Ground Clutter Processing for Airborne Radar in a Compressed Sensing Context

Philippe Mesnard
Thales Systèmes Aéroportés
CREST ENSAE ParisTech
CIFRE contract, Université Paris Saclay, France
philippe.mesnard@fr.thalesgroup.com

Cyrille Enderli
Thales Systèmes Aéroportés
78990 Elancourt, France
cyrille-jean.enderli@fr.thalesgroup.com

Guillaume Lecué
CREST ENSAE ParisTech
91764 Palaiseau Cedex, France
guillaume.lecue@ensae.fr

***Abstract*—Changes in airborne radar processing context implies more and more improvements in terms of radar time management. New principles like Compressed Sensing are currently studied to take into account modern situations of multiple echoes, provided some dedicated signal is sent by the radar. However, the presence of ground clutter in the signal received by the radar affects deeply the efficiency of the reconstruction treatment. Then the principle retained here has to be extended. Moreover, fine range and Doppler resolutions are required for identification. Thus we define and build a more accurate model of ground clutter. We show that our original model satisfies necessary hypothesis for reconstruction and clutter separation procedure.**

I. Introduction

The process conventionally used to estimate parameters of targets detected by the radar is matched filtering, it is optimal for the detection of a single point target [1], [2]. Compressed sensing [5], [6] would define a new treatment for the radar, also performing in the multi-target case.

When the expected metric resolution is accurate, each reflecting element in the wave propagation field is modeled as a set of multiple point targets, each of them generating an echo of the wave emitted by the radar, modulated in time by a delay proportional to its distance from the radar, in frequency by the Doppler effect proportional to its relative velocity, and in amplitude by a factor depending on its local electromagnetic signature. A multiple target scene can thus be represented in a 2D discrete grid, indexed by testable Doppler frequencies and time delays, in which the amplitude of each localized point target is spotted. Apart from these points, any other coordinate of the grid is affected to a zero value, meaning that no reflecting point has been detected at the corresponding range and velocity. Since the grid is generally large, the number of reflecting elements appears to be very low in comparison to the size of the research space so that the grid is represented by a sparse matrix. That's why Compressed Sensing is adapted to such situation.

But this hypothesis is valid when only few airborne objects, like planes, are present in the radar emission field. However, a large surface of the ground beneath the airborne platform often intercepts the electromagnetic wave. This defines a geometrical and physical intersection in which many point elements reflect, with non negligible amplitudes, the signal back to the radar. Therefore the exact representation of a global reflecting scene including both planes and ground clutter would be composed of many nonzero points in the Doppler-delay discrete grid, so that the hypothesis of sparsity for the scenario in the research space is no more valid. From this perspective, one wish to extend the mathematical context of Compressed Sensing to a procedure that ables the separation of the ground clutter and the so-called "useful signal" referring to airborne targets. This can be tractable provided the sub-matrix representing the clutter within the grid supports dedicated algebraic conditions. The approach retained here is the "sparse plus low rank procedure". One main objective of this paper is to check if one can build a model for the reflecting ground to make it representable by a low rank matrix.

Current geometrical and physical modeling for wave echoes from the ground is not accurate enough, because of geometrical approximations to simplify the modeling. For example, iso-velocity hyperbola on the ground are often approximated by their asymptotes. More exact modeling is however required to assess the algebraic properties of the underlying range-Doppler matrix. For this purpose, the complete writing of the corresponding sparse plus low rank procedure for the reconstruction and separation of airborne and ground echoes is being set up. Finally, we compare the representation of the ground with the ones already obtained for current model and test its mathematical properties as a matrix.

II. Compressed Sensing : its application to airborne radar and its limits in presence of ground clutter

A. Compressed Sensing applied to step frequency waveforms

We model a signal as a train of N pulses of duration T, transmitted with a repetition period T_R, whose carrier frequency varies from one pulse to the next. We denote, for all $n \in [\![1; N]\!]$, by f_n the frequency jump such as the carrier frequency of the nth emitted pulse is $f_0 + f_n$. The shape of these signals leaves a large margin of choice in their definition parameters and make them adaptable to different types of missions. Moreover, these waveforms are relatively simple to generate and a radar emitting such a signal is more resistant to jamming and counter measures [3] [4]. It is in this perspective that we focus our study towards this type of waveforms. The different frequencies assigned to the signal pulses are defined from a frequency quantum Δ_f and chosen from a set of N frequencies $\{k\Delta_f; k = 1, ..., N\}$, so that the signal frequency bandwidth is $B = N\Delta_f$.

