A Dual Filtering Approach in MEMS based Dynamic Attitude Estimation

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Abstract—The problem considered in this paper is the design of a low cost MEMS based attitude estimation unit to be used in ultralight, experimental and sport pilot aircrafts as auxiliary safety tool in VFR flight conditions. The proposed approach relies on a new data fusion scheme based on a dual Kalman filter design and on acceleration–based switch criteria. Attitude information is extracted from the states of the filters under the restriction of non aerobatic uses that allows the introduction of a priori limits in roll and pitch angles.

I. INTRODUCTION

Attitude estimation plays a role of great importance in several control applications concerning, for instance, walking robots, unmanned vehicles and satellite orientation. Another traditional area where attitude and heading information is vital, concerns military, commercial and general aviation. Here the shift from old systems based on gimballed spinning-wheel gyros plagued by several limitations and by a MTBF of the order of 600 hours to modern Ring Laser Gyro (RLG) or Interferometric Fiber Optic Gyros (IFOG) systems based on the Sagnac–effect has already been performed. Inertial navigation was advanced by Germans in 1940 to the level required for missile guidance (V2) and the last applications in this area have concerned the first 70 missions of the Ariane launcher. In commercial aviation the last mechanical gyros have been mounted on the Boeing 747 while the subsequent 757 model has been endowed with one of the first commercially available RLG navigation systems [1].

While previous systems can be considered as high end solutions that find their natural market in performance–driven applications, the low end market is given by the increasing number of potential users that could take advantage of reliable heading and navigation systems but cannot afford the prices of RLG or IFOG technologies. In the field of light aviation the categories of ultralight and experimental aircrafts and the recently introduced (September 2004) category of Sport Pilot aircrafts constitute a market only marginally interested to certified navigation systems because of the high costs of the devices meeting the Federal Aviation Administration (FAA) Technical Standard Orders (TSO) and of the auxiliary role that heading and navigation systems play in the Visual Flight Rules (VFR) conditions mandatory for most aircrafts of these classes. On the other hand the availability of Instrumental Flight Rules (IFR) instruments like artificial horizons in aircrafts not certified for this type of flight constitutes a remarkable additional safety degree which could have contributed to save the life of several pilots that have inadvertently encountered Instrumental Meteorological Conditions (IMC).

This challenge can be won by Micro-Electro-Mechanical-Systems (MEMS) based systems. In fact MEMS constitute a rapidly emerging technology already playing an important role in the current growth and wider deployment of low cost navigation systems. Whether for hand-held devices or vehicle mounted systems, new applications of MEMS appear every day either to complement systems based on satellite derived data, like GPS, GLONASS and GALILEO or to implement fully autonomous systems. MEMS devices, whose market increase is estimated at 30% per annum in Japan, offer intrinsically lower cost and high reliability solutions but require a non trivial degree of ingenuity to deal with their noise and drift limitations.

The basic MEMS components required to develop an Inertial Measuring Unit (IMU) are accelerometers and rate gyro whose limiting factors are essentially given by noise and drift respectively. The direct use of the integral of the rate gyro to measure attitude is thus unthinkable because of drifts that can reach several hundreds degrees per hour (against 1 degree per hour of RLGs) and accelerometers can be used to estimate attitudes only in absence of vehicle accelerations. The solution generally adopted consists in implementing a data fusion algorithm in order to use the attitude estimated from the accelerometers in steady portions of the trajectory to trace and compensate the rate gyro bias and the standard tool to perform this fusion can be considered extended Kalman filtering [2], [3], [4], [5].

This paper describes the results obtained in designing a low cost artificial horizon to be used as auxiliary tool in VFR conditions and on non aerobatic aircrafts. These design goals have allowed to take advantage of the associated limits to the roll and pitch angles and to obtain, thanks to a new design based on dual Kalman filtering and to an acceleration driven switch criterion, a good overall performance.

The paper is organized as follows. Section II reports the design goals and the MEMS selection criteria that have been followed. Section III describes the implementation of the dual Kalman filtering scheme that has been adopted and the switch criterion. Section IV describes, by means of an example, the performance of the prototype system. Short concluding remarks are finally given in Section V.
II. DESIGN PHILOSOPHY AND SENSOR SELECTION

The target that has been considered limits the required range of both roll and pitch angles since no aerobatic maneuvers are expected; for this reason the roll angle excursion has been limited to $\phi = \pm 60^\circ$, corresponding to a load factor of 2 g, and the same limit has been adopted also for the pitch angle $\theta$. The choice of the accelerometers has then been performed by taking into account this limit and considering also a premium the minimization of the number of components required for the acquisition of their output. On the basis of these constraints the Analog Devices ADXL202 has been considered as particularly interesting because of the presence of an on–board Analog to Duty Cycle (ADC) converter with a resolution of approximately 14 bits; this allows an easy interfacing with any microcontroller without any necessity of additional AD converters. The noise density of the ADXL202 is (typically) $200 \mu g \sqrt{Hz}$ so that for the intended bandwidth of $10$ Hz a noise of approximately $0.8 \mu g$ rms can be expected.

