structure can be described similar to [15] as:

$$\mathbf{M}\left(\mathbf{\Theta}_k, \mathbf{x}_k, k\right) \ddot{\mathbf{x}}_k + \mathbf{g}\left(\mathbf{\Theta}_k, \mathbf{x}_k, \dot{\mathbf{x}}_k, k\right) = \mathbf{u}_k \tag{1}$$

$$\mathbf{\Theta}_{k+1} = \mathbf{\Gamma}\left(\mathbf{\Theta}_k, \mathbf{x}_k, \dot{\mathbf{x}}_k, k\right) \tag{2}$$

$$\mathbf{y}_k = \mathbf{h}^*\left(\mathbf{\Theta}_k, \mathbf{x}_k, \dot{\mathbf{x}}_k, k\right) \tag{3}$$

Eq. (1) is the nonlinear equation of motion in discrete time domain $t_k = k\Delta t,\ k \in \mathbb{N}$. $\mathbf{M}(\cdot) \in \mathbb{R}^{m\times m}$ is the mass matrix and $\mathbf{g}(\cdot) \in \mathbb{R}^m$ the force vector of elastic and damping forces. These can depend on the nodal displacement $\mathbf{x} \in \mathbb{R}^m$, the nodal velocity $\dot{\mathbf{x}} \in \mathbb{R}^m$ and the time step k. The damage parameter $\mathbf{\Theta} \in \mathbb{R}^p$ describes the change of structural integrity (loss of stiffness, loss of mass, etc.) by location and damage extent. For Structural Health Monitoring this is the parameter which needs to be reconstructed. Moreover, the damage parameter $\mathbf{\Theta}$ usually has also influence on the equation of motion. $\mathbf{u} \in \mathbb{R}^m$ is the vector of the external acting loads on the structure. The number of degrees of freedom (DOF) is m. The nonlinear function $\mathbf{\Gamma}(\cdot) \in \mathbb{R}^p$ describes the evolution of the damage parameter in Eq. (2). Eq. (3) is the measurement equation which links the model quantities (displacement, velocities and system parameters) with the output $\mathbf{y} \in \mathbb{R}^n$ of the measurement device by means of the function $\mathbf{h}^*(\cdot) \in \mathbb{R}^n$. The number of measurements equals n.

If the structure can be assumed to be linear, the equation of motion becomes:

$$\mathbf{M}\left(\mathbf{\Theta}_{\boldsymbol{k}}\right) \ddot{\mathbf{x}}_k + \mathbf{C}\left(\mathbf{\Theta}_{\boldsymbol{k}}\right) \dot{\mathbf{x}}_k + \mathbf{K}\left(\mathbf{\Theta}_{\boldsymbol{k}}\right) \mathbf{x}_k = \mathbf{u}_k \tag{4}$$

Here, the structural mass matrix $\mathbf{M}$, the structural stiffness matrix $\mathbf{K}$ and the damping matrix $\mathbf{C}$ still depend on the damage parameter $\mathbf{\Theta}$.

Mostly the evolution of the damage parameters $\mathbf{\Theta}$ and the structural dynamic vibrations occur on two different time scales. Compared to the structural vibrations, the evolution of damage is a rather slow process. Thus, the damage parameter $\mathbf{\Theta}$ seems to remain constant during a short time span of data acquisition [15].

Now, a state space model can be defined, in which the unknown damage parameter vector is the state vector. The evolution of it is modeled by a Gaussian Markov process, also called random walk process [11]:

$$\mathbf{\Theta}_{k+1} = \mathbf{\Theta}_k + \mathbf{w}_k \tag{5}$$

$$\mathbf{y}_k = \mathbf{h}\left(\mathbf{\Theta}_k, [\mathbf{U}]_k, \mathbf{x}_0, \dot{\mathbf{x}}_0, k\right) + \mathbf{v}_k\ , \tag{6}$$

where $\mathbf{w}_k \in \mathbb{R}^p$ is zero-mean white process noise with covariance $\mathbf{Q}_k$, $\mathbf{w}_k \sim N(\mathbf{0}, \mathbf{Q}_k)$. Here, the measurement equation in Eq. (6) is defined slightly different as above. Unlike Eq. (3), the output measurement data are obtained depending on the initial nodal displacement and velocity $\mathbf{x}_0$ and $\dot{\mathbf{x}}_0$ and vector $[\mathbf{U}]_k = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k]^T$, which describes the external load input from the time step $k = 1$ to the current time step k. So the equation of motion is implicitly included in the nonlinear measurement equation $\mathbf{h}(\cdot) \in \mathbb{R}^n$. $\mathbf{v}_k \in \mathbb{R}^n$ represents the measurement noise with covariance $\mathbf{R}_k$, $\mathbf{v}_k \sim N(\mathbf{0}, \mathbf{R}_k)$. For reasons of clarity and without loss of generality, in the remainder of this paper it is assumed that $\mathbf{x}_0 = \mathbf{0}$ and $\dot{\mathbf{x}}_0 = \mathbf{0}$.