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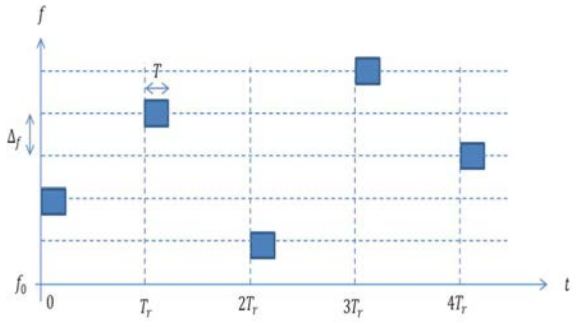

Fig. 1: Time-frequency graph of a step frequency waveform

The blue boxes on Figure 1 mean that the carrier frequency is constant over the duration T for each pulse of the transmitted signal.

We expect to reconstruct range delays and Doppler parameters of reflecting targets in a bi-dimensional discrete grid, with the associated electromagnetic signature. The testable delays, $\{\tau_l = \frac{l}{B}, 0 \leq l \leq BT\}$, figure on the Y axis, whereas the testable Doppler frequencies, $\{\nu_m = \frac{m}{NT_R}, 0 \leq m \leq N\}$, lie on the X axis.

The concept of CS was defined by E.Candès and D.Donoho in 2004 [7] [5] (see [6] for an exposition). This theory aims at the reconstruction of particular signals, called sparse. In practice, such a signal is represented by a large vector of size P, which we will denote by α, the majority of the coefficients are zero. We note s the number of non-zero elements of α, such as $s \ll P$, corresponding to the pseudo-norm L_0 of vector α. We write $||\alpha||_0 = s$.

The theory shows that such signals can be sampled linearly with a very low sampling rate, provided the information system that we are looking for satisfies the assumptions of CS. To do this, let us take the grid used for targets reconstruction : if we concatenate the columns of this table one above the others, we obtain a vector-column, constituted of N blocks of BT elements. Each block corresponds to a tested frequency ; within a block, an index represents a tested time.

Let $k \in [\![0; N * BT - 1]\!]$ be the index of one of the coefficients of the vector, and consider its Euclidean division by N: $k = mN + l$. $m \in [\![0; N - 1]\!]$ corresponds to the index of the block in which the k-th element of the vector is located. The Doppler frequency associated with this element is therefore $\nu_m = \frac{m}{NT_R}$. Let also $l \in [\![0; BT - 1]\!]$ be the number of the element k within the block of subscript m, it is associated with the delay $\tau_l = \frac{l}{B}$.

We can therefore find both the delay and the frequency associated with a coefficient of the vector directly from its coordinates. The values of these coefficients depend on the amplitude of the corresponding target, it is zero in the absence of a target characterized by these parameters. the total number of points detected by the radar is s, and α constitutes an inventory of the present targets, with their parameters and their amplitude. In this sense, the vector α can be interpreted as the

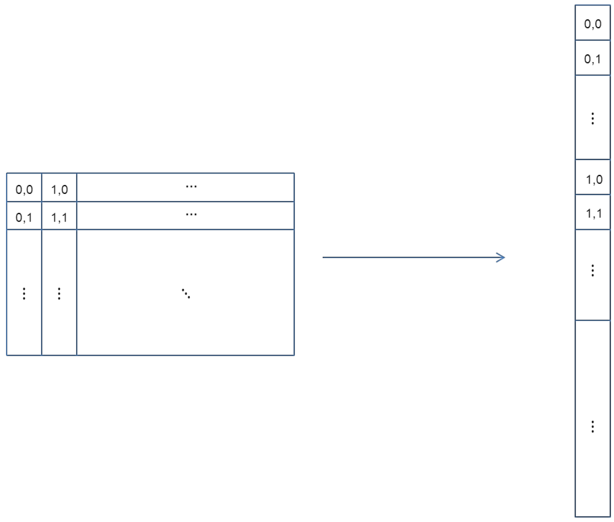

Fig. 2: Reconstruction grid recast into a vector for CS processing

”scenario” of the scene observed by the radar. We generally have a small number of targets to detect compared with the size of the grid search. Moreover, our observations are linear measurements of such signals. We therefore fall into the realm of CS. It should also be noted that this formulation takes into account the possible presence of several targets, unlike matched filtering. CS postulates the existence of a system of measurements, which we represent by a vector denoted by u, of dimension M, which can be expressed linearly from the information sought and an additional Gaussian thermal noise w from the radar:

$$u = \Phi\alpha + w \tag{1}$$

The Gaussian assumption on the noise is classical in radar context.

