

# IMPROVING THE PERFORMANCE OF TURBO CODES FOR AWGN, RMF AND B-RMF CHANNELS

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## ABSTRACT

*This paper presents three methods of increasing the performance of turbo codes, depending on the type of channel considered. In case of AWGN channels, a CRC aided decision criterion for the MAP algorithm is proposed. In case of RMF channels, a new chaotic channel interleaver is proposed for the bit interleaved coded modulation. For the B-RMF channels, a new composite combining receiver diversity technique is introduced.*

**Keywords:** turbo codes, interleaved codes, modulation coding, diversity methods

## 1. INTRODUCTION

The excellent performance of turbo codes is due to the iterative decoder design, which enables the two component MAP (Maximum a Posteriori) convolutional decoders to exchange extrinsic information on the decoded bits [1]. This architecture offers low error rates for AWGN (Additive White Gaussian Noise) channels, but is not very effective for fast RMF (Rayleigh Multiplicative Fading) channels. In order to lower the error rates in the presence of fast fading, a turbo coded BICM-ID (Bit Interleaved Coded Modulation-with Iterative Decoding) scheme is employed. In this situation, a bitwise random interleaver is inserted between the turbo encoder and the digital modulator. The bitwise interleaver allows an iterative demodulation - decoding process, in which soft LLR (Log Likelihood Ratios) are exchanged between the maximum likelihood receiver and the turbo decoder [2].

The slow Block-RMF (B-RMF) channel has constant fading coefficients for a block of bits (usually equal to the frame length). This situation is a severe flat fading scenario and can be overcome only through the use of transmission or reception diversity. Reception diversity offers greater performance than transmission diversity, because in most cases the CSI (Channel State Information) is available only at the receiver side [3]. The most popular receiver diversity methods are the linear combining ones, such as SC (Selection Combining), EGC (Equal Gain Combining) and MRC (Maximum Ratio Combining) [4].

In order to decrease the number of decoding iterations, the information bits can be appended with CRC (Cyclic redundancy Check) bits, which can provide additional information regarding the correct decoding of the information bits [5].

This paper presents three different methods that increase the performance of turbo coded systems. The proposed techniques depend on the channel type.

## 2. SYSTEM MODEL

This section describes the system model that was used in the simulations. The structure of the implemented transmitter is presented in Figure 1. The information bits  $x_k$  are passed through a turbo encoder and a CRC encoder. The resulting encoded bits  $y_k$  are obtained by appending the turbo coded bits with the CRC bits. In order to improve the performance under fading conditions, a BICM scheme is employed [6]. Thus, the encoded bits are scrambled by a bitwise interleaver and then mapped by a BPSK (Binary Phase Shift Keying) modulator.

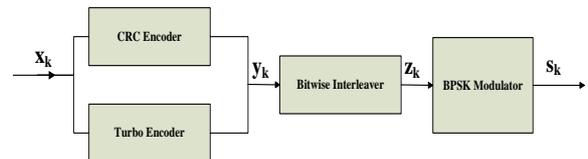


Figure 1: The structure of the transmitter.

The considered channel is depicted in Figure 2. In case of fading, the multiplicative coefficients  $a_k$  are values of Rayleigh random variables, which change their value for each symbol (fast RMF) or remain constant for an entire block of symbols equal to the frame length (slow B-RMF). In case of AWGN, the coefficients  $a_k$  are equal to 1 and the noise is determined only by the additive coefficients  $n_k$ , which are values of Gaussian random variables.

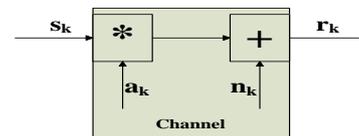


Figure 2: The considered channel.

The configuration of the receiver is illustrated in Figure 3 for a SIMO (Single Input Multiple Output) system with ‘m’ receive antennas. Each received symbol is passed through a matched filter, which provides the corresponding soft LLR<sub>i</sub> (Log Likelihood Ratio). Because BPSK modulation is used,

the  $LLR_i(k)$  for the  $i$ -th antenna and  $k$ -th bit is computed using equation (1):

$$LLR_i(k) = \frac{2a_{k,i}}{\sigma^2} r_{ki} \quad (1)$$

In the above relation,  $a_{k,i}$  represents the fading coefficient for the  $i$ -th antenna at time  $k$  and  $\sigma^2$  is the variance of the AWGN channel, which is equal to:

$$\sigma^2 = \frac{N_0}{2E_b R} \quad (2)$$

Where  $N_0$  is the one sided noise spectral density,  $E_b$  is the bit energy and  $R$  is the code rate.