III. Extended L_1-minimizing Kalman Filter

In the following, the concept of L_1-minimizing sparse reconstruction is incorporated into an Extended Kalman Filter framework. Loffeld et al. were the first to propose an L_1-minimizing Kalman filter approach for solving underdetermined sparse problems [16]. Here, this idea is adopted to stabilize the Extended Kalman Filter parameter estimation process for a large damage parameter space p and a low number of sensors n.

Structural damages due to e.g. cracks can often be interpreted as spatial singularities, as they lead to a stiffness reduction in a very local area of the system rather than a global stiffness reduction. Thus, it can be assumed that the unknown damage parameter vector $\mathbf{\Theta}$ is sparse. The sparsity will be considered as a constraint, which will be part of the state space model as an additional, nonlinear observation. It is promoted by the L_1-norm of the state vector:

$$\hat{\mathbf{y}}_k = \gamma_k = \left\|\mathbf{\Theta}_k\right\|_1 = \sum_{j=1}^{p} \left|\Theta_{j,k}\right| \tag{7}$$

Starting from $\gamma_0 = \left\|\mathbf{\Theta}_0\right\|_1$ the fictive measurement γ_k can now successively be decreased in each time step k by a scaling factor $\alpha < 1$,

$$\gamma_{k+1} = \alpha \left\|\mathbf{\Theta}_k\right\|_1 \tag{8}$$

The scalar Eq. (7) pushes down the L_1-norm of the state vector and thus leads to a sparse estimation of $\mathbf{\Theta}$. The now obtained augmented observation vector $\tilde{\mathbf{y}}_k$ reads as follows:

$$\tilde{\mathbf{y}}_k = \begin{bmatrix} \mathbf{y}_k \\ \gamma_k \end{bmatrix} = \begin{bmatrix} \mathbf{h}\left(\mathbf{\Theta}_k, [\mathbf{U}]_k, k\right) \\ \left\|\mathbf{\Theta}_k\right\|_1 \end{bmatrix} + \begin{bmatrix} \mathbf{v}_k \\ \nu_k \end{bmatrix} \tag{9}$$

$\nu_k \in \mathbb{R}$ reflects the uncertainty of the additional L_1-minimizing observation equation.

For estimating the states of the now obtained nonlinear state space model an Extended Kalman Filter is used. The EKF linearizes the nonlinear model in each time step around the *a posteriori* estimated state vector, using a first-order Taylor series approximation. After linearization the traditional prediction-correction algorithm of the Kalman Filter can be applied.

Starting from the initial conditions $\hat{\mathbf{\Theta}}_{0|0}$ and $\mathbf{P}_{0|0}$, a forecast of the state is made in the prediction step:

$$\hat{\mathbf{\Theta}}_{k|k-1} = \hat{\mathbf{\Theta}}_{k-1|k-1} \tag{10}$$

$$\mathbf{P}_{k|k-1} = \mathbf{P}_{k-1|k-1} + \mathbf{Q}_k \tag{11}$$

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In the corrector step the actual measurements $\mathbf{y}_k$ and the fictive measurements $\tilde{\mathbf{y}}_k$ are considered and compared with the prediction. The residual $\Delta\tilde{\mathbf{y}}_k$ is weighted by the Kalman gain matrix $\mathbf{K}_k$ and added to the prediction $\hat{\mathbf{\Theta}}_{k|k-1}$:

$$\Delta\tilde{\mathbf{y}}_k = \tilde{\mathbf{y}}_k - \mathbf{h}\left(\mathbf{\Theta}_{k|k-1}, [\mathbf{U}]_k, k\right) \tag{12}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \tilde{\mathbf{R}}_k\right)^{-1} \tag{13}$$

$$\hat{\mathbf{\Theta}}_{k|k} = \hat{\mathbf{\Theta}}_{k|k-1} + \mathbf{K}_k\Delta\tilde{\mathbf{y}}_k \tag{14}$$

$$\mathbf{P}_{k|k} = \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}\right)\mathbf{P}_{k|k-1} \tag{15}$$