The major assumption in this writing is that the measurements are significantly less numerous than the number of elements of α, which translates to $M \ll NBT$ in our case. Φ, the measurement matrix of the radar, is thus rectangular, of dimension $M \times NBT$. A line of Φ is associated with a measurement, while a column is associated with a pair of time-frequency parameters tested. It can be shown that the expression of this matrix depends on the characteristics of the waveform used; it is therefore known by the user of the radar. The purpose of the reconstruction is to estimate the vector α containing all the information of interest to us, knowing that we have the measured value u and that we know the measurement matrix Φ. Such a vector is solution of the matrix equation written above. We can implement a new treatment on the studied step frequency waveforms, in order to find the targets detected by the radar. The concept of CS allows us to assume the existence of several targets simultaneously detectable by the radar. The writing of the CS problem is based on the definition of exhaustive statistics for the target amplitude parameters, by calculating the probability density of the received signal with respect to the measurement of the noise.

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After having formulated the CS measurement model for the radar, we can choose several algorithmic procedures to numerically solve the problem of finding observed point targets from noisy measurements. Those procedures can be selected from classical CS methods on optimization programming, greedy or thresholding methods. We focus here on LASSO procedure, adapted to a classical CS modelling, and with a shape that will help us to extend these technics to the problematic of ground clutter separation from airborne targets echoes:

$$\min_{\alpha} \frac{1}{2}||u - \Phi\alpha||_2^2 + \gamma||\alpha||_1 \tag{2}$$

The first term represents the magnitude of noise, and is related to the reliability of an assumed solution α to the model, while the $||.||_1$ term, the convex relaxation of operator $||.||_0$, stands for the sparsity of a tested solution. The scalar γ is a calibration parameter set by a cross-validation procedure. Such a well-conditioned procedure ables to accurately reconstruct both support and amplitudes profile of multiple point targets detected scene. However, we won't focus any longer on this simple frame, and will retain the specific structure of LASSO (2) to include the ground clutter effect in our study

We mention that previous investigations on CS for radar have already been held, [8] being one of the first articles to propose such an application, making use of "Alltop" measurements. However, the link between the CS measures and the electromagnetic radar signal is not always explicit or guaranteed. One of the particularity of our work is to expose a specific waveform mathematically associated (via exhaustive statistics) to a matrix with 2-dimensional Fourier entries, so that it's consistent with CS procedures [9].

B. Impact of the ground clutter

The radar antenna is a circular electronic scanning antenna, that concentrates and directs the wave energy in the wished direction of emission. In fact, many lobes are generated by the antenna during the emission : the main lobe oriented by the fixed direction of emission, and secondary lobes of lower energy, sent in various directions around the main lobe. Each of them can be reflected by the ground beneath the radar. Even if the energy transported by a secondary lobe is significantly smaller than the one transported by the main lobe, any lobe can generate a strong amplitude echo towards the radar, following its relative direction with respect to the ground surface. Any point on the ground surface intercepted by a radar lobe will reflect the wave with a retro-diffusion amplitude given by the radar equation. In our model, each of these points is spotted by its angular coordinates, the bearing angle θ_g and the angle of site θ_s. We suppose that the ground is straight and horizontal and we consider that the plane remains horizontal during its flight, so that its altitude H remains constant. We now wish to carry out a discretization of the surface of the ground according to the curves representing points of the ground at equal distance from the radar and points of the ground having the same radial speed with respect to the radar.

The ground clutter is the sum of the echoes of all the wave-illuminated point on the ground, each of them being characterized by its time delay and Doppler frequency, proportional to its distance and speed from the radar respectively. The set of points located at a same distance from the radar is the surface of the sphere having as center the center of the antenna of the radar. The horizontal ground is thus partitioned into several concentric circular crowns associated with the testable times delays indexing the reconstruction grid. The determination of the points having the same radial velocity with respect to the radar is more complicated. Using the mathematical definition of the radial velocity of one point with respect to another, calculations show that the points we are looking for are located on the surface of a cone centered on the antenna center, which axis is the velocity vector of the aircraft and which angular aperture depends on the radial velocity under consideration. When the aircraft remains horizontal, the common axis of all these cones is parallel to the ground. By applying the conic theory on a geometric plane, the intersection of one of these cones with the ground is a hyperbola, all originating at a given point on the ground. Two distinct hyperbolas never intersect, and the more the associated radial velocity to a hyperbola is high, the more this hyperbola is flared and distant from their origin.