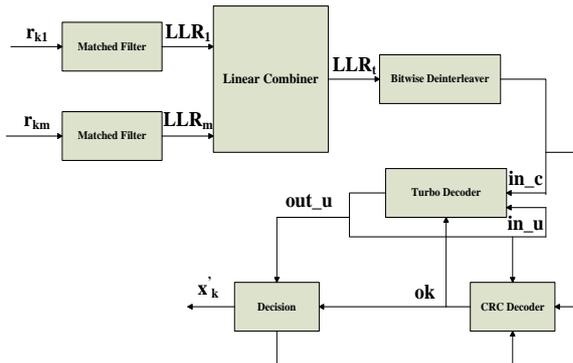


Figure 3: The structure of the receiver.

The computed  $LLR_i(k)$  from each antenna are processed by a linear combiner. The resulting  $LLR_i$  are deinterleaved, separated and passed through the turbo decoder and the CRC decoder, respectively. Because of the considered filter-matched structure, the BICM-ID receiver is simplified. In this situation there is no need to use a feedback bitwise interleaver connection between a soft demapper and the turbo decoder [7]. The iterations between the turbo decoder and the demodulator are ensured only through the  $in_u$  LLRs, which correspond to the a-priori information of the information bits. For the first inner BICM-ID iteration, the a-priori information is initialized to zero. For the next inner iterations, the a-priori information is given by the output LLRs generated by the turbo decoder. The turbo decoder also uses the  $in_c$  LLRs corresponding to the code bits. The structure of this decoder is presented in [8]. After each outer turbo iteration, the decoder sends the decoded bits to the CRC decoder block, which provides information regarding the correctness of the decoded sequence. If there is a match or the maximum number of iterations has been performed, then the outer (turbo) and the inner (BICM-ID) iterations are stopped. The decision block performs a hard decision on the decoded LLRs ( $out_u$ ). The decision block can also make some simple post-processing operations on the decoded bits in case that there is no match between the CRC and the decoded sequence.

### 3. PROPOSED IMPROVEMENT FOR AWGN CHANNELS

This paragraph contains the proposed improvement in case of AWGN channels. The standard turbo log-MAP decoding

algorithm computes for each received information bit  $u_k$  the following LLR:

$$out\_u(u_k) = \log \frac{P\{u_k = 1 | W\}}{P\{u_k = 0 | W\}} \quad (3)$$

Where  $W$  represents the received bits from the respective frame.

The decision block from Figure 3 performs a hard decision on the LLR from equation (3):

$$u_k = \frac{\text{sgn}(out\_u(u_k)) + 1}{2} \quad (4)$$

For AWGN channels with a high signal to noise ratio, a significant number of decoding errors is due to the bits with the minimum absolute value of the LLR from equation (3). In order to overcome this issue, a slightly modified decision block is proposed. Its behaviour can be described in the following manner:

- After each decoding iteration, the decision block performs a hard decision on the decoded sequence and the result is sent to the CRC decoder for validation
- In case the CRC validates the decoded sequence the iterations stop
- In case the CRC does not validate the sequence and the number of performed iterations is smaller than the maximum pre-defined number, then the decoder performs another iteration
- If the CRC does not validate the decoded sequence and the maximum number of iterations has been reached, then the sign of the LLRs with the smallest absolute value is changed. This is equivalent to changing the values of the bits with the smallest confidence degree. The modified result is sent back to the CRC decoder for validation. If the decoded sequence is validated, then the changes are kept and are discarded otherwise.

### 4. PROPOSED IMPROVEMENT FOR FAST RMF CHANNELS

This section contains the proposed improvement in case of fast RMF channels. The performance of a turbo code in a fast RMF channel can be substantially increased through the concatenation with a BICM-ID scheme. For AWGN channels, it is desirable to maximize the free Euclidian distance, which can be achieved using coding modulation techniques, such as TCM (Trellis Coded Modulation). For fast RMF channels, the main goal is to maximize the free Hamming distance, which can be accomplished through the use of BICM-ID [9]. The standard BICM-ID interleaver is the pure random interleaver [7]. The proposed interleaver design is based on the Henon chaotic mapping. This type of mapping associates a point of coordinates  $(x_n, y_n)$  to a point of coordinates  $(x_{n+1}, y_{n+1})$ :

$$x_{n+1} = y_n + 1 - ax_n^2 \quad (5)$$

$$y_{n+1} = bx_n \quad (6)$$

Where  $a$  and  $b$  are two constant parameters. For the set of values  $a=1.4$  and  $b=0.3$ , the mapping has a chaotic behaviour. One of the invariant points is the attractor of coordinates  $x=0.6313$  and  $y=0.1894$ .

