A WAVELET TRANSFORM-ARTIFICIAL NEURAL NETWORKS (WT–ANN) BASED ROTATING MACHINERY FAULT DIAGNOSTICS METHODOLOGY

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ABSTRACT

This paper outlines a Wavelet Transform (WT) based Artificial Neural Network (ANN) input data pre-processing scheme and presents the results of localized gear tooth defect recognition tests by employing this proposed methodology. The methodology consists of calculating Daubechies' 20-order (DAUB-20) mean-square dilation WTs of the data, and then selecting predominant wavelet coefficients distributed to certain levels of these WTs as inputs to ANNs for pattern recognition. The test results show that a fairly small sized backpropagation network trained with a reasonably small number of training sets can detect and classify various types or degrees of failures occurring on a spur gear pair successfully.

1. INTRODUCTION

Of the features that are available to a classification method, it is extremely beneficial to identify those that provide the most discriminative information and also identify those that may be redundant. By removing the useless features, classification becomes less computationally demanding. Additionally, when an ANN is considered as a classifier, limiting the number of features is critical to the reduction in the number of training samples required. Reducing the dimension of the input vector in an effective way will also enhance the network's *fault-tolerance* and *generalization* capabilities. These characteristics will lead the network to recognize the cases in the state of having corrupted or less number of training data, or to recognize cases, which has never met before.

When a rotating machinery vibration signal pre-processor to ANNs is considered, it should be noted that non-stationary or transitory characteristics such as, drift, trends, abrupt changes, and beginnings and ends of events are the most important part of the signal, since they can reveal abnormal changes representing a failure. Statistical or Fourier Transform (FT) based analysis is not suited to detecting them. Whereas wavelet transforms (discrete or continuous) pinpoint those localized features by means of their variable-sized windows. Discrete wavelet transform (associated with the "time-scale" type wavelets) can adapt to signals having a discontinuous structure more suitably. Furthermore, having been computed with fast algorithms and being able to reproduce the original signal make discrete WT practical in the analysis of machinery vibration signals, Engin *et al*. 1996 [5], Yeşilyurt and Ball 1997[12], Badi. *et al*. 1997 [2].

Daubechies wavelet transforms are discrete WTs and can be computed by fast algorithms (namely, Mallat's pyramid algorithm). They can provide compact input vectors with their powerful feature extraction capabilities. Additionally, Daubechies WT based mean-square wavelet maps show the distribution of each wavelet coefficient to the particular wavelet level. Analyzing the most informative levels by means of a pattern recognition technique can reveal sound clues for Fault Diagnostics of rotating machinery. Daubechies 20-order WT works with wavelets with more vanishing moments, which is significant for suppressing the details and highlighting the remainder. Hence choosing a large number, for example 20, will ensure Daubechies WT being computed in optimum speed and an effective feature extraction tool.

The performance of the methodology was tested with both numerically simulated and experimentally acquired spur gear vibration time signals. The seeded localized tooth defects were kept as small as possible to emulate impending failure realistically. This was important from the point of carrying out early fault detection applications. A multilayer ANN fed by the proposed WT based signal features identified the faulty signal features from the healthy ones very successfully. When the trained network was introduced with the feature vectors for two types of faults and their reference (healthy) states, the network classified them with very high success rates ranging from 70% to 100%.

A brief theory of wavelet transforms and their effective computation method with an emphasis on the considerations of the choice of wavelet families are presented in the following section. Then the proposed WT–ANN based fault diagnostics methodology is outlined. This is followed by the experimental results with related graphs, tables and discussions. Finally, the paper reaches the conclusion section, which draws the relevant conclusions of the research findings and suggests some directions for future investigations.

