COMPARISON OF LOCALIZATION ALGORITHMS USING TIME DIFFERENCE OF ARRIVAL

Dirk Czepluch, Franz Demmel
Rohde & Schwarz
dirk.czepluch@rohde-schwarz.com
franz.demmel@rohde-schwarz.com

Stefan Schmidt
Rohde & Schwarz
stefan-a.schmidt@rohde-schwarz.com

ABSTRACT

This paper performs a study on different TDOA based localization algorithms, in dispersive radio channels. The cross-correlation between the signals is performed under COST259 [1] channel model conditions. The scenario is an non-cooperative system and the spectrum of the signal is unknown. The different methods are localization based on intersecting hyperbolas, location on the conic axis (LOCA) and a divide and conquer (DAC) approach.

1. INTRODUCTION

This paper is a summary of different localization methods based on the time difference of arrival. We consider a uncooperative system. This means there is no communication between the transmitter and the receivers or trainings sequence of a given communication standard. This is the standard case in localization systems using by federal authorities.

The paper is organized in six sections. In the second Section we will show, how to get the time difference of arrival. The next three Sections discusses different methods to estimate the position of the unknown transmitter. This means in detail the localization method with intersecting hyperbolas, Section three, followed by the localization based on conic axes in Section four and finished with a divide and conquer approach in Section five. Section 6 presented the simulation results in detail. The last Section yields the conclusion of the paper.

2. MEASUREMENT OF TIME DIFFERENCE OF ARRIVAL

If a electromagnetic wave occurs at a transmitter, this will arrive at a set of different receivers at a different times. To make this more explicit let us define a set of receivers and a signal source (transmitter) given by its coordinates \( R_i = [x_i, y_i]^T \) for \( i = 1, \ldots, N_r \), where \( N_r \) denotes the number of receivers, and \( T = [x_0, y_0]^T \) for the transmitter respectively. The range distances \( r_i \) for \( i = 1, \ldots, N_r \) from each receiver to the transmitter \( T \) are given as

\[
r_i \triangleq \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} \quad \text{for} \quad i = 1, \ldots, N_r
\]

Now the time difference of arrival between receiver \( i \) and receiver \( j \) can be written as

\[
t_{ij} \triangleq \frac{r_j - r_i}{c} \quad \text{for} \quad i = 1, \ldots, N_r \quad \text{and} \quad j > i
\]

where \( c \) denotes the speed of light. Now let \( x_i(t) \) and \( x_j(t) \) for \( t = 1 \ldots K \) be time series measured at receivers \( R_i \) and \( R_j \) respectively. One common method to determine \( t_{ij} \) is given by means of the standard cross-correlation function

\[
R_{x_i x_j}(\tau) = E[x_i(t)x_j^*(t - \tau)]
\]

The argument \( \tau \) that maximizes equation (3) provides an estimate of \( t_{ij} \). In future the \( t_{ij} \) will be described as \( \Delta t \). Note that in literature there is a concept called generalized cross correlation which aims at improving the accuracy of the time delay estimate through appropriate pre-filtering of the time series \( x_i(t) \) and \( x_j(t) \) before cross-correlation. For example, one common approach is to choose the pre-filters in a way that the signals passed to the correlator are accentuated at those frequencies at which the signal-to-noise ratio (SNR) is highest. Unfortunately the generalized cross-correlation methods require knowledge of the signal spectrum, which is not given in case of the non-cooperative location problem considered in this work. Therefore the signal spectrum would have to be estimated in order to apply these methods, which in turn would lead to a large amount of additional signal processing. This is the reason why most practical non-cooperative location systems rely on the standard cross-correlation procedure as stated in (3) and we do not consider the generalized cross-correlation further in this work. Nevertheless we refer to [2] for a very good summary of the most important generalized cross-correlation techniques.

3. INTERSECTING HYPERBOLAS

The first localization method based on time difference of arrival we presented in this paper based on the intersecting of
hyperbolas. It can be easily shown that the calculated time
difference between two receivers the transmitter possible
source location is existing of a hyperbola. The sign of the
time difference leads to the hyperbola axis.

