Abstract—In this article we will present a detailed study of recursive extended least squares method (RELS) to identify without bias of the models (system + disturbance). The idea is simultaneously to identify the model of the system and the model of the disturbance to obtain asymptotically a white error of prediction, which guarantees an unbiased estimate of the parameters system. Then we choose an application of this algorithm to identify the parameters of auto regressive model AR associated to the speech signal.

I. INTRODUCTION

This method was developed to be able to identify without bias for ARMAX models which they have the following structure:

\[
A(z)y(t) = B(z)u(t) + C(z)e(t)
\]  

(1)

Let us recall that algorithm RLS supposes the additive disturbance as a white noise, on the other hand in the case of the RELS considers that the noise added to the system is a colored noise. This noise can be considered as being the result of the filtering of a white noise.

The idea is simultaneously to identify the model of the process and the model of the disturbance, to be able to obtain asymptotically a white prediction error [3].

II. THE STRUCTURE OF ARMAX MODEL

The ARMAX model has the following structure:

\[
\begin{align*}
A(z) &= \sum_{i=0}^{n} a_i z^{-i} \\
B(z) &= \sum_{i=0}^{m} b_i z^{-i} \\
C(z) &= \sum_{i=0}^{p} c_i z^{-i}
\end{align*}
\]

(5) \hspace{1cm} (6) \hspace{1cm} (7)

With \(a_0 = 1, b_0 = 0, c_0 = 1\)

The equation (4) will thus be written:

\[
Y(z) = \frac{B(z)}{A(z)} U(z) + \frac{C(z)}{A(z)} E(z)
\]

(8)

We put:

\[
H_1(z) = \frac{B(Z)}{A(Z)} \quad ; \quad H_2(z) = \frac{C(Z)}{A(Z)}
\]

With

\(H_1(z)\): Transfer function of the system.

\(H_2(z)\): Transfer function of the filter.

III. DETERMINATION OF THE PARAMETERS

We thus can, to think in an intuitive way that if we increase the number of observations; the problem will be reduced to the resolution of a system of linear equations.
In the case of a disturbed system, we carry out N measurements of observations, we can write according to the equation (2), a matrix form:

\[ y(t) = \Theta^T \varphi(t) + e(t) \]  

(9)

Let us put these notations [5]:

\[ \Theta = [a_1, a_2, \ldots, a_n, b_1, b_2, c_1, c_2, \ldots, c_p] \]

\[ \varphi(t) = -y(t) - y(t-n)u(t-1) - u(t-n)u(t-1) - \ldots - u(t-n)u(t-p) \]

With

\[ \Theta^T: \text{ vector of the parameters to be identified } \]

\[ \varphi^T(t): \text{ vector of the data. } \]

\[ y(t): \text{ output signal of system } \]

\[ e(t): \text{ affected white noise to the system } \]

T: indicate transposes it of a vector or a matrix.

We define the error of prediction in a priori as being the difference between the output of the system and the output of the model:

\[ \varepsilon(t) = y(t) - \hat{y}(t) \]  

(10)

\[ \hat{y}(t) = \hat{\Theta}(t-1)^T \varphi(t) \]  

(11)

With \( \hat{\Theta}(t-1) \): Represent the estimated parameters.

The problem of the ordinary method of least squares is due to the non measurable of e (t), there for it should be estimated. According to the equations (2), and (3), we write:

\[ e(t) = y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) - b_1 u(t-1) - \ldots - b_m u(t-m) - c_1 e(t-1) - \ldots - c_p e(t-p) \]  

(12)

We call \( \varepsilon(t) \) the estimator of e (t), from where Eq.12 will be written in the form:

\[ \varepsilon(t) = y(t) + \hat{a}_1 y(t-1) + \ldots + \hat{a}_n y(t-n) - \hat{b}_1 u(t-1) - \ldots - \hat{b}_m u(t-m) - \hat{c}_1 e(t-1) - \ldots - \hat{c}_p e(t-p) \]  

(13)

The priori adjustable prediction in the case of extended least squares is obtained from Eq.10, by replacing the known parameters by the estimated parameters:

\[ \hat{\Theta}(t) = \begin{bmatrix} \hat{a}_1 | \hat{a}_2 | \ldots | \hat{a}_n | \hat{b}_1 | \hat{b}_2 | \ldots | \hat{b}_m | \hat{c}_1 | \hat{c}_2 | \ldots | \hat{c}_p \end{bmatrix} \]

\[ \hat{\varphi}(t) = \begin{bmatrix} -y(t) & -y(t-n) & u(t-1) & u(t-n) & \ldots & u(t-p) \end{bmatrix} \]