2. WAVELET TRANSFORM

As known, the basic idea in time-frequency representations is that *two* parameters are needed: one called a, which refers to frequency; the other called b, which indicates the position in the signal. Thus a general time-frequency transform of a signal x will take the form,

$$x(t) \leftrightarrow \psi(a,b) = \int_{-\infty}^{\infty} \overline{\psi_{ab}}\, x(t)\, dt \qquad \textbf{Eq. 1},$$

where ψ_{ab} is the analyzing function and $\overline{\psi_{ab}}$ (indicated also as ψ^*) is its complex conjugate. In wavelet transform, the analyzing function ψ is defined as,

$$\psi_{ab}(t) = a^{-1/2}\, \psi\left(\frac{t-b}{a}\right) \qquad \textbf{Eq. 2}.$$

Combining the **Equations 1** and **2**, the basic formula for the continuous WT (CWT) can be obtained as,

$$W_x(a,b) = a^{-1/2} \int_{-\infty}^{\infty} \overline{\psi}\left(\frac{t-b}{a}\right) x(t)\, dt \qquad \textbf{Eq. 3}.$$

In discrete WT (DWT), the two parameters a and b which are for scaling and translating, respectively can be defined as functions of level j and position k

$$a = 2^{-j} \quad j \in Z, b = a.k \quad k = 0...n\text{-}1 \qquad \textbf{Eq. 4}.$$

Then the analyzing function ψ becomes

$$\psi_{j,k} = 2^{j/2}\psi(2^j t - k) \qquad \textbf{Eq. 5},$$

where ψ called *mother wavelet* and $\psi_{j,k}$ called *daughter wavelet*. Here the level j determines how many wavelets are needed to cover the mother wavelet, and the number k determines the position of the wavelet and gives the indication of time. It is possible to decompose any arbitrary signal $x(t)$ into its wavelet components. The approach is similar to the harmonic analysis in Fourier transform except that, instead of breaking a signal down into harmonic functions of different frequencies and amplitudes, the signal is broken dawn into wavelets of different scale (level), different positions and the corresponding amplitudes of wavelets.

This can be put into a simpler explanation to stress the similarity of approaches between the two transforms, as follows. The Fourier transform breaks down a signal by frequency, and the wavelet transform breaks down a signal into components of different scales by comparing the signal to wavelets of different sizes. In both cases, this is done by integration: multiplying the signal by the analyzing function (sines and cosines or wavelets) and integrating the product, Hubbard 1996 [9]. FT and WT are both linear and square-integrable functions, derived from group representation theory (from different groups). The essential difference between the two is in the way the frequency (scaling) parameter a is introduced in the analyzing function. In both cases, b is simply a time translation.

If the analyzing function of short-time Fourier transform (STFT) or Gabor transform, both of which involve Fourier transform, is expressed in the similar way with the analyzing function of the wavelet transform in **Equation 2**, it would be as follows,

$$\psi_{ab}(t) = e^{jt/a}\, \psi(t-b) \qquad \textbf{Eq. 6}.$$

The relation between the two (STFT and WT) can be seen clearly in **Figure 1**. Here ψ is a window (or analyzing) function and the a-dependence is a modulation ($1/a$ ~ frequency). Now, how these transforms work can be understood easily: The window in STFT has constant width, but the lower a, the larger the number of oscillations in the window, **Figure 1 (a)**. The effect of a on the analyzing function ψ of WT is a dilation (a >1) or a contraction (a <1). The shape of the function is unchanged, it is simply spread out or squeezed, **Figure 1 (b)**, Antoine 1996 [1]. This unique characteristics of analyzing a signal with an adjustable window enables wavelet transform to detect small, hidden or sudden changes more accurately.

