Seismic wave separation by SVD and (F-K) combined filters

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Abstract— Different surface seismic surveys have been recorded on an experimental hydrogeological site that has been developed for several years near Poitiers city. The acquisition of usable seismic data is particularly difficult for low-depth reservoirs underlying a thick weathered zone, such as the aquifer studied herein. We propose to show the benefit of combining F-K and SVD-ICA wave-separation methods in order to remove all the energetic wave-field (surface, air and refracted waves) in order to provide a better imaging of the reflecting wave-field. The results obtained are calibrated on well seismic data which confirm the validity of the processing.

I. INTRODUCTION

Seismic explorations and especially near surface study require information conveyed by reflecting waves. Separation of interfering wave-fields is a crucial step to enhance reflecting waves quality. Nonetheless, in near surface experimentation’s, this stage is really difficult and requires high-resolution methods, both in the acquisition and in the processing of data. In the paper, we propose to show the benefit of combining two different wave-separation methods in order to extract structural information from data recorded in near surface acquisitions. Since reflected waves are partially hidden, one aim is to remove all the energetic wave-field in order to provide a better imaging of the reflecting wave-field. To achieve it, we use the conventional F-K method to filter surface waves and converted refracted waves. The F-K filter is then combined with the SVD-ICA method (Singular Value Decomposition jointly coupled with Independent Component Analysis), which has been developed recently as an improvement of the classical SVD (Vrabie et al 2004).

After a short description of the (F-K) and SVD-ICA filters, we present the field example.

II. (F-K) - SVD FILTERS

We consider that seismic signals are recorded on an array of scalar sensors (singular component sensor. These signals are generally described as summation of several events related to the different sources (reflected, refracted, converted waves, surface waves) propagating through the media. The observed scalar signal depending on time t and space x (a seismic section) is described as:

\[ r(t,x) = \sum_i a_i(t) * s_i(t,x) \]  (1)

where \( a_i(t) \) is the wavelet of source i, \( s_i(t,x) \) the propagation vector of the source and \( * \) the symbol for the convolution product. Signal \( r(t,x) \) can be described in dual domains associated to time and distance variables as:

- \( S_i(f,x) = \text{FT}_{t}[s_i(t,x)] \), the distance - frequency space representation;
- \( S_i(f,k) = \text{FT}_{x}[S_i(f,x)] \), the frequency – wavenumber representation (2D Fourier transforms on time and distance variables).

Filtering in the frequency -wavenumber domain : F-K filter.

In the following field case, f-k filter will be used to cancel dispersive surface waves. One wave propagating in a non-dispersive medium (with constant velocity V) can be expressed in time distance domain as: \( r(t,x) = w(t – x/V) \). Modulus of the 2D Fourier Transform of \( r(t,x) \) gives the representation of the wave in the f-k plane as:

\[ R(f,k) = \text{FT}_{t,x} [w(t – x/V)] = W(f) \cdot \delta(k + f/V). \]  (2)

In f-k plane, \( \delta(k + f/V) \) represents the straight-line \( k + f/V = 0 \) passing through the origin with slope \(-1/V\). In a dispersive medium, the dispersion law is not linear. The wave velocity depends on frequency. Group velocity and phase velocity of the wave can be estimated respectively as \( v_g = \frac{df}{dk} \) and \( v_\phi = \frac{f}{k} \).

In case of a seismic section with one dispersive event, respectively in time-distance domain and frequency – wavenumber domain signal, \( r(t,x) \) is written as:

\[ r(t,x) = w(t – x/V) e^{-ix \phi} \]  (3)

\[ R(f,k) = W(f) \cdot \delta(k + f/V + \phi/2\pi). \]  (4)

In f-k plane, a dispersive wave is characterized by a non-straight line passing through the wavenumber axis at position \( k_0 \) non-equal to zero. In order to select dispersive waves by a masking filter in f-k plane (fan filter, strip filter), it is necessary to apply a group velocity correction and a phase shift correction (translation of \( k_0 \) value on the wavenumber axis).