For the proposed damage parameter estimation strategy only the measurement equation is nonlinear and needs to be linearized:

$$\mathbf{H}_k = \begin{bmatrix} \left.\frac{\partial \mathbf{h}\left(\mathbf{\Theta}_k, [\mathbf{U}]_k, k\right)}{\partial \mathbf{\Theta}}\right|_{\mathbf{\Theta}=\hat{\mathbf{\Theta}}_{k|k-1}} \\ \left.\frac{\partial \|\mathbf{\Theta}_k\|_1}{\partial \mathbf{\Theta}}\right|_{\mathbf{\Theta}=\hat{\mathbf{\Theta}}_{k|k-1}} \end{bmatrix} \tag{16}$$

The derivative of $\mathbf{h}(\cdot)$ with respect to $\mathbf{\Theta}$ can be either approximated by the finite difference method or determined exactly by using the system-output sensitivity for linear structures. A detailed description of the output-sensitivity calculation can be found e.g. in [17].

The Jacobian matrix of the L_1-minimizing constraint

$$\frac{\partial \|\mathbf{\Theta}\|_1}{\partial \mathbf{\Theta}} = \left[\frac{\partial \|\mathbf{\Theta}\|_1}{\partial \Theta_1}\frac{\partial \|\mathbf{\Theta}\|_1}{\partial \Theta_2}\cdots\frac{\partial \|\mathbf{\Theta}\|_1}{\partial \Theta_p}\right], \tag{17}$$

can be obtained by:

$$\frac{\partial \|\mathbf{\Theta}\|_1}{\partial \Theta_j} = \operatorname{sign}\left(\mathbf{\Theta}_j\right) \tag{18}$$

The determination of the partial derivative in each time step k is computationally very expensive. In order to save computing time, this can be performed just in every second or third step. This also helps to stabilize the filter process in the beginning.

As usually structural damage has no direct impact on the measurements at the same specific time step k, it is advisable to extend the physical measurement $\mathbf{y}_k$ and to process a bloc of l physical measurements in each Kalman Filter step:

$$\begin{bmatrix} \mathbf{y}_k \\ \mathbf{y}_{k+1} \\ \mathbf{y}_{k+2} \\ \vdots \\ \mathbf{y}_{k+l} \end{bmatrix} = \mathbf{h}^l\left(\mathbf{\Theta}_k, [\mathbf{U}]_k, k, l\right) + \mathbf{v}_k^l \tag{19}$$

By this *en bloc* processing, the filter is no longer operating in real time but with a time lag $t_l = l\Delta t$ in the past.

IV. Proof-of-Concept

In order to demonstrate the functionality of the proposed damage identification strategy a proof-of-concept simulation study is performed. The observed mechanical structure is a simple square aluminum plate of $1\text{m} \times 1\text{m}$ edge length and 2mm thickness. It is clamped on all sides. The structural dynamics of the plate due to external forces are described by a finite element model. The plate is modeled by 121 quadratic shell elements and 144 nodes (each with 6 degrees of freedom), see Fig. 1. The employed structural responses

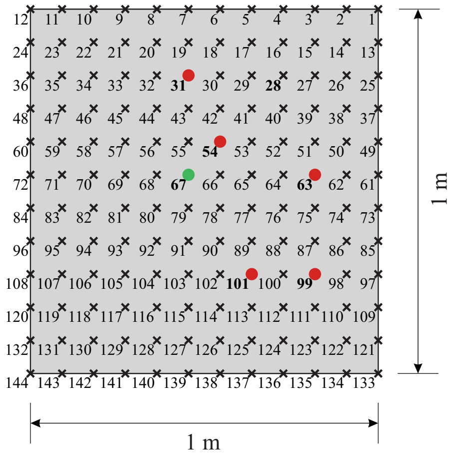

Fig. 1. Node numbering of the finite element plate model; node 31, 54, 63, 99 and 101 are acceleration measurement positions; node 67 is the structure excitation position

are simulated acceleration measurements perpendicular to the plate plane. The obtained simulated measurement signals are low-pass filtered by a cut-off frequency of 200Hz. Thus, for damage detection only the low frequency content of the time signals is employed. White Gaussian noise, with a standard deviation of three percent of the maximum measurement value, is added to the simulated outputs to imitate real acceleration measurement data. Throughout all investigations shown in this paper, only five accelerometers are used.

A widely used approach to introduce structural damage on a substructure or element level which represents the changes of the structural stiffness $\Delta\mathbf{K}$ compared to a reference model $\mathbf{K}_0$ is:

$$\Delta\mathbf{K} = \sum_j \mathbf{K}_j\mathbf{\Theta}_j \tag{20}$$

where $\mathbf{K}_j$ is the jth substructure or element stiffness matrix, respectively. By determination of the unknown correction parameters $\mathbf{\Theta} = [\Theta_1, \Theta_2, \ldots, \Theta_p]$ the damage can be localized and quantified.