The choice of the angular rate gyros is quite critical because of their high drift, undesired sensitivity to acceleration (almost all rate gyros are based on the detection of Coriolis force) and additive output noise. The dual-sensor design of the Analog Devices ADXRS150 rejects external $g$–forces and vibrations (always present in light aircrafts) in a very effective way; in fact vibration rectification for frequencies up to $20$ kHz is on the order of $0.00002(\circ/s)/(m/s^2)^2$ and does not depend significantly on frequency.

Other advantages of this sensor concern the signal conditioning electronics directly implemented on the chip that reduce the number of external components and preserve signal integrity. The output of the ADXRS150 is analog so that either an AD converter or a microcontroller endowed with an on–board AD converter with a resolution of at least 12 bits is required. The drift, is very significant since its range is of $600$ mV over the whole temperature interval against a nominal signal range of $3750$ mV. The ADXRS150 incorporates an accurate temperature sensor and a reference voltage source that can be used to compensate the temperature drift; the use of a three points calibration technique allows to reach a drift of the order of $300^\circ$/hour while more sophisticated polynomial interpolations can lead, if properly performed, even to $60 - 70^\circ$/hour. The additive rate noise density is a feature less significant than drift; the nominal value is $0.05(\circ/s)\sqrt{Hz}$ rms which is two orders of magnitude worse than the drift of RLGs. Finally the range of the device, $\pm 150^\circ/s$ is more than adequate for non aerobatic maneuvers.

III. FILTERING AND DRIFT TRACING

The attitude estimation system that has been developed relies on three accelerometers aligned along the roll ($x$), pitch ($y$) and yaw ($z$) axes and on two rate gyros aligned along the roll and pitch axes. The models considered for the accelerometers are of the type

$$\dot{a}_i(t) = w_i(t)$$  \quad (1)

where $w_i(t), i = \{x, y, z\}$ denote zero mean Gaussian white noises since accelerations are due to unknown maneuvers and are consequently treated as disturbances. To simplify the notation in the following only the roll angle $\phi(t)$ will be considered; the acquisition of the pitch angle, $\theta(t)$ is absolutely similar. The acceleration measures are corrupted by additive noise so that, for every accelerometer,

$$a_{io}(t) = a_i(t) + v_i(t)$$ \quad (2)

where $v_i(t), i = \{x, y, z\}$ denote zero mean Gaussian white additive observation noises. The absence of accelerations or the presence of negligible ones can be detected by means of the condition

$$\|a(t)\|_2 = \sqrt{a_x^2(t) + a_y^2(t) + a_z^2(t)} \approx g.$$ \quad (3)

When (3) is satisfied it is possible to evaluate $\phi(t)$ from $a_z(t)$ (at least inside the previously defined range) by means of the relation

$$\phi(t) = \arcsin(a_z(t))$$ \quad (4)

where $a_z(t)$ is measured in $g$ units. It is important to observe that the nonlinear relation (4) can be approximated very well by a linear relation of the type

$$\phi(t) = c_z a_z(t)$$ \quad (5)

(the error is $1.35^\circ$ at $30^\circ$) and that, within this limit, also the noise whiteness is affected only marginally by the linearization; while $\phi(t)$ will be actually computed by means of relation (4), we will make reference, in the following, to the linearized observation equation

$$\phi(t) = c_z a_z(t) + \nu(t).$$ \quad (6)

Remark 1: A two stage filtering procedure (hardware + Kalman filtering) is actually performed on the accelerometer outputs. These operations, despite their positive influence on the global performance, are standard and of marginal interest in the context of this paper; they have been mentioned only for completeness.

Remark 2: Since every ADXL202 contains two accelerometers with orthogonal axes, two independent measures of $a_z(t)$ are available. A data fusion with weights calibrated on the measured noise characteristics of the used devices is thus possible. Another possibility concerns the use of this redundant information for fault detection purposes. Also these aspects fall outside the main purpose of this paper.