Filtering by Singular Value Decomposition coupled with Independent Component Analysis (SVD-ICA)

After propagation through the media, the received signal \( r_i(t) \) on sensor \( i \) results from the superposition of NS waves \( [a_{1i}(t),...,a_{NSi}(t)] \) via the transfer functions \( s_{ip}(t) \).
\[ r(t) = \sum_{p=1}^{NS} \int s_{p}(t - \tau) \cdot a_{p}(\tau) + b_{p}(t) \quad \text{with } i = 1, N_{c} \quad (5) \]

where \( b_{p}(t) \) is a noise supposed to be Gaussian, white and centered and \( N_{c} \) the number of sensors. With signals sampled in time, we write the received signals in a data matrix as:

\[ r = [r_{i}]_{i=1, \ldots, N_{c}} = [r_{j,i}]_{j=1, \ldots, N_{c}} \in \mathbb{R}^{N_{c} \times N_{t}} \quad (6) \]

The Singular Value Decomposition of the time-space data matrix \( r \) provides two orthogonal matrices \( u \) and \( v \) and one diagonal matrix \( \Delta \) made up of singular values (Vrabie et al. 2004). The initial data matrix is expressed as:

\[ r = u \Delta v^{T} = \sum_{k=1}^{N} \lambda_{k} u_{k} v_{k}^{T} \quad \text{with } N = \{ \min N_{c}, N_{t} \} \quad (7) \]

where \( u = [u_{1}, \ldots, u_{N_{c}}] \) is a \( N_{c} \times N \) orthogonal matrix made up of left singular vectors \( u_{k} \) giving the amplitude in the real case (amplitude and phase in the complex case), therefore called propagation vectors; \( v = [v_{1}, \ldots, v_{N_{t}}] \) is a \( N \times N_{t} \) orthogonal matrix made up of right singular vectors \( v_{k} \) giving the time dependence, hence named normalized wavelets; \( \Delta = \text{diag}(\lambda_{1}, \ldots, \lambda_{k}, \ldots, \lambda_{N}) \) a \( N \times N \) diagonal matrix with the diagonal entries ordered \( \lambda_{1} \geq \ldots \geq \lambda_{k} \geq \ldots \geq \lambda_{N} \geq 0. \)

The product \( u_{k} v_{k}^{T} \) is an \( N_{c} \times N_{t} \) unitary rank matrix named the \( k \)th singular image of data matrix \( r \). Therefore, \( r \) is given by the sum of all the \( k \)th singular images multiplied by their correspondent \( k \)th singular values \( \lambda_{k} \). The rank of the matrix \( r \) is the number of non-zero singular values in \( \Delta \). In the noise free case, if the recorded signals are linearly dependent (for example if they are equal to within a scale factor; that means one wave with an infinite velocity) the matrix \( r \) is of rank one and the perfect reconstruction requires only the first singular image (Freire and Ulrich 1988). If the \( N_{c} \) recorded signals are linearly independent, the matrix \( r \) is full rank and the perfect reconstruction requires all singular images.

Using the SVD filter, separation between the signal and the noise subspace is given by:

\[ r = r_{SG} + r_{NOISE} = \sum_{k=1}^{NS} \lambda_{k} u_{k} v_{k}^{T} = \sum_{k=NS+1}^{N} \lambda_{k} u_{k} v_{k}^{T} \quad (8) \]

The signal subspace \( r_{SG} \) is characterized by the first NS higher singular images (associated to the first NS higher singular values). It gives roughly the waveform of the dominant wave, its energy and its amplitude repartition on the sensors. The reminder subspace \( r_{NOISE} \) contains the waves with a low degree of sensor-to-sensor correlation and the noise (Vrabie et al. 2004, Mars et al. 2004).

Nonetheless, normalized wavelets \( u \) describing signal subspace and propagation vector \( v \) are orthogonal. This orthogonality condition forces the wavelet to be a mixture of waves, therefore errors can appear in the signal subspace. Idea proposed by Vrabie et al. (2004) was to find a new matrix \( \tilde{u} = uR \) by subspace rotation where \( \tilde{u} \) are the most independent possible (ICA technique). \( R \) is given by a diagonalization of the cumulant of \( u \). As a result, SVD-ICA provides better results rather than classical SVD since it relaxes the constraint of orthogonality. Details on SVD-ICA can be found in Vrabie et al. (2004) with examples on synthetic datasets.

Hence, in practice, before performing the SVD or SVD-ICA filtering, a flattening operation on the initial data is applied to obtain an infinite apparent velocity for the selected wave (Mari et al. 1997). For refracted waves, the flattening pre-processing is obtained by time shifting the data, the time shifts are derived from the picking of the first arrival times. For dispersive surface waves, the flattening is obtained by group velocity and phase shift corrections.

Whatever the filter used (F-K or SVD-ICA) the wave separation procedure is the following one:

- flattening operation
- filtering by the selected filter
- inverse flattening operation.