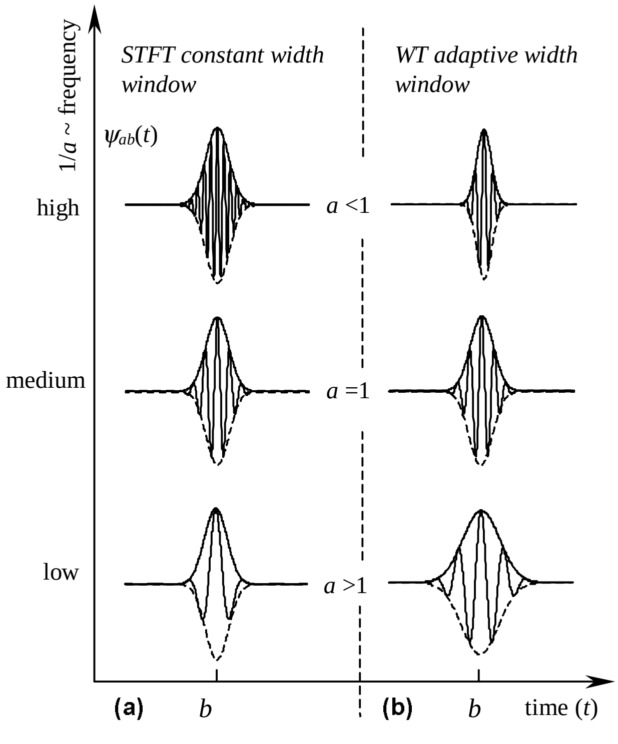

Figure 1. Varying the scale parameter a in the case of the short-time Fourier transform, **(a)**; and the wavelet transform, **(b)**.

2.1 Choice of Wavelet Families

In the wavelet analysis of signals, investigations on the matter of “which wavelet to choose” have been in continuous progress. This is because while a single algorithm (Fourier transform) is appropriate for all stationary signals, the transient signals are so rich and complex that a single analysis method (whether of time-scale or time-frequency) or one single type analyzing wavelet cannot serve them all. This and following subsections introduce several well-known wavelet families and their characteristics briefly, and then outline the two main approaches (continuous and discrete) to the wavelet computations in order to justify the reason for preferring a particular type of WT for effective and fast machinery vibration signal analysis. For different wavelet applications to diagnostics refer to Engin *et al*. 1996 [5].

There are different types of wavelet families whose qualities vary according to several criteria. For example, **Figure 2** shows the three analyzing wavelets used at three decreasing values of scale with a shift parameter: Morlet wavelets **(a)**, applied as *continuous* wavelet transform; and Daubechies D4 & D20 wavelets **(b,c)**, applied as *discrete* wavelet transform.

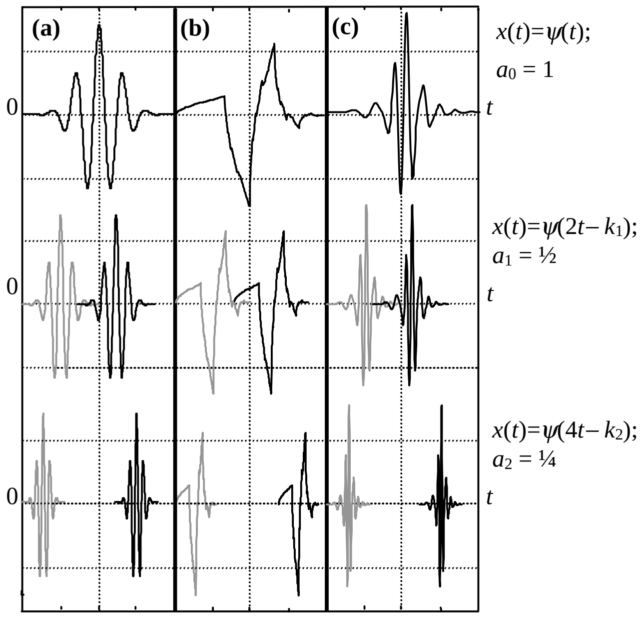

Figure 2. The Morlet wavelet **(a)**, and Daubechies D4 and D20 wavelets **(b, c)** at three decreasing values of scale (a_0 =1, a_1 =½ and a_2 =¼) with shift (k_0 =0 and $|k_0| < |k_1| < |k_2|$).

The main criteria in choosing a family of wavelets are as follows, Misiti *et al*. 1997 [11],

- The support of wavelet function ψ, scaling function ϕ, and their Fourier transforms: the speed of convergence at infinity to 0 of these functions when the time or the frequency goes to infinity, which quantifies both time and frequency localization.
- The symmetry, which is useful in avoiding dephasing in image processing (which is out of the scope of present study).
- The number of vanishing moments for ψ or for ϕ (if it exists), which is related to reducing the polynomial degree of time series being analyzed and is useful for compression purpose.
- The regularity, which is useful for getting nice features like smoothness of the reconstructed signal or image.