The rate gyro can be described by means of a model of the type

$$\omega_g(t) = \omega(t) + b_g(t)$$ \quad (7)

where $\omega_g(t)$ denotes the observed gyro output, $\omega(t)$ the true angular rate, $w_g(t)$ is a zero mean white and Gaussian noise and $b_g(t)$ the gyro drift bias at time $t$ that is assumed to be a random walk process i.e. a process described by the output of an integrator driven by a zero mean white Gaussian noise, $n_g(t)$ [6]:

$$\dot{b}_g(t) = n_g(t).$$ \quad (8)
A model describing the dynamics of the roll angle and of the gyro bias is thus given by the following differential equations
\[
\begin{align*}
\dot{\phi}(t) &= \omega_y(t) - b_y(t) + w_1(t) \\
\dot{b}_y(t) &= w_2(t)
\end{align*}
\] (9)
and the data fusion with the accelerometer observation of \(\phi(t)\) can be obtained by using, as observation of the state of system (9), relation (6). By defining the state and noise vectors \(x(t)\) and \(w(t)\)
\[
\begin{align*}
x(t) &= \begin{bmatrix} \phi(t) \\ b_y(t) \end{bmatrix}, \quad w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix},
\end{align*}
\] (10)
and the output
\[
y(t) = c \alpha_z(t) + v(t)
\] (11)
we can thus write the state space model
\[
\begin{align*}
\dot{x}(t) &= A x(t) + B u(t) + w(t), \\
y(t) &= C x(t) + v(t)
\end{align*}
\] (12)
where \(A, B\) and \(C\) can be immediately deduced from (9)–(11) and \(u(t) = \omega_y(t)\). Once that a sampling interval \(T\) has been selected, its discrete–time version (assuming negligible variations in \(u(t)\) during \(T\)) is
\[
\begin{align*}
x(t + 1) &= \Phi x(t) + G u(t) + w(t), \\
y(t) &= C x(t) + v(t)
\end{align*}
\] (13)
where
\[
\Phi = e^{A T}, \quad G = \int_0^T e^{A (T - \tau)} B d\tau.
\] (14)
The associated standard Kalman filter is thus given by
\[
\begin{align*}
\dot{x}(t + 1) &= \Phi \dot{x}(t) + G u(t) + K(t) \varepsilon(t), \\
\hat{y}(t) &= C \hat{x}(t)
\end{align*}
\] (15)
where \(\varepsilon(t)\) denotes the innovation i.e.
\[
\varepsilon(t) = y(t) - C \hat{x}(t)
\] (16)
and \(K(t)\) denotes the (time dependent) gain of the filter. A traditional implementation of the data fusion algorithm would stop here with a criterion to select \(K(t)\) on the basis of the unmodeled disturbance, i.e. of the acceleration. It can however be observed that, despite the fact that, during most flights, the accelerations are negligible for long periods, this is only a generic observation that can be easily violated (e.g. during prolonged turns). A more robust approach could be based on a switch scheme for the filter gain on the basis of the measured total acceleration vector as proposed in [2] or the adoption, along the same philosophy, of a fuzzy function to commute between gains optimized for acceleration and non acceleration conditions. These solutions have, in fact, been tested with positive results; the best results have, however, been obtained by adopting a more computationally demanding scheme based on the simultaneous use of two Kalman filters whose (constant) gains have been optimized for the the conditions of reliability or unreliability of the estimation of \(\phi(t)\) on the basis of \(\alpha_z(t)\).

The gain of the first filter, optimized for acceleration conditions, can be designed on the basis of the knowledge of the covariances \(Q\) and \(r\)
\[
\begin{align*}
Q &= E[w(t) w^T(t)] = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}, \\
r &= E[v^2(t)],
\end{align*}
\] (17)
that can be evaluated only with some difficulty, particularly for what concerns \(q_2\) and \(r\) that depends on the actual value of the accelerations. The approach that has been followed, due to Mehr [7], does not require this information since leads directly to the estimation of the stationary value of \(K(t)\) starting from an initial estimate, \(K_0\), of \(K\) and from the knowledge of the associated innovations.

The steps of this interesting algorithm, that requires only the availability of input/output sequences of the process and a (also crude) initial estimate of \(K\) are the following (for the considered system of order 2).