The Henon based interleaver can be obtained by generating a succession of  $L$  sets of points, with coordinates given by equations (5) and (6). The initial point is close to the unstable attractor described above. The final permutation is obtained in the form of the indexes resulted from sorting the  $L$  generated points after the  $x$ - axis. The generated interleaver has a better normalized dispersion than the standard random interleaver. Furthermore, its design is fully deterministic, thus the permutation can be generated locally at the receiver, without the need of storing the obtained indexes.

## 5. PROPOSED IMPROVEMENT FOR SLOW B-RMF CHANNELS

In this paragraph a new composite combining technique is proposed for slow B-RMF channels. The performance of turbo codes on slow B-RMF channels can be increased provided that diversity is employed. SISO (Single Input Single Output) systems cannot combat block fading and no matter the error correction code used, the obtained error rates are not a measure of the error correction capability of the code, but rather a measure of the channel [10]. A significant coding gain can be obtained through the use of linear reception diversity combining, which can be used both for transmission and reception diversity. This paper assumes only reception diversity SIMO (Single Input Multiple Output) systems, with perfect CSI and BPSK modulation. The most common diversity combining techniques are SC, EGC and MRC. For  $m$  receive antennas, SC selects from the  $m$  available LLRs only one, using the equation:

$$LLR_{i\_SC}(k) = \text{sgn}(\max_{i=1,m} LLR) \max_{i=1,m} (abs(LLR_i(k))) \quad (7)$$

Where  $\max_{i=1,m} LLR$  is the selected LLR with the maximum absolute value.

EGC and MRC combine the  $m$  available LLRs into a single LLR, as shown in relations (8) and (9).

$$LLR_{i\_EGC}(k) = \sum_{i=1}^m LLR_i(k) \quad (8)$$

$$LLR_{i\_MRC}(k) = \sum_{i=1}^m LLR_i(k) abs(a_{k,i}) \quad (9)$$

This paper proposes a new composite combining (CC) diversity method, which takes advantage of the benefits of the three combining methods described above. For each symbol, SC, EGC and MRC are applied, thus obtaining three distinct LLRs. The final LLR is given by the maximum absolute value of the three previously obtained LLRs. The sign of the selected LLR is also taken into account.

## 6. SIMULATION RESULTS

This section presents the results of the performed simulations, which have been conducted for AWGN, RMF and B-RMF channels. The turbo code used was a symmetrical one, with the generator polynomial equal to  $G=[1\ 5/7]$ . The selected turbo interleaver was an S-Random interleaver of length 1024. The length of the BICM-ID interleaver was 4104, which is equal to the frame length, considering the post-interleaver trellis termination and the CRC-16 code.

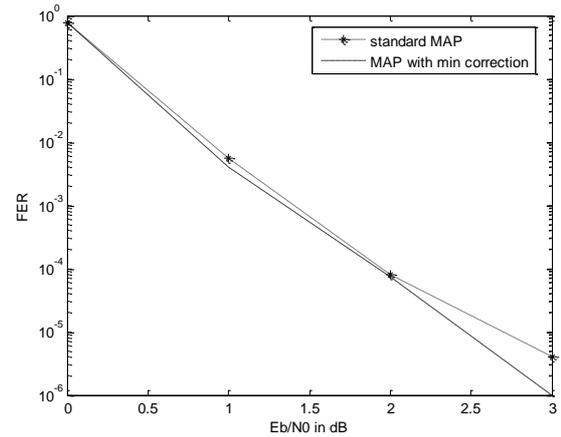


Figure 4: FER for AWGN.

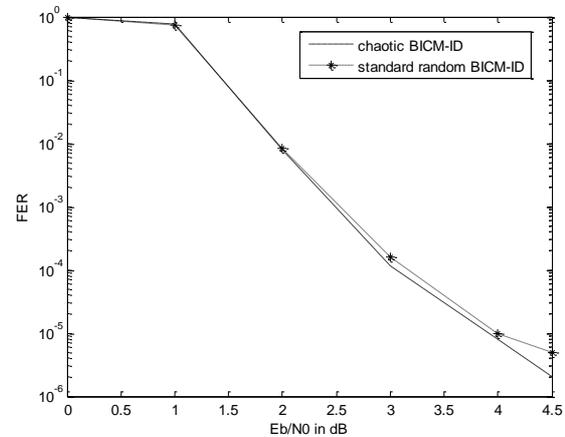


Figure 5: FER for fast RMF.