These are associated with two properties that allow fast algorithm and space-saving coding:

- The existence of a scaling function ϕ.
- The orthogonality or the biorthogonality (when the wavelets used in the deconstruction and reconstruction are different) of the resulting analysis,

and perhaps less important ones:

- The existence of an explicit expression of ϕ (if exists) and ψ.
- The ease of tabulating.

Daubechies wavelets constitute perhaps the most popular wavelet family among the others. These wavelets have no explicit expression except for D2, which is the simplest and certainly the earliest wavelet introduced by Haar in 1911. Several significant characteristics of Daubechies wavelets are as follows,

- The support length of ψ and ϕ is $2N$–1, where N is the number of coefficients (e.g. for D4 this is 2·4–1 =3.)
- The number of vanishing moments of ψ is N, which is significant for suppressing the details and highlighting the remainder. Hence choosing a large N, for example D20, will ensure Daubechies wavelets an effective feature extraction tool.
- Most Daubechies wavelets (usually from D2 to D20) are not symmetrical. For some, the asymmetry is very pronounced. There is a trade off between the symmetry and computation simplicity. D20 wavelets, however, are not very far from symmetry and their computations are not much more costly (in terms of time and computer storage) than the other wavelets having smaller number of coefficients. There are some other efforts on this subject like Daubechies proposition of modifying her wavelets such that their symmetry can be increased while retaining great simplicity. For example, *symlet wavelets* as "near symmetric" wavelets were developed. These wavelets have very similar features that Daubechies wavelets have and do not offer more advantages. Besides, symlets are only near symmetric; consequently some authors do not call them symlets. Symmetry, however, is not important in signal processing; it is a concern in 2d data, i.e. in image processing.

- The analysis is orthogonal, which alleviates the computation burden by allowing highly efficient algorithms to be devised.

Another popular family is Morlet wavelets. Morlet wavelets can be expressed explicitly as follows, Bentley and Grant 1995 [3],

$\psi(t) = \exp(j\omega_0 t)\exp(-t^2/2)$ **Eq. 7**,

where ω_0 is selected according to the frequency range of the signal being analyzed. Alternatively, it can be expressed as, Misiti *et al*. 1997 [11],

$\psi(t) = C\exp(-t/2)\cos(5t)$ **Eq. 8**,

where the *C* constant is used for normalization in view of reconstruction. Both expressions give the same shape of wavelet function: a sinusoid windowed by a Gaussian function, as was plotted in **Figure 2 (a)**. The popularity of Morlet wavelets is very much due to the direct connection between scale and frequency. Morlet wavelets can be legitimately called a time-frequency transform. In applications where it is desirable to analyze the time-frequency nature of the signal the Morlet wavelets are particularly useful. Being symmetrical, having an explicit expression and providing an exact time-frequency analysis make Morlet wavelets really useful. On the other hand they suffer from several drawbacks such as absence of scaling function ϕ and being not orthogonal so they cannot be computed with fast algorithms. The other disadvantage is that they do not give exact reconstruction when the inverse wavelet transform is performed, Bentley and Grant 1995 [3].

There are some other wavelets in use such as Mexican hat wavelets, two-humped wavelets, chirp wavelets, Meyer wavelets, Battle-Lemarie wavelets and so on. These wavelets have advantages and disadvantages compared to each other and they present successful results according to the suitability of purpose or applications. For example, two-humped wavelets were used to detect chords in recorded music and chirp wavelets were defined to improve description of heart sound signals (refer to Grossmann and Martinet 1987 [8] and Bentley and McDonnell 1994 [4], for the development of these wavelets).

