DISTRIBUTED AND SEQUENTIAL SENSING FOR COGNITIVE RADIO NETWORKS

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Outline

- Cognitive radios (CRs) and spectrum sharing
  - Motivation and context

- Collaborative and distributed CR sensing
  - RF interference spectrum cartography
  - Channel gain cartography

- Sequential CR sensing
  - … if time allows …
What is a cognitive radio?

- Fixed radio
  - *policy-based*: parameters set by operators

- Software-defined radio (SDR)
  - *programmable*: can adjust parameters to intended link

- Cognitive radio (CR)
  - *intelligent*: can sense the environment & learn to adapt [Mitola’00]

- Cognizant receiver: sensing
- Agile transmitter: adaptation
- Intelligent DRA: decision making
  - radio reconfiguration decisions
  - spectrum access decisions
Spectrum scarcity problem

- Fixed spectrum access policies have useful radio spectrum pre-assigned.
- Inefficient occupancy.

US FCC
Dynamical access under user hierarchy

- Primary Users (PUs) versus secondary users (SUs/CRs)

- Spectrum underlay
  - restriction on transmit-power levels
  - operation over ultra wide bandwidths

- Spectrum overlay
  - constraints on when and where to transmit
  - avoid interference to PUs via sensing and adaptive allocation
Motivating applications

- Future pervasive networks: efficient spectrum sharing

**Licensed networks**
- Cellular, PCS band
- Improved spectrum efficiency
- Improved capacity

**Secondary markets**
- Public safety band
- Voluntary agreements between licensees and third party
- Limited QoS

**Third party access in licensed networks**
- TV bands (400-800 MHz)
- Non-voluntary third party access
- Licensee sets a protection threshold

**Unlicensed networks**
- ISM, UNII, Ad-hoc
- Automatic frequency coordination
- Interoperability
- Co-existence

☑ more users/services  ☑ higher rates  ☑ better quality  ☑ less interference
Efficient sharing requires sensing

- Multiple CRs jointly detect the spectrum [Ganesan-Li’06 Ghasemi-Sousa’07]

- Benefits of cooperation
  - spatial diversity gain mitigates multipath fading/shadowing
  - reduced sensing time and local processing
  - ability to cope with hidden terminal problem

- Limitation: existing approaches do not exploit spatial dimension

Source: Office of Communications (UK)
Cooperative PSD cartography

- **Idea:** collaborate to form a spatial map of the RF spectrum

**Goal:** Find PSD map $\Phi(x, f)$ across space $x \in \mathbb{R}^2$ and frequency $f \in \mathbb{R}$

- **Specifications:** coarse approx. suffices

- **Approach:** basis expansion of $\Phi(x, f)$

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Modeling

- Transmitters
  \( TX_s, \ s = 1, \ldots, N_s \)

- Sensing CRs
  \( CR_r, \ r = 1 : N_r \)

- Frequency bases
  \( b_\nu(f), \ \nu = 1 : N_b \)

- Sensed frequencies
  \( f_k, \ k = 1 : K \)

- Sparsity present in space and frequency
Space-frequency basis expansion

- Superimposed Tx spectra measured at CR $r$

\[
\Phi_r(f) = \sum_{s=1}^{N_g} \gamma_{sr} \Phi_s(f) + \sigma_r^2 = \sum_{s=1}^{N_g} \gamma_{sr} \sum_{\nu=1}^{N_b} \theta_{sv} b_{\nu}(f) + \sigma_r^2
\]

- Average path-loss $\gamma_{sr} = \mathbb{E}(|H_{sr}(f)|^2) = \gamma_0 \left(\frac{d_0}{||x_s-x_r||}\right)^{-\alpha}, \alpha \in [2, 5]$