The CRC-16 code was not applied on the whole information bit sequence, but rather on small 16 bits groupings. As a result, for the considered scenarios the redundancy increases with 1024 bits (roughly 33%), equal to the interleaver length. The modulation used was BPSK, whereas the maximum number of iterations for both the turbo encoder and the BICM-ID receiver was set to 12. Additionally, the turbo decoding algorithm used was log-MAP. The simulations for the AWGN channel are depicted in Figure 4, those for the fast RMF channel are illustrated in Figure 5 and those corresponding to the slow B-RMF channel are represented in Figure 6, for  $m=2$  receive antennas and in Figure 7 for  $m=3$  receive antennas. The variance of the fading was considered equal to  $1/2$ .

The simulations for the AWGN channel show that the decision check based on the minimum absolute value of the LLRs increases the performance of the turbo code. Furthermore, the simulations for the fast RMF channel illustrate that the proposed chaotic bitwise interleaver for the BICM-ID provides better performance than the standard random one. The simulations conducted for the B-RMF yield a clear improvement of the considered SIMO systems over the SISO one.

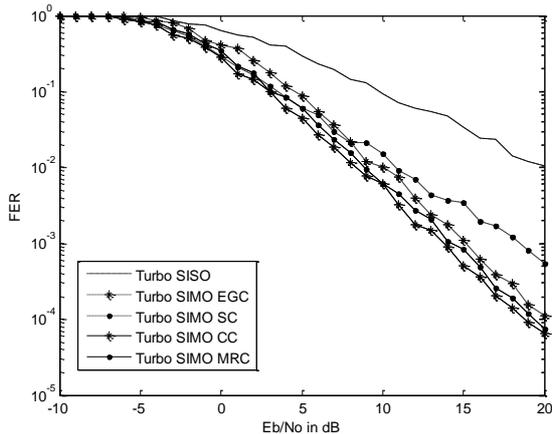


Figure 6: FER for slow B-RMF for  $m=2$ .

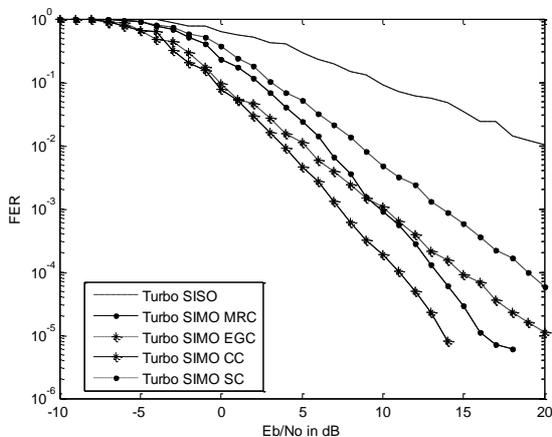


Figure 7: FER for slow B-RMF for  $m=3$ .

In addition, the proposed composite combining (CC) method offers lower frame error rates than the standard linear combining ones, not only for two receive antennas, but also for three receive antennas.

The SISO system performs far worse in slow-fading environments compared to fast fading scenarios. This is due to the fact that in fast fading, the redundancy employed by an error correcting code coupled with interleaving can help recover the symbols lost due to the deep fade. This is not possible in slow fading, as in this instance a deep fade lasts for the entire frame length, resulting into the corruption of the whole frame [6].

## 7. CONCLUSIONS AND FUTURE WORK

This paper proposes three new methods that lead to increased performance for turbo codes, depending on the type of channel. For AWGN channels, a superior coding gain can be obtained through a modification of the decision procedure of the log-MAP decoding algorithm. For fast RMF channels, lower error rates result from the use of a chaotic Henon mapping BICM-ID interleaver.

In case of slow B-RMF channels, simple linear diversity combining techniques ensure a better performance compared to the SISO system. Furthermore, the composite combining

method takes advantage of the benefits offered by the standard combining methods and provides a more reliable turbo encoded system. Future work should address to the study of the diversity combining methods in conjunction with higher order modulations and in frequency selective fading scenarios.

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