2.2 Continuous vs. Discrete Wavelet Transform

Including machinery vibration signal any signal processing performed on a computer must be discrete – that is it has been measured at discrete time intervals. Therefore, the Continuous Wavelet Transform (CWT) is also operating in discrete time. What make CWT “continuous” and what distinguishes it from Discrete Wavelet Transform (DWT) is the scales at which it operates. Unlike the DWT, the CWT can perform at every scale up to some maximum scale with minimum increment, which can be determined by trading off the need for detailed analysis with available computational horsepower. The CWT is also continuous in terms of shifting: during computation, the analyzing wavelet is shifted smoothly over the full domain of the analyzed signal. Calculating wavelet coefficients at every possible scale is a fair amount of work, and it generates an awful lot of data, Misiti *et al*. 1997 [11]. It has been shown that by choosing scales and positions based on powers of two (so-called dyadic scales and positions) the analysis will be much more efficient and just as accurate. Such an analysis is performed by the *discrete wavelet transform.* A very efficient way to implement this scheme using low-pass and high-pass filters was developed by S. G. Mallat in 1988 (see Mallat 1989 [10]).

The discretized CWT yields magnitude and phase plots with *n* samples along the time-shift axis (where *n* is the number of samples in the analyzed signal) and $j \times m$ samples along the scale axis (where *j* is referred to the number of octaves and *m* is the number of voices per octave). In this form wavelet transform exhibits a large degree of redundancy. The redundancy in scale can be removed if a single analysis per octave is performed and redundancy in the time-shift variable can be reduced to a minimum by subsampling the signal after each octave analysis. In conclusion, Daubechies orthogonal wavelets, which are computed by Mallat’s tree algorithm (Fast Wavelet Transform), make discrete wavelet analysis very much practicable. Together with being a discrete WT, Daubechies wavelets also offer very competitive feature extraction capabilities for vibration signal analysis. They can provide compact input vectors with their powerful feature extraction capabilities. Additionally, Daubechies WT based mean-square wavelet maps show the distribution of each wavelet coefficient to the particular level. Hence, this type of wavelets has been considered to be the most appropriate method in the analysis of rotating machinery vibration signals.

3. THE DIAGNOSTICS METHODOLOGY

The feature extraction scheme proposed for the fault diagnostics methodology here is based on calculating mean-square D20 WT map of the vibration signal to introduce the characterizing mean-square wavelet amplitudes of the critical levels to the ANNs. The block diagram for the methodology is sketched in **Figure 3**.

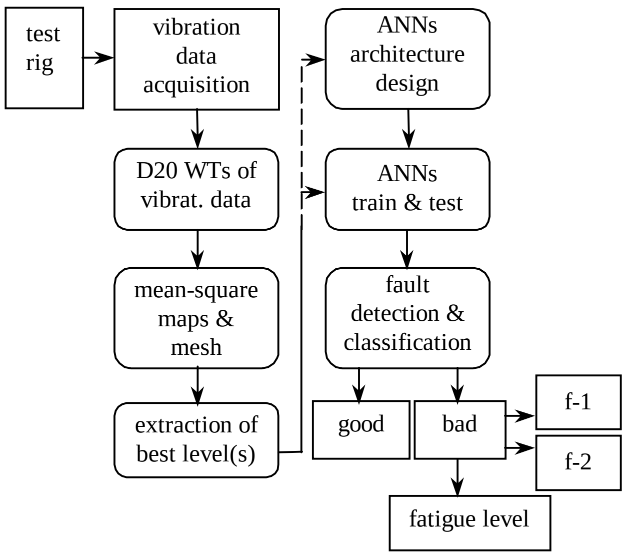

Figure 3. Block diagram of the feature extraction scheme for gear fault diagnosis.

The diagram details the steps of the fault diagnosis methodology as part of the established Condition Monitoring set-up, detailed in Engin 1998 [7]. It combines the two types of fault experiments; the impulsive (blip and shaved faults) and bending fatigue failures. The classification results for example failures are presented in the following subsections. As noted, the performance of the scheme was tested with standard backpropagation ANNs.