The method of RELS remains the same one as the RLS, except that the matrix \( \Phi_N \) and the vector \( \Theta \) which were modify. So that, the equations allowing the algorithm of the extended least squares will be:

\[ \varepsilon(t) = y(t) - \hat{\Theta}^T(t-1) \varphi(t) \]  

(14)

\[ \hat{\Theta}(t) = \hat{\Theta}(t-1) + \hat{p}(t) \varphi(t) \varphi^T(t) \hat{p}(t-1) \]  

(15)

\[ \hat{p}(t) = \hat{p}(t-1) - \hat{p}(t-1) \varphi(t) \varphi^T(t) \hat{p}(t-1) + 1/ \hat{p}(t) \varphi(t) \varphi^T(t) \hat{p}(t-1) \]  

(16)

Initialization concerning the matrix \( \hat{p}(t) \) and the vector of the parameters \( \hat{\Theta}(t) \) is done often as follows:

\[ \hat{\Theta}(0) = \begin{cases} \Theta & \text{a priori} \\ 0 & \text{else} \end{cases} \]

\[ \hat{p}(0) = C.I \text{ such as } C: \text{ constant } \]

I: matrix identity.

IV. STATISTICAL PROPERTIES OF THE ESTIMATOR

To have a satisfactory estimator, is needed that its bias is null, the estimate \( \hat{\Theta} \) is unbiased, \( E[\hat{\Theta}(t)] = \Theta_0 \), because the introduced noise is uncorrelated with the elements of the matrix \( \Phi_N \).

V. EXAMPLE OF SIMULATION

Let us consider a stable physical system of type ARMAX having the following transfer functions:

\[ \frac{H_1(z)}{H_2(z)} = \frac{1 + 2z^{-1}}{1 + 0.3z^{-1} + 0.8z^{-2}} \]

\[ H_3(z) = \frac{1 + 0.1z^{-1} + 0.4z^{-2}}{1 + 0.3z^{-1} + 0.8z^{-2}} \]

With

\[ a_0 = 1; a_1 = 0.3; a_2 = 0.8 \]

\[ b_0 = 0; b_1 = 1; b_2 = 2 \]

\[ c_0 = 1; c_1 = 0.1; c_2 = 0.4 \]

The implementation of algorithm RELS on PC, by using the programming language MATLAB, gave us the following results:
VI. APPLICATION TO THE SPEECH SIGNAL

Let us consider the output signal as a signal of speech which is in the form of a data file which corresponds to the following sentence: (Un loup s'est jeté immédiatement sur la petite chèvre) (See Fig.4).

The model which corresponds to the disturbed signal of speech having the auto regressive structure AR of the form: [6].

\[ y(t) = \sum_{i=1}^{n} a_i y(t-i) + v(t) \]

With

- \( n \): represent the order of the model
- \( v(t) \): filtered white noise.

The use of the algorithm of recursive extended least squares RELS to identify the parameters of model AR, gives the following results:

| Table 3. Estimate of the parameters of a speech signal in the presence of the noise |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| \( a_0 \)                       | \( a_1 \)                       | \( a_2 \)                       | \( a_3 \)                       | \( a_4 \)                       | \( a_5 \)                       | \( a_6 \)                       |
| \( -0.0000 \)                    | \(-0.7964\)                     | \(-0.1461\)                     | \(-0.0053\)                     | \(-0.0373\)                     | \(-0.0004\)                     | \(-0.0093\)                     |
| \( 1.0000 \)                    | \(-0.7964\)                     | \(-0.1461\)                     | \(-0.0053\)                     | \(-0.0373\)                     | \(-0.0004\)                     | \(-0.0093\)                     |
| \( 1.0000 \)                    | \(-0.7964\)                     | \(-0.1461\)                     | \(-0.0053\)                     | \(-0.0373\)                     | \(-0.0004\)                     | \(-0.0093\)                     |
| \( 1.0000 \)                    | \(-0.7964\)                     | \(-0.1461\)                     | \(-0.0053\)                     | \(-0.0373\)                     | \(-0.0004\)                     | \(-0.0093\)                     |
| \( 1.0000 \)                    | \(-0.7964\)                     | \(-0.1461\)                     | \(-0.0053\)                     | \(-0.0373\)                     | \(-0.0004\)                     | \(-0.0093\)                     |

Fig.4. Representation of a speech signal
VII. CONCLUSION

In the presence of a random disturbance corresponding to model ARMAX, the error tends asymptotically towards a white noise, which guarantees unbiased estimate of the parameters for a stable physical system.

The identification of the parameters of a speech signal by algorithm RELS gives acceptable results under conditions without noise but these performances are degraded quickly in the presence of noise.

In the representation time frequency of the disturbed signal, some of the parameters of the speech are masked by the noise [7]. In such a case the parameters of the clean signal cannot be estimated starting from the disturbed signal and are thus considered as dubious.

REFERENCES