Without loss of generality we assume the receivers to be
located at x-axis with distance of \(a\) and the transmitter is lo-
cated at \([x_0, y_0]\). Then the time difference between the two
receivers is determined through

\[
\Delta t = \sqrt{(-\frac{a}{2} - x_0)^2 + y_0^2} - \sqrt{(\frac{a}{2} - x_0)^2 + y_0^2}
\]

where \(c\) denotes the speed of light. After transforming this
equation this finally ends up with

\[
1 = \frac{x_0^2}{c^2 \Delta t^2/4} - \frac{y_0^2}{a^2/4 - c^2 \Delta t^2/4}
\]

which is recognized as the equation of a hyperbola. In
figure below the hyperbola is plotted for the two cases. The
first scenario is a three receiver scenario, which shows the
problem that three receivers are not enough for unambiguous
estimation. The figure (1) shows the solution of the problem,
while using four receivers.

4. LOCATION ON THE CONIC AXIS - LOCA

In this section we will present another localization method
based of time differences. The method is called the "Loca-
tion on the Conic Axis (LOCA)" algorithm given in [2].
Schmidt shows that the range differences from the transmitter
to three known receivers, which can be obtained by multiply-
ing the measured TDOAs with the speed of light \(c\), provide
a straight line of position, which is the major axis of a gen-
eral conic (conic axis). The conic axis connects the foci of
a conic. Moreover it is shown that the three receivers lie on
the conic and one of the foci is the location in question, see
figures (4) and (??). Therefore his algorithm is referred to as
LOCA (Location on the Conic Axis).

Now we describe the approach in more detail. Let \(R_i \triangleq
[x_i, y_i]^T\) for \(i = 1, 2, 3\) and \(T \triangleq [x_0, y_0]^T\) define the coordinates
of the receivers and the transmitter respectively. Moreover let \(r_i\) denote the range between receiver \(R_i\) and transmit-
ter \(T\). Then with

\[
\begin{align*}
\Delta t_1 &\triangleq r_j - r_i \quad \text{and} \quad a_i^2 &\triangleq x_i^2 + y_i^2 \\
r_1^2 &\triangleq (x_0 - x_1)^2 + (y_0 - y_1)^2 \\
r_2^2 &\triangleq (x_0 - x_2)^2 + (y_0 - y_2)^2 \\
r_3^2 &\triangleq (x_0 - x_3)^2 + (y_0 - y_3)^2
\end{align*}
\]

\[
\Delta t_{ij} \triangleq r_j - r_i \quad \text{and} \quad a_i^2 \triangleq x_i^2 + y_i^2 \quad \text{for} \quad i, j = 1, 2, 3
\]
So far we have considered two cases. In the first case only three receivers are available and the fix can be estimated by determining the foci of a conic (LOCA). In the second case three receivers are available and the fix can be estimated by determining the foci of a conic (LOCA). In the second case three receivers are available and the fix can be estimated by determining the foci of a conic (LOCA).

Now solving for range sums one gets

\[ r_2^2 - r_1^2 = \Delta_{12}(r_1 + r_2) \]

\[ = 2x_0(x_1 - x_2) + 2y_0(y_1 - y_2) + (a_2^2 - a_1^2) \]  \hspace{1cm} (7a)

\[ r_3^2 - r_2^2 = \Delta_{23}(r_2 + r_3) \]

\[ = 2x_0(x_2 - x_3) + 2y_0(y_2 - y_3) + (a_3^2 - a_2^2) \]  \hspace{1cm} (7b)

\[ r_1^2 - r_3^2 = \Delta_{31}(r_3 + r_1) \]

\[ = 2x_0(x_3 - x_1) + 2y_0(y_3 - y_1) + (a_1^2 - a_3^2) \]  \hspace{1cm} (7c)