A. Single and multiple damage scenarios

In a first simulation study the stiffness of element no. 81 has been decreased by 20%. The plate is excited by an impulse force load perpendicular to the surface at node no. 67 of known time history. The obtained simulated acceleration time data are now used for structural damage identification. Fig. 2 and Fig. 3 compare the damage parameter estimation results for the proposed Extended Kalman Filter method with and without additional L_1-minimizing observation. Both figures display the parameter estimation results at the end of simulation time. Fig. 2 shows that for all elements the corresponding damage parameters are close to zero. Except the true damage parameter is significantly larger. Thus, the damage is localized and quantified. It is obvious that no clear damage estimation result can be achieved without L_1-minimizing observation (see Fig. 3). Even though the reconstructed damage parameter error for the damage element no. 81 is not too big, many more element stiffness changes (reduction and increase) are identified. On the other hand, it can be clearly distinguished

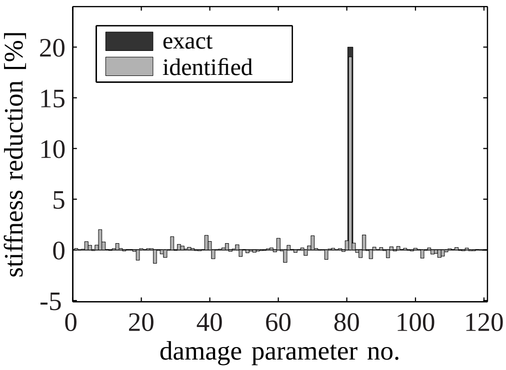

Fig. 2. Estimation result with L_1-minimizing observation: Damage is localized and quantified

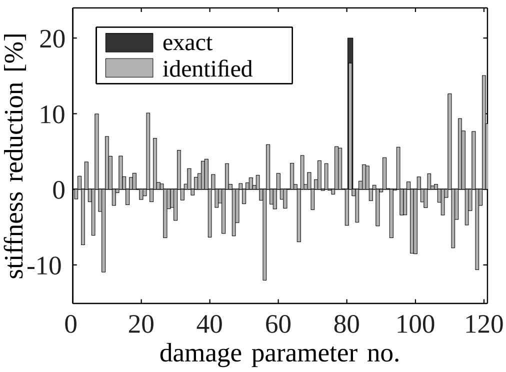

Fig. 3. Kalman filter estimation without additional L_1-observation: No clear damage pattern can be obtained

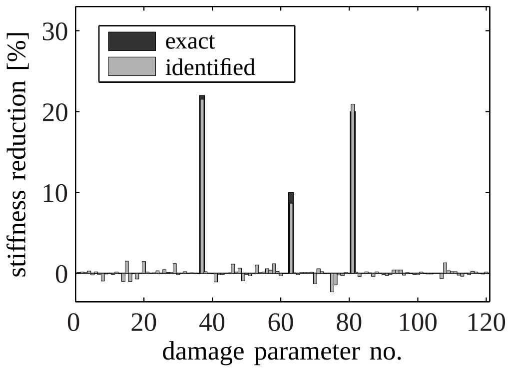

Fig. 4. Multiple damage scenarios: three damaged elements no. 37, no. 63 and no. 81

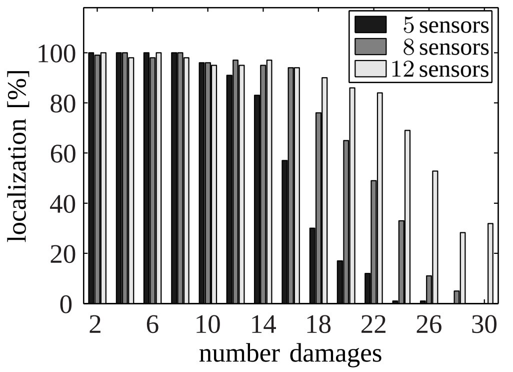

Fig. 5. Monte Carlo simulation: Influence of the sensor number and the damage number on localization reliability

between damaged and undamaged elements if the additional observation is used, as L_1-minimization promotes this sparse solution.

In a next step a multiple damage scenario is investigated. Here the plate structural damage is modelled by a stiffness reduction of various elements with different amount. Which means that the damage parameter vector needs to be less sparse than in the case of a single element stiffness change. Fig. 4 shows the damage identification results at the end of simulation time for three damaged elements. In this case a clear damage identification, similar as before is obtained.