Until now, the iso-speed curves were only defined in an approximate way in industrial work on the ground clutter. The exact calculation of their parameters thus constitutes a novelty and allows a more precise modeling of ground echoes before being able to test its algebraic properties and take them into account in the simulations realized with dedicated electromagnetic signals. As the radar balance shows, the amplitudes of ground elements depend on their surface. This is why we have determined the expression of area of each of the infinitesimal surfaces resulting from the division previously introduced. This is why we determine the expression of area of each of the infinitesimal surfaces resulting from this division. Moreover, to satisfy the modeling of the ground clutter as the sum of the echoes of scattering centers, we associate each of these surfaces with their geometric barycenter, which we also calculate analytically. The barycenter of the surfaces are represented by blue stars in the next figure, while the iso-distant and iso-velocity curves are respectively plotted in green and red:

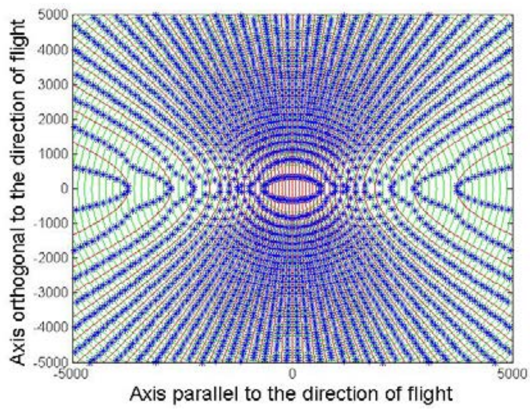

Fig. 3: Repartition of resolution cells barycenter

Using the Cartesian and angular coordinates of these points, we can recalculate all the factors in the expression of the amplitude of these points (distance, antenna gain, reflectivity). We rely on fixed flight parameters for numerical simulations: a platform altitude of 5000*ft*, a carrier speed of 240*m/s*, a viewing angle of the radar given by bearing angle $\theta_g^0 = 30^o$ and angle of site $\theta_s^0 = -1^o$. Fig. 4 and 5 are geometrical modelings of ground clutter, that is sensed by the radar through a waveform that is generally ambiguous in range and velocity. Next figure shows the ambiguous clutter map obtained from Fig. 5 and a periodic waveform with pulse repetition frequency :

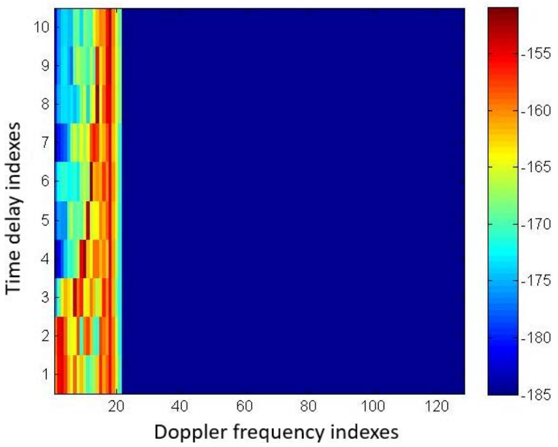

Fig. 4: Ambiguous ground clutter map

The figure we obtained is consistent with real data analyzed previously. We can notice the clear area beyond the Doppler corresponds to the speed of the aircraft, as well as the vertical line corresponds to the Doppler of the elements of the ground illuminated by the main lobe.

The new modeling thus being validated, we can check to test its mathematical properties and to use it in the simulations using more general waveforms such as frequency-hopping waveforms. The previous observations suggest how to run the whole radar reconstruction procedure in presence of ground clutter. Since the index separating the areas with and without clutter is given by the known flight parameters, we can divide the research into two parts : we apply classic CS techniques on the area where only airborne target are likely to appear, and we apply an extended method of CS for the area with ground clutter. We can then address the principle of this extension.