- Frequency bases $b_{\nu}(f) = \text{rect}(f - f_{\nu})$

- Linear model in $\theta_{sv}$

\[
\phi = \begin{pmatrix}
\Phi_1(f_1) \\
\Phi_1(f_K) \\
\Phi_2(f_1) \\
\Phi_2(f_K) \\
\vdots \\
\Phi_{N_r}(f_1) \\
\Phi_{N_r}(f_K)
\end{pmatrix} = \begin{pmatrix}
b_1(f_1)\gamma_{11} & \cdots & b_{N_b}(f_1)\gamma_{N_s1}
\vdots & \ddots & \vdots
b_1(f_K)\gamma_{1N_r} & \cdots & b_{N_b}(f_K)\gamma_{N_sN_r}
\end{pmatrix} \begin{pmatrix}
\theta_{11} \\
\theta_{21} \\
\vdots \\
\theta_{N_s1} \\
\theta_{12} \\
\vdots \\
\theta_{N_s2} \\
\vdots \\
\theta_{N_sN_b}
\end{pmatrix} = B\theta
\]
Sparse linear regression

- Seek a sparse $\theta$ to capture the spectrum measured at $CR_r$

Lasso

$$\hat{\theta} = \arg \min_{\theta} \| \varphi - B\theta \|_2^2 + \lambda \| \theta \|_1$$

- Soft threshold shrinks noisy estimates to zero

- Effects sparsity and variable selection
- Improves LS performance by incorporating a priori information

Distributed recursive implementation

- Consensus-based approach
  - solve locally
    
    \[
    \hat{\theta} = \arg \min_{\theta_r \geq 0} \| \varphi_{rt} - B_r \theta_r \|_2^2 + \frac{\lambda}{M} \| \theta_r \|_1 \\
    \text{s.to} \quad \theta_r = \theta_{r'}, \quad \forall r' \in \mathcal{N}_r
    \]

Constrained optimization using the alternating-direction method of multipliers (AD-MoM)

Exchange of local estimates \( \theta_r \)
RF spectrum cartography

- 5 sources
- $N_s = 121$ candidate locations, $N_r = 50$ cognitive radios

As a byproduct, Lasso localizes all sources via variable selection
MSE performance

- Error between estimate $\hat{\theta}$ and $\theta$
- Monte Carlo MSE versus analytical approximations

- PA1 with known $C_e$
- PA2 with bounds for $C_e$
Tracking performance

- Normalized error $\frac{||\hat{\theta} - \theta||}{||\theta||}$

- Non-stationarity: one Tx exits at time-slot $t=650$

![Graph showing batch solutions and distributed online updates with non-stationarity event at $t=650$.]
Simulation: PSD map estimation

- Centralized sensing
- No fading
- $l=25$
- $J=15$

Transmitters: $\Phi_i(f)$

Sensors: $\Phi_m(f)$
Centralized sensing without fading

- \( L_1 \) norm minimization yields a sparse solution

- "true" Tx spectrum
- BP solution

\[ \Phi_i(f) \]

- LS solution yields spurious peaks

- \( L_1 \) norm minimization yields a sparse solution
Distributed consensus with fading

- Starting from a local estimate, sensors reach consensus
Spline-based PSD cartography

Q: How about shadowing?  
A1: Basis expansion w/ coefficient-functions

\[
\Phi(x, f) = \sum_{\nu=1}^{N_b} g_\nu(x) b_\nu(f)
\]

- \( b_\nu(f) \): known bases accommodate prior knowledge
  - overcomplete expansions allow for uncertainty on Tx parameters

- \( g_\nu(x) \): unknown dependence on spatial variable \( x \)
  - learn shadowing effects from periodograms at spatially distributed CRs

Smooth and sparse coefficient functions

- Twofold regularization of variational LS estimator

\[
\begin{align*}
\min_{\{g_\nu \in \mathcal{S}\}} & \quad \frac{1}{N_r N} \sum_{r=1}^{N_r} \sum_{n=1}^{N} \left( \varphi_{rn} - \sum_{\nu=1}^{N_b} g_\nu(x_r) b_\nu(f_n) \right)^2 \\
+ & \lambda \sum_{\nu=1}^{N_b} \int_{\mathbb{R}^2} \left\| \nabla^2 g_\nu(x) \right\|_F^2 \, dx + \mu \sum_{\nu=1}^{N_b} \left\| \left[ g_\nu(x_1), \ldots, g_\nu(x_{N_r}) \right]^T \right\|_2.
\end{align*}
\] (P1)