B. Monte Carlo simulation

In section IV-A some selected damage identification results have been shown. However, for a statistical validation a Monte Carlo simulation is performed. In this study three different sensor setups (5, 8 or 12 sensors) are compared. For each setup 5000 trials with different damage scenarios are carried out. In each of the 5000 trials multiple damages are introduced in the structure by reducing the stiffness of various elements. The damage locations are chosen randomly with uniform distribution over all elements. The damage extent is also a random parameter with Gaussian distribution (mean value: 25% stiffness reduction; standard deviation: 5%).

The obtained damage localization results are displayed in Fig. 5. An estimated damage pattern is only defined as correct if all stiffness-reduced elements have been detected correctly. It can be seen that for a larger number of damages the localization reliability decreases. However, if more sensors are used this reliability can be improved.

C. Model error compensation

As the proposed damage identification strategy is a model-based approach, modeling errors will have an impact on the reconstruction results. For most practical applications there are some modeling parameters which are subject to uncertainties, e.g. the global modulus of elasticity, the mass density or the correct definition of the boundary conditions.

In order to compensate possible modeling errors, such model parameters can also be integrated in the estimation process. Thus, the algorithm will fit the unknown model parameters to the measurement output data. To this end, the parameter vector $\mathbf{\Theta}$ needs to be extended by these model parameters:

$$\tilde{\mathbf{\Theta}} = [\Theta_1, \Theta_2, \cdots, \Theta_p, \Theta_1^m, \cdots, \Theta_n^m] \tag{21}$$

Here the first p values are the damage parameters as previously defined. The last n parameters describe now the global model parameters.

Fig. 6 shows a damage reconstruction result by using an incorrect structural model. The model used in the reconstruction process varies from the one, which is employed to create the measurement data, not only in terms of the structural damage but also in terms of a modulus of elasticity and mass density.

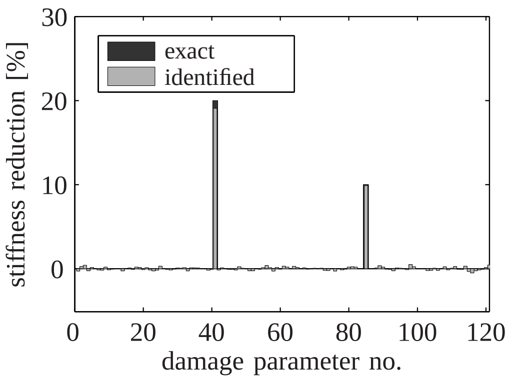

Fig. 6. Damage identification by using an incorrect structural model

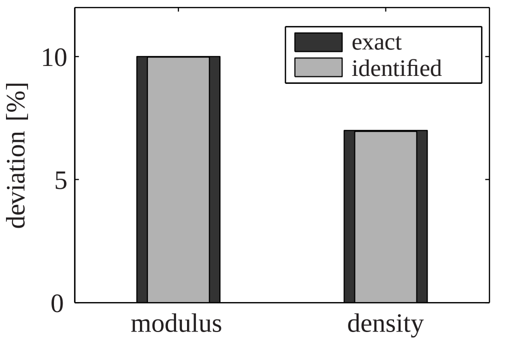

Fig. 7. Estimation of initially wrong model parameters (Young's modulus and mass density)

The deviation is 7% in mass density and 10% in modulus. However, a very clear estimation of the damage pattern can be obtained. The damage elements no. 41 and 85 are identified correctly and also the damage extend is reconstructed properly. Additionally, the model parameter mass density and modulus of elasticity have been identified, as shown in Fig. 7.

V. Conclusion

In this contribution a new time domain method for damage detection has been proposed. The local character of damage justifies the use of sparse reconstruction strategies for the ill-posed inverse problem. Sparsity of the estimated state vector of damage parameters is ensured within the Extended Kalman Filter by adding a fictive non-linear L_1-minimizing observation.

It has been shown that the proposed reconstruction method is able to determine the damage location and extent simultaneously. In contrast to the Extended Kalman Filter process without additional L_1-observation a clear damage pattern is obtained. This was shown for single damage scenarios as well as for multiple damage events. A statistical validation has been performed by means of a Monte Carlo simulation. Considering the damage parameter space of size 121 in the demonstrated study, the number of sensors using only 5 to 12 accelerometers is significantly lower than the parameter space. Moreover, modeling error can be compensated by including the model parameters, which are subject to uncertainties. Besides the unknown model parameter, this approach can also be used to reconstruct damage under changing environmental and operational conditions (EOC), if the EOC sensitive parameters are also included in the parameter vector.

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