III. Extension of Compressed Sensing techniques to separate airborne targets from ground clutter

A. Reconstruction and separation procedure set up

Consider the decomposition $x = L + S$. The expected procedure supposes both that S is sparse and that L is a matrix of low rank. Thus, if S can be represented as well by a sparse matrix as by a sparse vector, L only supports a matrix form. Since the two data are summed to each other, S has also to be represented by a matrix, by homogeneity. Then the program to be determined admits two matrix variables. This also implies changes in the expression of the measure operator, previously defined as the matrix Φ. Especially, the size of the reconstruction grid of the mixed airborne targets-ground clutter are represented on the left part of Figure 4 is not $N \times BT$ anymore. Indeed, we only consider in the current procedure the Doppler index below the index referering to the pplane velocity.

Just as the classic CS program uses the convex relaxation $||.||_1$ to express the sparsity of the sought solution, the sparse plus low rank approach also employ the convex relaxation of the matrix rank function (non convex), i.e. the nuclear norm $||.||_*$, defined as the sum of the singular values modules of the matrix. First studies on this problematic consisted in separating sparse and low rank components from a given matrix [12]. Such work is not sufficient for us since we only know the reduced measurement vector obtained from the scenario to be decomposed. However, more recent papers [10] [11] able to deal with this extension. The sparse plus low lank program we aim to apply to the radar reconstruction and separation writes:

$$\min_{L,S \in \mathbb{C}^{p_1 \times p_2}} \frac{1}{m} \sum_{j=1}^{N} \left(y_j - < \tilde{\Phi}_j, L + S > \right)^2 + \mu \, ||L||_* + \gamma \, ||S||_1 \tag{3}$$

where p_1 and p_2 are two integers satisfying $P = p_1 p_2$, where P is the number of testable parameters in the clutter area. For any $j \in [\![1; N]\!], \tilde{\Phi}_j \in \mathbb{C}^{p_1 \times p_2}$ and $<,>$ nominates the Frobenius scalar product on $\mathbb{C}^{p_1 \times p_2}$. The coefficients μ and γ are obtained by a cross-validation procedure. Here, for calculation requirements necessary to to fit the expected model, p_1 stands for the total number of testable delays, still equal to NBT, while p_2 represents the number of testable Doppler frequencies below the plane velocity index (equal to 24 when we set the value $240m/s$ for the speed of plane).

It appears that, if p_1 and p_2 refer to the numbers of testable time delays and Doppler frequencies, if, for any $1 \leq j \leq N$, $\tilde{\Phi}_j$ is the recast of the $j-$th row of Φ into a complex $p_1 \times p_2$ matrix and if $<,>$ refers to the Frobenius scalar product on $\mathbb{C}^{p_1 \times p_2}$, then the quadratic term $\sum_{j=1}^{N} \left(y_j - < \tilde{\Phi}_j, L + S > \right)^2$ in (3) corresponds to the L_2-norm of the noise vector $||y - \Phi vec(x)||_2^2$ in CS program formulation (2), where $vec(x)$ refers to the suitable recast of the sum $L+S$ into a vector. The factor $\frac{1}{2}$ affected the quadratic term is replaced by $\frac{1}{m}$ for computational means. Moreover, the previous measurement operator Φ of size $m \times NBT$ is reduced to the dimension $m \times 24BT$ (since we only consider $24 < N$ Doppler indexes in the clutter area), and then recasted into m matrices of size $BT \times 24$.

We note that, in absence of noise, i.e. for $L = 0$ in (3), we can retrieve the classical CS formulation (2) in the previous sparse and low rank program. Indeed, we have an identity between the L_1-norm for matrix S and L_1-vectorial norm for corresponding recasted vector, and another identity between the sum with Frobenius scalar products of matrices $\tilde{\Phi}_j$ and S, and the global euclidean norm with Φ and S in (2). The significant difference between both procedures is that the number of testable parameters has reduced since we focus on a limited zone. Although we extended the classical CS process with (2), this new approach can also meet the case where the research space dimension is lower than the number of stored measurements. This is not the case with our current parameters, but we can suppose it could be real provided the plane velocity is sufficiently low and if we also track a limited area of time delays. We can apply (3) in this situation too.

Due to mathematical tools in [10] and [11], the fact that any $\tilde{\Phi}_j, 1 \leq j \leq N$ comes from some Fourier operator is relevant to the existence of convergent iterative procedures solving 3, and therefore computing the researched features in S.