Proposition: optimal \( g_\nu(x) \) admits kernel expansion

\[ g_\nu(x) = \sum_{r=1}^{N_r} \zeta_{\nu r} k(\Delta x_r) \]

\[
k(\Delta x_r) := \left\| \Delta x_r \right\|^2 \log(\left\| \Delta x_r \right\|)
\]
Estimating kernel parameters

- Need \( \zeta_\nu = (\zeta_{\nu 1}, \ldots, \zeta_{\nu N_b}) \), \( \nu = 1, \ldots, N_b \)

- Group Lasso on (P1) equivalent

\[
\min_\zeta \frac{1}{2} \| y - X\zeta \|^2 + \mu \sum_{\nu=1}^{N_b} \| \zeta_\nu \|^2
\]

- \( X \) depends on kernels and bases

- Case \( X = I \) admits closed-form solution

\[
\zeta_\nu^* = \frac{y_\nu}{\|y_\nu\|^2} \left( \|y_\nu\|^2 - \mu \right) + \mu \|y_\nu\|
\]

- \( \zeta_\nu = 0 \rightarrow g_\nu(x) = 0 \ \forall x \rightarrow b_\nu(f) \) not included
Simulation: PSD atlas

- $Nr=100$ CRs, $Nb=90$ bases (raised cosines), $Ns=5$ Tx PUs

- Frequency bases identified by enforcing sparsity
- Power distribution across space revealed by promoting smoothness
Cartography for CR sensing

- Power spectral density (PSD) maps
  - Capture ambient power in space-time-frequency
  - Can identify “crowded” regions to be avoided

- Channel gain (CG) maps
  - Time-freq. channel *from any-to-any point*
  - CRs adjust Tx power to min. PU disruption
Cooperative CG cartography

- Wireless CG (in dB)

\[ G_{x \rightarrow y}(t) = G_0 - 10\gamma \log_{10}(||x - y||_2) + s_{x \rightarrow y}(t) \]

- TDMA-based training yields CR-to-CR shadow fading measurements

\[ \tilde{s}_{x_{j \rightarrow x_r}}(t) = s_{x_{j \rightarrow x_r}}(t) + \epsilon_{x_{j \rightarrow x_r}}(t) \]

\[ \tilde{s}_r(t) \triangleq [\tilde{s}_{x_{1 \rightarrow x_r}}(t), \ldots, \tilde{s}_{x_{r-1 \rightarrow x_r}}(t), \tilde{s}_{x_{r+1 \rightarrow x_r}}(t), \ldots, \tilde{s}_{x_{N_r \rightarrow x_r}}(t)]^T \]

- **Goal:** Given \( \{\tilde{s}_r(t)\}_{\forall r, \tau \geq 1} \), estimate \( s_{x \rightarrow y}(t) \) and \( G_{x \rightarrow y}(t) \) for any \( x, y \in A \)

Dynamic shadow fading model

- Shadowing in dB is Gaussian distributed

- *Spatial loss field-based* shadowing model [Agrawal et al. ’09]

\[ s_{x \rightarrow y}(t) = \frac{1}{||x - y||^{1/2}} \int_{x}^{y} \ell(u, t) du \]

- Spatio-temporal loss-field evolution [Mardia ’98] [Wikle et al. ’99]

\[ \ell(x, t) = \bar{\ell}(x, t) + \tilde{\ell}(x, t) \]

\[ \bar{\ell}(x, t) = \int_{A} w(x, u) \bar{\ell}(u, t - 1) + \eta(x, t) \]

- \( \bar{\ell}(x, t) \): spatio-temporally colored
- \( \tilde{\ell}(x, t) \): temporally white and spatially colored
- \( w(x, u) \): known, captures interaction between \( \bar{\ell}(x, t) \) and \( \bar{\ell}(u, t - 1) \)
- \( \eta(x, t) \): zero-mean Gaussian, spatially colored, and temporally white