4. WAVELET APPLICATIONS

As was the case when introducing other signal processing methods, the customary approach is followed, and the proposed WT–ANN based fault diagnostics methodology was presented with the numerically simulated data. For this purpose a simple MATLAB® program was coded to simulate vibration time signals representing three different health states of a typical 36-tooth spur gear. The resultant vibration signals representing the reference, having fault-1 (similar to gear with one tooth giving a "blip") and fault-2 (similar to gear having a "shaved" tooth) are displayed in **Figure 4 (a-c)**, respectively.

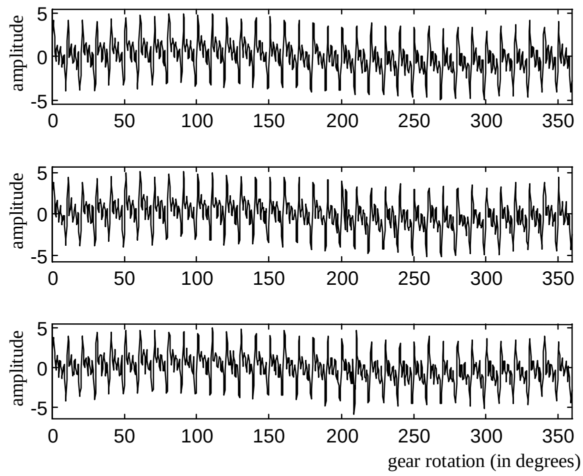

Figure 4 Numerically simulated vibration signals; the reference **(a)**, first fault **(b)**, and second fault **(c)**.

All three kinds of signals carried an amplitude modulated main sine and several other sine functions with higher frequencies giving the meshing frequency and its first three harmonics. The fault-1 and fault-2 were introduced as localized sine functions (enveloped with fast decaying exponential components), i.e. starting with a relatively high amplitude and ending with a very low amplitude at around 205° and 210° of rotation angles. The signal and seeded fault-1, fault-2 and the noise components of the signal are plotted separately in **Figure 5 (a-d)**, respectively.

While the first fault was designed to last 7 or 8 samples (giving a sharp impulse), the second lasted around 15 samples (simulating nearly half of a tooth was shaved), corresponding to the duration of ~2.6 and ~5.2 degrees in **Figure 5 (b)** and **(c)**, respectively. The embedded faults are tried to be kept fairly small (around half of the signal amplitude) to simulate the conditions of developing failures realistically. Hence, as **Figure 4** illustrates the variations between the signals are hardly distinguishable by eye.

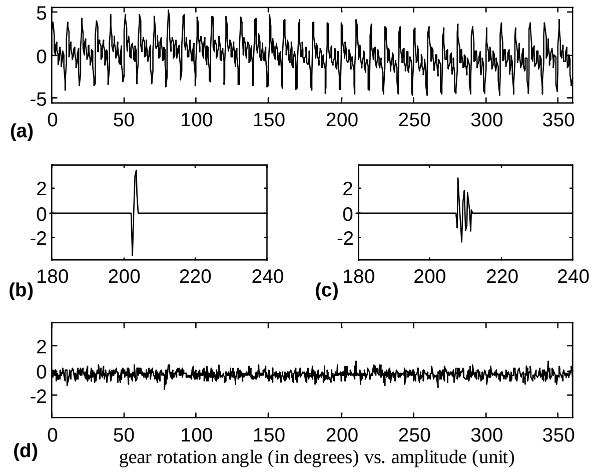

Figure 5. The reference signal **(a)**, and Fault-1 **(b)**, Fault-2 **(c)**, noise components **(d)**.

40 copies of each health state (healthy, fault-1 and fault-2) were taken and each was distorted by a random noise signal as shown in **Figure 5 (d)**. Then their D20 wavelet transform based mean-square wavelet maps were computed for feature extraction. The 3d mean-square mesh diagrams of these maps (for the three signals plotted in **Figure 4 (a-c)**) are presented in **Figure 6** to **8**, respectively.