Yet, we still have to establish that the echoes from the ground clutter can be represented by a low rank matrix. That matrix won't have strictly a low rank but it will admit an enough decreasing singular value decomposition, so that 3 may be applied to our radar context.

B. Validation of the sparse and low rank approach

We found out that a clear dissociation appears on the map previously obtained between the ground clutter below a certain index and the so-called clear zone beyond this index and in which only airborne targets could appear.

These results justify the sparse plus low rank procedure 3, that can readily be implemented for ground clutter separation from the useful targets signals, while the simpler Compressed Sensing procedure would be used on the clear zone. This is conceivable insofar as the limit between these two zones is defined by a threshold entirely characterized by the flight parameters. Our objective here is to be able to achieve the global reconstruction of the reflective objects and the dissociation of the ground and possible airborne targets in the first zone, even if there are many possible scenarios where airborne and ground clutter targets occur at the same Doppler shift. The sparse and low rank approach is supposed to be also efficient in such situations.

We validated our model of ground clutter by establishing a grid obtained from a classic pulse train radar signal. Such a signal is not adapted to the global Compressed Sensing approach. We suppose now that the radar emits a step frequency waveform. Let us calculate the *SVD* of the matrix representing the area of ground clutter in the search space indexed by the reconstruction grid, obtained from the emission of dedicated step frequency signal.

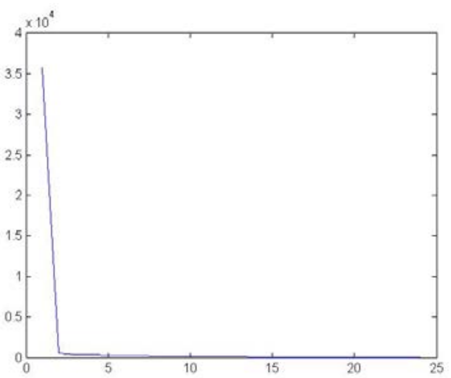

Fig. 5: Singular Value Decomposition of the ground clutter matrix

This figure shows a decrease in the singular values of the matrix in agreement with those of the so-called low rank matrices for which the sparse plus low rank procedure has proved conclusive. This motivates the application of this approach to the detection, location and identification of airborne targets in the presence of clutter in the context of Compressed Sensing radars. Moreover, in order to know if the components separation can be effective, we have to test some incoherence conditions on matrices U and V obtained from the ground matrix *SVD* [12]. Those matrices should verify some incoherence conditions with the sparse scenario basis, which means that their columns vectors have small norms.

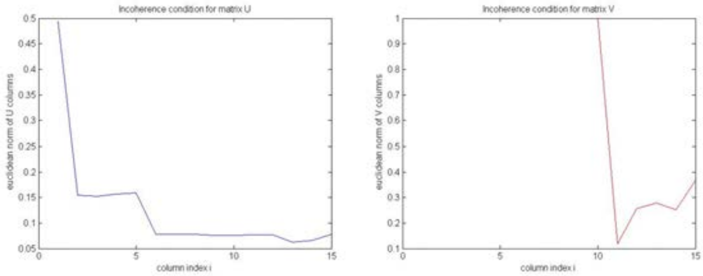

Fig. 6: Incoherence test for matrix V

The figures 6 and 7 indicate that columns from U and V have small norms, which is encouraging for our work. However, the computed matrix UV^* has 0.47 for maximum amplitude value, which is not that small. The fact that U and V could not be mutually so incoherent can affect our separation perspectives. However, it concerns only necessary conditions, so we can still hope for efficient reconstructions in presence of ground clutter.

IV. Conclusion

We have justified a mathematical approach to handle with the radar procedure of reconstructing and separating from ground clutter the airborne objects that reflected the emitted electromagnetic radar signal. We defined a more accurate model for the ground clutter, in order to make it suitable for the expected resolution of reconstruction and adapted to the sparse plus low rank procedure. Checking the obtained result has able to valid the use of the considered approach with the new model of ground reflection and for emission of a radar signal consistent with Compressed Sensing context. Future work using effective algorithms will be used to rigorously validate this approach for the detection, location and identification of multiple targets in the presence of ground echoes. We currently try to develop proximal gradient method and extend ADMM for the two-fold variables problem to verify the reconstruction, after having run a cross-validation procedure.

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