Now solving for range sums one gets

\[ r_1 + r_2 = 2x_0 \frac{x_1 - x_2}{\Delta_{12}} + 2y_0 \frac{y_1 - y_2}{\Delta_{12}} + \frac{a_2^2 - a_1^2}{\Delta_{12}} \]  \hspace{1cm} (8a)

\[ r_2 + r_3 = 2x_0 \frac{x_2 - x_3}{\Delta_{23}} + 2y_0 \frac{y_2 - y_3}{\Delta_{23}} + \frac{a_3^2 - a_2^2}{\Delta_{23}} \]  \hspace{1cm} (8b)

\[ r_3 + r_1 = 2x_0 \frac{x_3 - x_1}{\Delta_{31}} + 2y_0 \frac{y_3 - y_1}{\Delta_{31}} + \frac{a_1^2 - a_3^2}{\Delta_{31}} \]  \hspace{1cm} (8c)

which in turn leads to

\[ (r_2 + r_3) - (r_3 + r_1) = r_2 - r_1 = \Delta_{12} \]

\[ = 2x_0 \left[ \frac{x_2 - x_3}{\Delta_{23}} - \frac{x_3 - x_1}{\Delta_{31}} \right] + 2y_0 \left[ \frac{y_2 - y_3}{\Delta_{23}} - \frac{y_3 - y_1}{\Delta_{31}} \right] \]

\[ + \frac{a_3^2 - a_2^2}{\Delta_{23}} - \frac{a_1^2 - a_3^2}{\Delta_{31}} \]  \hspace{1cm} (9)

Finally, after multiplying (9) with the common denominator and rearranging one obtains

\[ [x_1\Delta_{23} + x_2\Delta_{31} + x_3(\Delta_{12} - \Sigma)]x_0 \]

\[ + [y_1\Delta_{23} + y_2\Delta_{31} + y_3(\Delta_{12} - \Sigma)]y_0 \]

\[ = \frac{1}{2}[\Delta_{12}\Delta_{23}\Delta_{31} + a_1^2\Delta_{23} + a_2^2\Delta_{31} + a_3^2(\Delta_{12} - \Sigma)] \]  \hspace{1cm} (10)

where

\[ \Sigma = \Delta_{12} + \Delta_{23} + \Delta_{31} \]  \hspace{1cm} (11)

Equation (9) can be written in a more compact form as

\[ Ax + By = C \]

where \( A, B \) and \( C \) are constants, determined by the receiver locations \( R_i \) for \( i = 1, 2, 3 \) and the measured range differences \( \Delta_{12}, \Delta_{23}, \Delta_{31} \). Thus it is established that the transmitter \( T = [x_0 \ y_0]^T \) lies on the straight line given by \( Ax + By = C \).

In [1] it is shown that equation \( Ax + By = C \) together with the three receiver locations \( R_i \) for \( i = 1, 2, 3 \) uniquely define a conic whose axis (= line, where the foci of the conic lie on) is given by \( Ax + By = C \). Moreover the transmitter location \( T = [x_0 \ y_0]^T \) is given as one of the two foci. This means that once we have determined the coefficients \( A, B \) and \( C \) from the measured range differences \( \Delta_{12}, \Delta_{23}, \Delta_{31} \) and the receiver locations \( R_i \) for \( i = 1, 2, 3 \) for one receiver triad (= set of three receivers) there are two ways to proceed in order to get a fix (= estimate of the transmitter position). If more than three receivers are available we can determine the coefficients \( A', B' \) and \( C' \) for a second receiver triad and the fix will be given as the intersection of the two conic axes defined by \( Ax + By = C \) and \( A'x + B'y = C' \).