**Algorithm 1 (Mehra) - Estimation of the optimal gain \(K\)**

1) Compute the matrix
\[
M = \begin{bmatrix} C \Phi & C \Phi^2 \end{bmatrix}.
\] (18)

2) Compute the matrix
\[
\Lambda = (M^T M)^{-1} M^T \left[ \begin{array}{c} \gamma_1 + C \Phi K_0 \gamma_0 \\ \gamma_2 + C \Phi K_0 \gamma_1 + C \Phi^2 K_0 \gamma_0 \end{array} \right].
\] (19)

where
\[
\gamma_i = E[\varepsilon(t) \varepsilon(t - i)].
\] (20)

3) Set \(k = 0\) and assign arbitrary initial values \(K_k = K_0\) and \(\delta P_k = \delta P_0\), where \(\delta P_0\) is a \(2 \times 2\) positive definite matrix.

4) Compute the matrix
\[
\delta P_{k+1} = \Phi \left[ \delta P_k - (\Lambda + \delta P_k C^T) \left( \gamma_0 + C \delta P_k C^T \right)^{-1} \right. \\
\left. \times \left( \Lambda^T + C \delta P_k \right) + K_k \Lambda + \Lambda K_k^T - \gamma_0 K_k K_k^T \right] \Phi^T.
\] (21)

5) Set \(\delta P_k = \delta P_{k+1}\) and compute the gain
\[
K_k = (\Lambda + \delta P_k C^T) \left( \gamma_0 + C \delta P_k C^T \right)^{-1}.
\] (22)

6) Repeat steps 4 and 5 until convergence of \(K_k\).

**Remark 3:** For the practical implementation of Algorithm 1 the autocovariances of the innovation \(\gamma_i, \ i = 0, 1, 2\) are replaced by the sample quantities
\[
\hat{\gamma}_i = \frac{1}{N} \sum_{t=1}^{N-1} \varepsilon(t) \varepsilon(t - i)
\] (23)
where \(N\) denotes the number of available samples.

**Remark 4:** Of course the sequences to be used for computing \(K\) must concern situations of non negligible acceleration.
Fig. 1. The Sport Pilot aircraft (Flight Design CT 2K) used for flight tests

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Speed (Km/h)</th>
<th>Acceleration (g)</th>
<th>Φ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>732</td>
<td>180</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1500</td>
<td>180</td>
<td>1.0348</td>
<td>14.9</td>
</tr>
<tr>
<td>682</td>
<td>180</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6000</td>
<td>180</td>
<td>1.0348</td>
<td>14.9</td>
</tr>
<tr>
<td>980</td>
<td>180</td>
<td>1.133</td>
<td>28.1</td>
</tr>
<tr>
<td>3750</td>
<td>180</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>960</td>
<td>180</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The gain of the second filter has not been computed by means of the Mehra algorithm but by minimizing the mean square error between the gyro drift estimation and its true value. The switch between the estimates of $\phi(t)$ obtained by the two Kalman filters has been performed by comparing the norm of $a(t)$ (3) with $g$ and assuming, as in [2], negligible acceleration when

$$\|a(t)\|_2 - g < 3\sigma_a$$

(24)

where $\sigma_a$ denotes the observed standard deviation of $a(t)$ in steady state conditions. As pointed out in [2] it is possible, by applying suitable inputs, to obtain non–zero acceleration situations maintaining $a(t)$ for finite intervals of time on the sphere with radius $g$; these abnormal conditions have some (limited) interest in robotics more than in aviation.

IV. EXPERIMENTAL RESULTS

The experimental results reported in this section concern a simulation that has been performed on the basis of actual noise and drift measurements performed on the accelerometers and gyros of the system that has been realized and on the analytical reconstruction of paths followed during test flights.

This modality has been preferred to the report of test flight data because no high accuracy reference attitude recording system was available on the CT 2K aircraft used for tests (Fig. 1) and also because analytical simulations allow to perform tests with artificially increased noise and drift levels.

The test path consists in the sections reported in Table I. The estimated roll angle is reported in Fig. 2 where the true values are indicated with dashed lines. The estimation error, shown in Fig. 3, is less than 2° during both accelerated and non accelerated segments and reaches high levels only during the transition from one condition to the other.

V. CONCLUSIONS

The attitude estimation system described in this paper is based on low cost MEMS accelerometers and rate gyros and relies on the switch between the states of two parallel Kalman filters that perform the fusion between accelerometers and gyros data and are optimized for accelerated and non accelerated conditions.

The design takes advantage of the limited excursion of roll and pitch angles that can be assumed for non aerobatic aircrafts and the obtained results can be considered as good.

The update rate of the prototype is 0.1 s and can be improved by using more performing microcontrollers. Future tests concerning more critical flight conditions have been programmed on a CAP10 aircraft. The future availability of accurate on–board reference platforms will also allow to perform more accurate evaluations of the system sensitivity to unwanted disturbances due to engine vibrations and turbulence.

REFERENCES