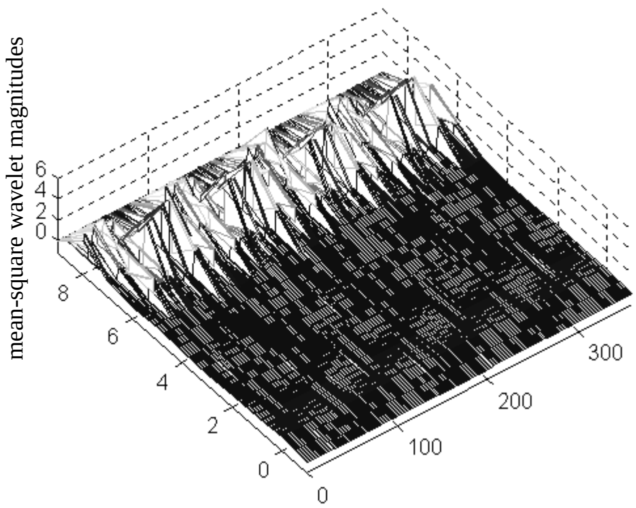

Figure 6. Wavelet mean-square mesh diagram for the numerically simulated reference vibration time signal given in Figure 4 (a).

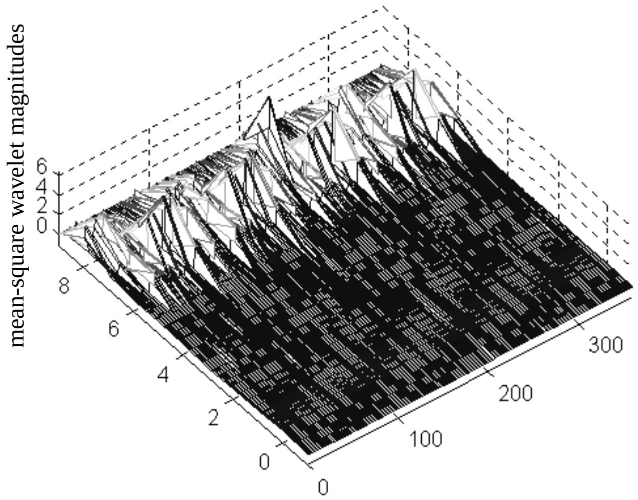

Figure 7. Wavelet mean-square mesh diagram for the numerically simulated fault-1 (similar to the blip fault) vibration time signal given in Figure 4 (b).

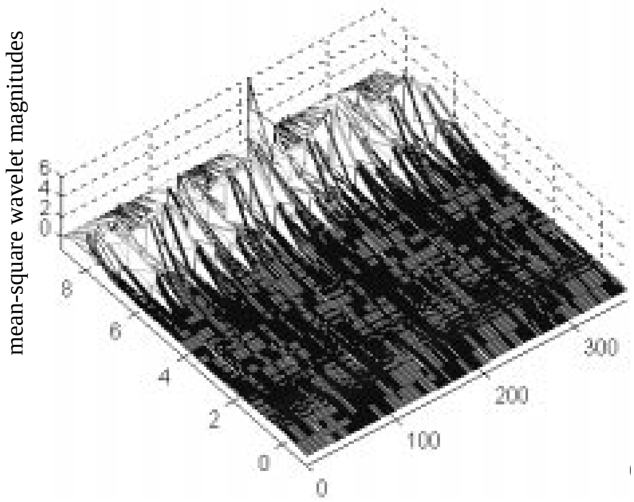

Figure 8. Wavelet mean-square mesh diagram for the numerically simulated fault-2 (similar to the shaved fault) vibration time signal given in Figure 4 (c).

As was outlined in **section 3**, wavelet magnitudes distributed in high levels detected the changes indicating the health of the signal sufficiently. For this example, when the computed *A* matrices (carrying the mean-square wavelet map information) are studied it is seen that while levels 7 and 8 are mostly indicative for fault-1, levels 8 and 9 are more indicative for fault-2. This is due to the fact that the fault-1 involved lower fault frequencies compared to the fault-2. Therefore, while levels 7 and 8 are sufficient for fault-1, for fault-2 higher levels, i.e. levels 8 and 9 seemed to be more descriptive. And only several (about six) wavelet magnitudes in those levels reveal the differences between the health patterns.