If on the other hand only three receivers are available three different cases have to be considered. In the first case the conic is an ellipse. Then the focus which does not correspond to the transmitter position \( T \) would produce the negative range differences \( -\Delta_{12}, -\Delta_{23}, \) and \( -\Delta_{31} \), if the transmitter had its location there, see figure 8. In the second case the conic is a hyperbola. Then a transmitter being located at either of the foci would produce the same range differences, see figure 6. This means that in the case of the conic being a hyperbola, an ambiguity arises and it is not possible to decide mathematically which of the two foci corresponds to the transmitter position \( T \). In the third case one of the foci has moved to infinity and the conic is a parabola [2].

Overall, only the case of the conic being a hyperbola leads to an ambiguity in determining the transmitter location \( T \).

5. Divide and Conquer Approach - DAC

So far we have considered two cases. In the first case only three receivers are available and the fix can be estimated by determining the foci of a conic (LOCA). In the second case...
more than three receivers are available and the fix can be estimated through cutting the conic axes corresponding to the different receiver triads.

Another approach for the case that \( N > 3 \) receivers are available would be to determine a fix for each receiver triad by applying LOCA which leads to a set of \( \binom{N}{3} \) fixes. In the next step these \( \binom{N}{3} \) fixes should be combined in an optimal way in order to produce a reliable estimate of the source location. In this section we will show how the so called divide and conquer approach proposed in [3] can be applied to obtain a maximum likelihood (ML) estimate of the source location by optimally combining the \( \binom{N}{3} \) appropriately weighted fixes.

For the mathematical background we refer to [3].

5.1. Formation of the DAC estimate

In order to form the DAC estimate one has to divide the data vector \( \mathbf{x} \) in \( m \) possibly overlapping sub vectors \( \mathbf{x}_i \) for \( i = 1, \ldots, m \). We have to make sure that each element of \( \mathbf{x} \) is represented in at least one \( \mathbf{x}_i \). In the next step for each sub vector \( \mathbf{x}_i \) an estimate \( \hat{\theta}_i \) is determined via maximum likelihood, for example through solving (3). In general \( \theta_i \) can be a sub vector of \( \theta \) given as

\[
\theta_i = S_i \theta
\]

where \( S_i \) is a selection matrix. \( S_i \) can be obtained from an identity matrix by deleting the rows \( j \) corresponding to the elements \( j \) of \( \theta \) that one does not want to estimate via \( \mathbf{x}_i \). The DAC estimate \( \hat{\theta}_{DAC} \) can then be obtained by linearly combining the estimates \( \hat{\theta}_i \) according to [4,5]

\[
\hat{\theta}_{DAC} = (S^T W S)^{-1} S^T W \hat{\theta}
\]

where \( S \) and \( \hat{\theta} \) are constructed through concatenating the \( \hat{\theta}_i \) and \( S_i \) as

\[
\hat{\theta} \triangleq \begin{bmatrix} \hat{\theta}_1 \\ \vdots \\ \hat{\theta}_m \end{bmatrix}, \quad S \triangleq \begin{bmatrix} S_1 \\ \vdots \\ S_m \end{bmatrix}
\]

and \( W \) is a positive definite weighting matrix, chosen in a way that the mean squared error of the estimate \( \hat{\theta}_{DAC} \) is minimized.

5.2. Bias and Variance

In this subsection we will derive bias and covariance of the DAC estimate in (13) for the case that the data vector \( \mathbf{x} \) is Gaussian distributed. Let's start with the following definitions
\( \mu_i \triangleq E\{x_i\} \quad (15) \)
\( C_i \triangleq E\{[x_i - \mu_i][x_i - \mu_i]^T\} \quad (16) \)
\( C_{ij} \triangleq E\{[x_i - \mu_i][x_j - \mu_j]^T\} \quad (17) \)
\( J_i \triangleq \partial \mu_i^T \partial \theta_i C_i^{-1} \partial \mu_i \partial \theta_i \quad (18) \)