State-space model

- Basis-expansion representation for $\bar{\ell}(x, t)$ and $w(x, u)$

$$
\bar{\ell}(x, t) = \sum_{k=1}^{\infty} \alpha_k(t) \psi_k(x) \\
w(x, u) = \sum_{k=1}^{\infty} \beta_k(x) \psi_k(u)
$$

- Retain $K$ terms and sample at $\{x_r \in A\}_{r=1}^{N_r}$

  ➢ state equation

$$
\alpha(t) = T\alpha(t-1) + \Psi^\dagger \eta(t)
$$

- Recall loss field model

$$
\tilde{s}_{x \rightarrow y}(t) = \sum_{k=1}^{\infty} \left[ \frac{1}{\|x-y\|^{1/2}} \int_{x}^{y} \psi_k(u) du \right] \alpha_k(t) 
\approx \phi_{x \rightarrow y, k}^T \alpha(t)
$$

  ➢ measurement equation

$$
\tilde{s}(t) = \Phi \alpha(t) + \tilde{s}(t) + \epsilon(t)
$$
Tracking via Kriged Kalman Filtering

Idea: estimate $\alpha(t)$ (and hence $s_{x \rightarrow y}(t)$) via Kalman filtering (KF) spatially interpolate with Kriging (KKF) to account for $\tilde{s}(x, t)$

- Conditioned on $\tilde{s}_{1:t} \triangleq \{\tilde{s}(\tau)\}_{\tau=1}^t$, $s_{x \rightarrow y}(t)$, $x, y \in \mathcal{A}$ is Gaussian

\[
\hat{s}_{x \rightarrow u}(t) \triangleq \mathbb{E}\{s_{x \rightarrow u}(t)|\tilde{s}_{1:t}\} = \phi_{x \rightarrow u}^T \hat{\alpha}(t|t) + c_{\tilde{s}}^T(x, u) \Sigma^{-1} [\tilde{s}(t) - \Phi \hat{\alpha}(t|t)]
\]

\[
\text{var}\{s_{x \rightarrow u}(t)|\tilde{s}_{1:t}\} = \sigma_{\tilde{s}}^2 - c_{\tilde{s}}^T(x, u) \Sigma^{-1} c_{\tilde{s}}(x, u) +
\]

\[
+ \left[\phi_{x \rightarrow u}^T - c_{\tilde{s}}(x, u) \Sigma^{-1} \Phi\right] P(t|t) \left[\phi_{x \rightarrow u} - \Phi^T \Sigma^{-1} c_{\tilde{s}}(x, u)\right]
\]

\[
c_{\tilde{s}}(x, y) \triangleq \mathbb{E}\{\tilde{s}(t) \tilde{s}_{x \rightarrow y}(t)\}
\]

- Estimated CG map: $\hat{G}_{x \rightarrow y}(t) = G_0 - 10\gamma \log_{10}(||x - y||) + \hat{s}_{x \rightarrow y}(t)$
Distributed implementation

- Prediction step locally but correction step collaboratively

\[
\hat{\alpha}(t|t) = \hat{\alpha}(t|t-1) + P(t|t)\Phi^T\Sigma^{-1}[\tilde{s}(t) - \Phi\hat{\alpha}(t|t-1)]
\]

- Distributed solution via alternating direction method of multipliers (AD-MoM)

- Kriging can be distributed likewise via AD-MoM and consensus

Simulation: map estimation performance

(a) True CG map.

(b) True shadow fading map.

(c) Estimated CG map.

(d) Estimated shadow fading map.
Tracking of PU power and position

- **Given** maps \( g_r(t) \triangleq [g_{x_1 \rightarrow x_r}(t) \ldots g_{x_{N_s} \rightarrow x_r}(t)]^T \), \( \{x_s \in \mathcal{A}\}_{s=1}^N \) candidate PU positions

\[
\pi_r(t) = g_r^T(t)p(t) + z_r(t)
\]

- **Estimate** sparse power vector

\[
p(t) \triangleq [p_1(t) \ldots p_{N_s}(t)]
\]