### III. FIELD CASE

The seismic data have been recorded on an experimental hydrogeological site that has been developed for several years near Poitiers city. The concerned aquifer, 20 to 130 meters in depth, consists of tight karstic carbonates of Middle Jurassic age. It lies on the borderline, named the “Poitou threshold”, between the Paris and the Aquitaine sedimentary basins. Around 30 wells have been drilled on this site. Most wells dispose of documented drilling records and logs of various nature, among which gamma-ray, temperature, acoustic. In addition, two wells were entirely cored. Wellbore and surface seismic data were also acquired in the vicinity of selected wells. The acquisition of usable seismic data is particularly difficult for low-depth reservoirs underlying a thick weathered zone, such as the aquifer studied herein.

Different surface seismic surveys were however attempted with different acquisition schemes. The selected recording spread is composed of 48 single geophones. The sources used were either a detonating impulse source or a mini vibrator system. The distance between 2 adjacent geophones is 5 m. The time sampling interval is 0.25 ms and the recording length is 0.5 s. For a given recording spread several shot points were fired. A direct shot and a reverse shot were fired in line to obtain 2D seismic images. Several shots were fired cross line to obtain 3D seismic images.

Figure 1 shows a direct in-line shot point obtained with the impulse source. The data are presented both in the time distance domain and in the F-k domain. The main energetic wave fields are the low and high apparent velocity pseudo Rayleigh waves and the direct and refracted body waves.
The shape of the refracted wave shows that the refractor is strongly disturbed. This is confirmed by the analysis of the picked times of the refracted arrivals. Refraction signals enable to get an image (not shown here) of the top surface of the aquifer. This is useful in the present case as the aquifer top surface is strongly disturbed as a result of the presence of caves of karstic origin filled in by clayey surface erosion material.

The wave separation procedure has been done in several different steps. Each step includes:

- the extraction of a given wavefield by a specific filter (F-K or SVD-ICA)
- the subtraction of the estimated wavefield to the input section to obtain a residual section
- the residual section becomes the input section for the following step

In a first step, the procedure has been used to extract by F-K filter the direct wave and the slow (low apparent velocity) Rayleigh wave.

In a second step, the procedure has been applied to extract the fast (high apparent velocity) Rayleigh wave.

The residual section exhibits negative apparent velocity events which are converted refracted waves which have been extracted in the third step (figure 4).

The separation procedure has then been applied to extract the refracted wave (SVD-ICA filtering). Normal Move-Out corrections (NMO) and deconvolution by spectrum balancing have been applied to the residual section. The seismic section thus obtained is shown in Figure 5. It clearly shows infinite apparent velocity events which are associated with reflected waves. The reflected wave which appears at 0.75 ms is associated with a reflector situated inside the reservoir at a depth of 100 m.
Figure 6 shows a cross-line shot point obtained with the vibrator source. The lateral offset of the source is 115 m. The shot point shows mainly a strong energetic aliased air wave.

An internal mute has been used to cancel the air wave. The same wave separation procedure has been applied to the data to extract the refracted wave (SVD-ICA filtering, figure 7), to cancel the surface waves (F-K filtering) and to extract the reflected events (shown in figure 8, after NMO corrections and deconvolution).

Offset/Vertical Seismic Profiles (VSP, Mari et al. 1997) were also acquired in the vicinity of one well in order to identify major reservoir markers. The SVD-ICA filtering has been used to separate the down-going and up-going wavefields of the VSP datasets.

Weathered zone obstacle was overcome thanks to buried sources. The frequency bandwidth is large and reaches 800 Hz. Figure 9 (upper part) shows the VSP sections in the 30-840 Hz bandwidth (on the left) and in the 20-140 Hz bandwidth (on the right). This way, aquifer bottom limit was identified as well as two reservoir markers, which were confirmed as two major drains of heterogeneous rock structure by production and acoustic logs.

Figure 8 is a composite section which shows the comparison between results obtained with the impulse source (offsets ranging between 0 and 50 m) and the vibrator source (offsets ranging between 50 and 300 m).

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Figure 9 (lower part) shows the in-line seismic section (figure 5) after depth conversion. The VSP data are inserted in the central part of the section. VSP data confirm the efficiency of the wave separation procedure used to extract reflected events and the possibility to plane 2D or 3D surface seismic surveying to study the aquifer reservoir.

IV. CONCLUSION

Figure 9 (lower part) shows the in-line seismic section (figure 5) after depth conversion. The VSP data are inserted in the central part of the section. VSP data confirm the efficiency of the wave separation procedure used to extract reflected events and the possibility to plane 2D or 3D surface seismic surveying to study the aquifer reservoir.

References