Assuming the \( \hat{\theta}_i \) to be ML estimates and sufficiently large Fisher information \( J_i \) we can apply (19) getting from [3]

\[
\hat{\theta}_{ML} \approx \theta + J_i^{-1} \partial \ln p(x, \theta) \partial \theta
\]

and come up with

\[
E\{\hat{\theta}_i\} \approx E\left\{ \theta_i + J_i^{-1} \partial \ln p(x_i, \theta_i) \right\}
\]

\[
= E\left\{ \theta_i + \frac{1}{\partial \theta_i} \partial \left(- \frac{1}{2} \ln \det(2\pi C_i)\right) \right\}
\]

\[
= E\left\{ \theta_i + \frac{1}{\partial \theta_i} \partial \left(- \frac{1}{2} [x_i - \mu_i]^T C_i^{-1} [x_i - \mu_i]\right) \right\}
\]

\[
= E\left\{ \theta_i + \partial \mu_i^T \partial \theta_i C_i^{-1} [x_i - \mu_i]\right\}
\]

\[
= E\{\theta_i\} + \partial \mu_i^T \partial \theta_i C_i^{-1} [E\{x_i\} - \mu_i]
\]

\[
= \theta_i
\]

Using (13) and \( E\{\hat{\theta}\} = S \theta \) the expectation of the DAC estimate \( \hat{\theta} \) can be computed as

\[
E\{\hat{\theta}_{DAC}\} = (S^T WS)^{-1} S^T W E\{\hat{\theta}\}
\]

and one finds that \( \hat{\theta}_{DAC} \) is unbiased. Finally from (13) and (27) the covariance \( C_{\hat{\theta}_{DAC}} \) of \( \hat{\theta}_{DAC} \) can be determined [4]

\[
C_{\hat{\theta}_{DAC}} = (S^T WS)^{-1} S^T WC_{\hat{\theta}} WS(S^T WS)^{-1}
\]

where in the step from (21) to (23) it has been used that \( C_i \) does not depend on \( \theta_i \). In order to obtain (24) from (23) we applied the chain rule in combination with the matrix equality in (??). Finally the linearity of the expectation operator \( E\{.\} \) has been used to get (25). Thus it has been established that \( \hat{\theta}_i \) is unbiased.

Now we will proceed with the determination of the covariance between \( \hat{\theta}_i \) and \( \hat{\theta}_j \), denoted as \( (C_{\hat{\theta}})_{ij} \). Applying (19) once again leads to

\[
(C_{\hat{\theta}})_{ij} \triangleq E\left\{ [\hat{\theta}_i - E\{\hat{\theta}_i\}][\hat{\theta}_j - E\{\hat{\theta}_j\}]^T \right\}
\]

\[
= E\left\{ \theta_i J_i^{-1} \partial \ln p(x_i, \theta_i) - E\{\hat{\theta}_i\} \right\}
\]

\[
= J_i^{-1} \partial \ln p(x_i, \theta_i) \left( \theta_j J_j^{-1} \partial \ln p(x_j, \theta_j) \right)
\]

\[
= J_i^{-1} \partial \mu_i^T \partial \theta_i C_i^{-1} C_{ij} J_j^{-1}
\]

which shows that the matrices \( \frac{\partial \mu_i^T}{\partial \theta_i} \) are invertible. Now plugging (30) in (27) we can express \( (C_{\hat{\theta}})_{ij} \) as
\[(C_\delta)_{ij} = J_i^{-1} \frac{\partial \mu^T}{\partial \theta_i} C_i C_{ij} \frac{\partial \mu_j}{\partial \theta_j} J_j^{-1} \]

\[= \left( \frac{\partial \mu_i}{\partial \theta_i} \right)^{-1} C_{ij} \left( \frac{\partial \mu_j^T}{\partial \theta_j} \right)^{-1} \]

(31)

Defining \( \Lambda_\delta \) as the block diagonal matrix with \( i \)th block \( \frac{\partial \mu_i}{\partial \theta_i} (31) \) can be written as

\[C_\delta = \Lambda_\delta^{-1} C \Lambda_\delta^{-T} \]

(32)

where

\[C = \begin{bmatrix} C_{11} & \ldots & C_{1m} \\ \vdots & \ddots & \vdots \\ C_{m1} & \ldots & C_{mm} \end{bmatrix} \]