- Sparse regression for tracking [Kim-Dall’Anese-GG’09]

\[
\hat{p}(t) = \arg \min_{p \geq 0} J_t(p), \quad J_t(p) \triangleq \left[ \frac{1}{2} \sum_{\tau=1}^{t} \mu^{t-\tau} \sum_{r=1}^{N_r} \left( \pi_r(\tau) - \hat{g}_r^T(\tau)p \right)^2 + \lambda_t \|p\|_1 \right]
\]
Simulation: PU power tracking

- Average tracking performance
  - Power MSE (avg. over all grid points) across time (KKF iterations)
  - Mean spurious power (avg. over all grid except PU points) vs. time
  - Area 200m x 200m
  - Parameters
    \[ N_s = 36, \quad N_r = 20 \quad \text{CR}, \quad d_{\text{comm}} = 125m \]
    \[ \text{var}\{\epsilon_{x_j \rightarrow x_r}(t)\} = 10, \quad \text{var}\{z_r(t)\} = 10^{-10} \]
  - Shadowing: 0-mean, std. dev. 10 dB
CG maps for resource allocation

- After having located the PU at $x_s$ with tx-power $P_s$ (dB); and rx-PU power $\Pi(x)$ at any $x$

- PU coverage probability: $P_{cov}(x) \triangleq \Pr\{\Pi(x) \geq \Pi_{\text{min}}\}$

$$P_{cov}(x) = Q\left(\frac{\Pi_{\text{min}} - P_s - G_0 + 10\gamma \log_{10} \|x_s - x\| - \hat{D}_{x_s \rightarrow x}}{\sigma_{s_{x_s \rightarrow x}}}\right)$$

- Coverage region not a disc (due to shadowing)

- CR interf. probability $P_{\text{int}}(x) \triangleq \Pr\{\Pi^{CR}(x) \geq I_{\text{max}}\}$

$$P_{\text{int}}(x) = Q\left(\frac{I_{\text{max}} - P_r - G_0 + 10\gamma \log_{10} \|x_r - x\|_2 - \hat{D}_{x_r \rightarrow x}}{\sigma_{s_{x_r \rightarrow x}}}\right)$$

- Interference regions not discs either
Coverage and interference maps

\[ C_s \triangleq \{ x \in A | P_{\text{cov}}(x) \geq 0.4 \}, C_I \triangleq \{ x \in A | P_{\text{int}}(x) \geq 0.01 \} \]

\[ P_s = 0 \text{dBW}, \quad \Pi_{\text{min}} = -60 \text{dBW}, \quad I_{\text{max}} = -40 \text{dBW} \]

Path loss-only

- disc-shaped and time-invariant

KKF-based

- captures spatial macro-diversity and spatio-temporal variations
Sequential sensing for multi-channel CRs

- Extra samples help detection/sensing but lower rate/throughput
  - Sensing-throughput tradeoff in batch single-channel [Liang et al’08]
  - Single-channel sequential CR sensing [Chaudhuri et al’09]
  - Multi-channel (e.g., OFDM) CR sensing [Kim-Giannakis’09]
Joint sensing-throughput optimization

- **Features**
  - Sense bands in parallel; stop sensing simultaneously (half-duplex constraint)
  - Throughput-optimal sequential sensing terminates when confident

- **Basic approach**: maximize avg. throughput under collision probability constraints to control Tx-CR interference to PUs (due to miss-detection)
  - Admits a constrained Dynamic Programming (DP) formulation
  - Reduces to an optimum stopping time problem
  - Optimum access: LR test w/ thresholds dependent on Lagrange multipliers

Simulated test case

- $M = 10$, $N = T = 100$, chi-square distributed channel gains
- Average performance over 20,000 runs per operating SNR
Concluding remarks

- Power spectrum density cartography
  - Space-time-frequency view of interference temperature
  - PU/source localization and tracking
- Channel gain cartography
  - Space-time-frequency links from any-to-any point
  - KF for tracking and Kriging for interpolation
- Parsimony via sparsity and distribution via consensus
  - Lasso, group Lasso on splines, and method of multipliers
- **Vision:** use atlas to enable spatial re-use, hand-off, localization, Tx-power tracking, resource allocation, and routing

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