Consequently, the program produced a set, that is to be the training file, containing 12 most energetic wavelets from levels 7 and 8 (each gave 6) computed for the reference and fault-1 signals, and from levels 8 and 9 (each gave 6) for fault-2 signal. Each health state was represented with 30 feature vectors, of which 10 were used for testing during the training. Another file was obtained in the same way containing 10 different (unused) vibration signal feature vectors for each state to be used in the recall mode. After various experiments, the MLFF (multilayer feed forward) neural network (trained with backpropagation algorithm) having 12 input neurons, 6 hidden neurons and 2 output neurons (trained so that to return 0 0 for reference, 0 1 for fault-1, and 1 0 for fault-2) yielded a very high success rate in classifying unmet 30 vibration signal features into healthy, fault-1 and fault-2 states. The network parameters used are presented in **Table 1**. The training error history is given in **Figure 9**, which plots the rms error between the training set targets and network outputs. The goal of backpropagation training algorithm is to drive the error to a minimum value. The plot shows that the rms error was dropped to under 0.01 within 10 000 iteration.

MLFF Network architecture			
Transfer function	**No. of inputs**	**Hidden layer**	**No. of outputs**
Sigmoid	12	3 – 3	2
Training parameters			
Max. iterations	**Learning rate (min)**	**Learning rate (max)**	**Momentum**
10 000	0.001	0.30	0.80

Table 1. Network architecture and training parameters used in the fault classification.

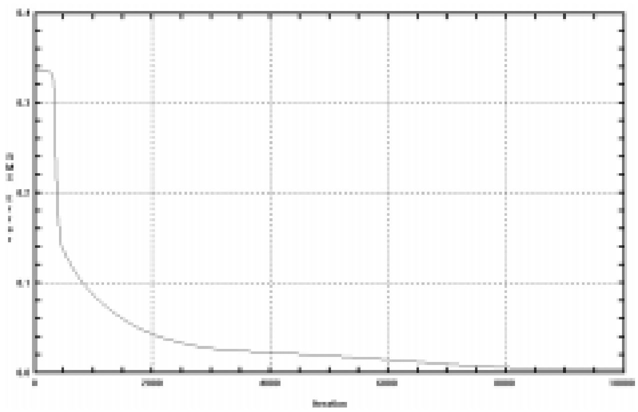

Figure 9. RMS error within the 10000 iteration.

5. CONCLUSIONS

Artificial neural networks have emerged as a very beneficial signal processing tool in many areas of application from pattern recognition to control. There exist various types of neural networks developed over the years. Among them multilayer feed forward network trained with backpropagation algorithm has been found as the most commonly used one. It is a self-adapting algorithm, that is, it learns a problem with examples then recognizes patterns, trends and hidden relationships within the similar problems represented by data with huge amount and complexity. A principled establishment methodology and an effective pre-processing scheme are two most important requirements for successful applications of neural networks. The experiments carried out on numerically and experimentally simulated data indicate that a well designed and trained neural network have the potential to estimate the condition of a rotating machinery component effectively by inspecting the encoded (in this case pre-processed with a D20 WT based feature extraction scheme) data of a single revolution of the component. This pre-processing has made the network's input vectors much more compact and hence easy to establish the relations between them, which leads to *fast* and *effective* signal classification.

Although the introduced spur gear localized tooth defects were reasonably realistic, the proposed automated Condition Monitoring and Fault diagnostics (CM/FD) methodology should be tested with the data acquired in real industrial conditions. Once similar good results obtained from these experiments, the methodology could be established as a part of an integrated on-line CM system, possibly devised with DSP based hardware operating with comprehensive and user-friendly software. As the literature indicates the wavelet transforms have already been implemented on DSPs. Consequently, ANNs fed by the proposed D20 WT based feature extraction scheme, which leads to a tremendously short training/classification time, could be perfectly used in on-line or real-time CM applications to avoid costly breakdowns.

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