(33)

Now let \( W = C_\delta^{-1} \) and plug (32) into (29) to obtain

\[C_{\delta, DAC} = \left[ S^T(\Lambda_\delta^{-1} C \Lambda_\delta^{-T})^{-1} S \right]^{-1} \]

\[= \left[ S^T \Lambda_\delta^{-T} \Lambda_\delta^{-1} S \right]^{-1} \]

\[= \left[ \frac{\partial \mu^T}{\partial \theta} C^{-1} \frac{\partial \mu}{\partial \theta^T} \right]^{-1} \]

(34)

where

\[\mu \triangleq E\{[x_1 \ldots x_m]^T\} \]

\[= [\mu_1 \ldots \mu_m]^T \]

(35)

is the CRLB. Thus it is shown that assuming sufficiently large Fisher information the DAC estimate approaches the CRLB if the parameter vectors \( \hat{\theta}_i \) and the sub vectors \( x_i \) have the same length.

### 5.3. Application of DAC to Range Difference Location

In order to apply the DAC to the range difference location problem, at first identify \( x \) as the vector of measured range differences

\[x \triangleq [\tilde{\Delta}_{12} \ldots \tilde{\Delta}_{1N} \ldots \tilde{\Delta}_{k(k+1)} \ldots \tilde{\Delta}_{kN} \ldots \tilde{\Delta}_{(N-2)N}]^T \]

(36)

In the next step we will divide \( x \) into \( M = \binom{N}{2} \) overlapping sub vectors \( x_i \in \mathbb{R}^{2 \times 1} \). In order to do so, at first number the receivers arbitrarily from 1 to \( N \). Then define sub vectors as

\[x_i^{(k)}(l, m) \triangleq \begin{bmatrix} \tilde{\Delta}_{kl} \\ \tilde{\Delta}_{km} \end{bmatrix}, \quad \text{for} \quad i = 1, \ldots, M \]

(37)

where \( k \in \{1, \ldots, N - 1\} \) and \( m \in \{l + 1, \ldots, N\} \). In order to illustrate these rather abstract definitions lets assume the case of \( M = 4 \) receivers. Then we have

\[x \triangleq [\tilde{\Delta}_{12} \tilde{\Delta}_{13} \tilde{\Delta}_{14} \tilde{\Delta}_{23} \tilde{\Delta}_{24}]^T \]

(38)

and

\[x_1^{(1)} \triangleq [\tilde{\Delta}_{12} \tilde{\Delta}_{13}], \quad x_2^{(1)} \triangleq [\tilde{\Delta}_{12} \tilde{\Delta}_{14}], \quad x_3^{(1)} \triangleq [\tilde{\Delta}_{13} \tilde{\Delta}_{14}], \quad x_4^{(1)} \triangleq [\tilde{\Delta}_{23} \tilde{\Delta}_{24}] \]

(39)

Now we can collect the vectors \( x_i^{(k)}(l, m) \) from (37) in the vector

\[x \triangleq [x_1^{(1)} \ldots x_4^{(1)} x_2^{(2)} x_3^{(2)} x_4^{(2)} x_3^{(3)} x_4^{(3)} x_2^{(4)} x_4^{(4)}] \]

(40)

and proceed with the identifications

\[\theta \triangleq [x_0 y_0]^T \]

\[\mu \triangleq E\{x\} \]

\[C \triangleq E\{(x - \mu)(x - \mu)^T\} \]

(41)

To compute the weighting matrix \( W \) in (13) according to section (5.2) the only thing that remains to determine is \( \frac{\partial \mu}{\partial \theta} \triangleq \frac{\partial E\{x^{(k)}\}}{\partial \theta} \). In order to do so, remember that

\[\Delta_{kl} \triangleq r_l - r_k \]

\[= \sqrt{(x_0 - x_l)^2 + (y_0 - y_l)^2} - \sqrt{(x_0 - x_k)^2 + (y_0 - y_k)^2} \]

\[\Delta_{km} \triangleq r_m - r_k \]

\[= \sqrt{(x_0 - x_m)^2 + (y_0 - y_m)^2} - \sqrt{(x_0 - x_k)^2 + (y_0 - y_k)^2} \]

(42)

which together with (41) yields

\[\frac{\partial \mu^{(k)}}{\partial \theta} \triangleq \frac{\partial E\{x^{(k)}\}}{\partial \theta^T} = \begin{bmatrix} \frac{\partial \Delta_{kl}}{\partial x_0} & \frac{\partial \Delta_{kl}}{\partial y_0} \\ \frac{\partial \Delta_{km}}{\partial x_0} & \frac{\partial \Delta_{km}}{\partial y_0} \end{bmatrix} \]

(43)
\[
\frac{\partial \Delta_{kl}}{\partial x_0} = \frac{x_0 - x_l}{\sqrt{(x_0 - x_l)^2 + (y_0 - y_l)^2}} - \frac{x_0 - x_k}{\sqrt{(x_0 - x_k)^2 + (y_0 - y_k)^2}} \\
\frac{\partial \Delta_{kl}}{\partial y_0} = \frac{y_0 - y_l}{\sqrt{(x_0 - x_l)^2 + (y_0 - y_l)^2}} - \frac{y_0 - y_k}{\sqrt{(x_0 - x_k)^2 + (y_0 - y_k)^2}} \\
\frac{\partial \Delta_{km}}{\partial x_0} = \frac{x_0 - x_m}{\sqrt{(x_0 - x_m)^2 + (y_0 - y_m)^2}} - \frac{x_0 - x_k}{\sqrt{(x_0 - x_k)^2 + (y_0 - y_k)^2}} \\
\frac{\partial \Delta_{km}}{\partial y_0} = \frac{y_0 - y_m}{\sqrt{(x_0 - x_m)^2 + (y_0 - y_m)^2}} - \frac{y_0 - y_k}{\sqrt{(x_0 - x_k)^2 + (y_0 - y_k)^2}}
\] (44)

Finally set
\[
S_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{for } i = 1, \ldots, M \tag{45}
\]
such that we can devise the following algorithm for range difference location

\textit{Algorithm 1:}

1. compute \( \hat{\theta}_i \) based on the sub-vectors \( \mathbf{x}_i \) for \( i = 1, \ldots, M \) using LOCA
2. compute \( \mu_i \) for \( i = 1, \ldots, M \) according to (43)
3. compute \( \mathbf{C}_\beta \) according to (32)
4. set \( \mathbf{W} = \mathbf{C}_\beta^{-1} \)
5. compute \( \hat{\theta}_{DAC} = [\hat{x}_0 \ \hat{y}_0]^T_{DAC} \) according to (13)

Note that \( [x_0 \ y_0]^T \) needed in step two for the computation of \( \mu_i \) for \( i = 1, \ldots, M \) is unknown, therefore one should use the estimate \( [\hat{x}_0 \ \hat{y}_0]^T = \frac{1}{M} \sum_{i=1}^{M} \hat{\theta}_i \) instead.

The performance of algorithm 1 has been simulated in section (6).

\section{6. SIMULATION RESULTS}

In this section we will describe the simulation results. The first and second pictures show the error contour plot of a LOCA system. With increasing the number of receivers we get a better overall RMS error.
In this work we investigated different localization methods based on the time of arrival. The time difference between two receivers was performed by a standard cross-correlation function. The simulation was influenced by COST259 channel model conditions. The main difference between the method based on intersecting hyperbolas and location on the conic axis (LOCA), is the number of necessary receivers. In the case of an ellipse used the LOCA method, it is enough to use three receivers for an ambiguity result. But with increasing the number of receivers we reduce the RMS error over the whole plane. The divide and conquer method gives a good approach to combine fixes.

8. REFERENCES


lit.dac